C. Other Secondary Particles

Tables II–VIII indicate that an appreciable number of other particles are also emitted. However, the individual statistics as well as the energy resolution are quite poor for most of these particles. In comparison to the yields of protons and alphas, the yields of H^2, H^3 and He^3 particles are considerable only for the light elements, Be and Al. The depression of the yields of these particles for heavier elements is primarily a Coulomb barrier effect. In particular for the H^2 and H^3 particles, their energies are approximately $\frac{1}{2}$ and $\frac{1}{3}$ respectively of the proton energy so that only the lower part of the spectrum is measured which is the part that is suppressed by the Coulomb barrier.

The results also indicate that secondary particles, fragments of $A>4$, are also emitted with high momenta. The yields seem to follow the curve for binding energy per nucleon, being high for the light and heavy elements and having a minimum for Ni. Most of these heavy fragments from heavy nuclei undoubtedly result from the fission process. However, it is conceivable, in the light of the spectra of the secondary alphas, that some of them are produced by the knock-out process.

VI. GENERAL OBSERVATIONS

The results obtained in this experiment for the proton and alpha yields are consistent with the predictions of an evaporation model. The positions of the maxima for each element and the change of the maxima as a function of atomic number are in accordance with the theory. Also, for Ag, at energies at about 6 Mev the distributions of protons and alphas seem to be isotropic. On the other hand, the high-energy tail of the proton distribution found for Ag indicates a competitive knock-out process such as has been treated by a nucleon cascade theory. However, the high-energy tail found for the alpha spectra of Ag as well as the large abundance of those alpha particles (~25 percent of the black tracks) indicate a knock-out process for alpha particles which does not have a theoretical interpretation as yet.

VII. ACKNOWLEDGMENTS

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Nucleon Anomalous Moment in a Cut-Off Meson Theory*

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The anomalous magnetic moments of the neutron and proton are calculated to fourth order by using a cut-off meson theory. The value obtained is 1.44 when one uses the same parameters as employed for fitting the meson-nucleon scattering data.

I. INTRODUCTION

A NUMBER of meson theory calculations have been carried out by using a cut-off model, the results of which seem to agree fairly well with experiment over the region of applicability. A summary of these results has been prepared.[1] This paper will deal with the calculation of the anomalous nucleon magnetic moments up to fourth order in the coupling constant. It will be seen that the f^4 corrections are small compared to the f^2 terms so that one has reason to believe that perturbation theory makes sense for this model. This point has been discussed in more complete detail for the general case.[2]

II. HAMILTONIAN

The Hamiltonian chosen corresponds to the derivative coupling of pseudoscalar mesons with a fixed extended source. The electromagnetic interactions do not include those involving the nucleon currents, which we assume to be small because of the elimination of high momentum mesons and hence large nucleon recoils.

The theory may be made gauge-invariant in an infinite number of ways. One such[3] is to modify the usual

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[1] G. F. Chew (to be published).

[2] G. F. Chew, Phys. Rev. 94, 1755 (1954).
[3] R. G. Sachs (private communication).

derivative coupling so that

$$(f/\mu)\int \tau^{+}\boldsymbol{\sigma}\cdot\nabla\phi(\mathbf{x})\rho(\mathbf{x})d\mathbf{x}\rightarrow(f/\mu)\int\tau^{+}\boldsymbol{\sigma}\cdot\nabla\times\left\{\phi(\mathbf{x})\exp\left(ie\int_{\Gamma}\mathbf{A}\cdot d\mathbf{s}\right)\right\}\rho(\mathbf{x})d\mathbf{x},$$

and

$$(f/\mu)\int \tau^{-}\boldsymbol{\sigma}\cdot\nabla\phi^{*}(\mathbf{x})\rho(\mathbf{x})d\mathbf{x}\rightarrow(f/\mu)\int\tau^{-}\boldsymbol{\sigma}\cdot\nabla\times\left\{\phi^{*}(\mathbf{x})\exp\left(-ie\int_{\Gamma}\mathbf{A}\cdot d\mathbf{s}\right)\right\}\rho(\mathbf{x})d\mathbf{x},\quad(1)$$

where $\mathbf{A}$ is the vector potential and ϕ the charged meson field. $\rho(\mathbf{x})$ is a form factor which makes the interaction nonlocal and Γ is a straight line contour from the origin to the point $\mathbf{x}$. However, since the magnetic moment will only receive contributions from circulating currents, the line integral $\int_{\Gamma}\mathbf{A}\cdot d\mathbf{s}$ is equal to zero for our present purposes. That is, if we choose $\mathbf{A}=-\frac{1}{2}\mathbf{r}\times\mathfrak{H}$, where $\mathfrak{H}$ is a uniform magnetic field, this is the case. It is also convenient to work in the interaction representation so that our interaction Hamiltonian finally becomes

$$H_{\text{int}}=H_1+H_2,$$

$$H_1=(f/\mu)\bar{\psi}(t)\int\tau_{\alpha}\boldsymbol{\sigma}\cdot\nabla\phi_{\alpha}(\mathbf{x},t)\rho(\mathbf{x})d\mathbf{x}\psi(t)-\delta m\bar{\psi}(t)\psi(t),\quad(2)$$

$$H_2=eT_{\mu\nu}\int d\mathbf{x}\mathbf{A}_T(\mathbf{x})\cdot\phi_{\mu}\nabla\phi_{\nu},$$

where

$$T_{\mu\nu}=1 \quad\text{for } \mu=1,\ \nu=2$$
$$=-1 \text{ for } \mu=2,\ \nu=1$$
$$=0, \text{ otherwise.}$$

Repeated indices are to be summed. The $\phi_{\alpha}(\mathbf{x},t)$ are the three real components of the meson field, while $\psi(t)$ is the nucleon field. In this approximation this latter is a function of time only (being fixed at the origin of the space co-ordinates) and consists of four states corresponding to the four spin and isotopic spin orientations. $\mathbf{A}_T(\mathbf{x})$ is the vector potential which is now transverse.

The Hamiltonian used here has omitted any description of the interaction of the nucleon current with the electromagnetic field. One may show that if this model is assumed to be the low-energy limit of a relativistic theory, then consistency requires that we also omit the effect of the Dirac moment of the proton in intermediate states.[4]

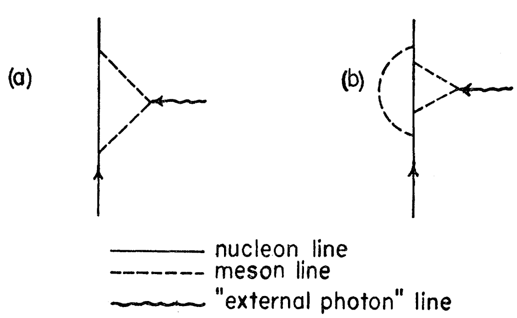

FIG. 1. Feynman diagrams for the anomalous magnetic moment calculation to fourth order. Diagram (a) gives a second order term and fourth order terms resulting from renormalization. Diagram (b) gives a fourth order contribution.

III. ANOMALOUS MOMENT TO ORDER $f^2/4\pi$

In the following, all references shall be to three-vectors, the fourth component being specifically written in or mentioned. Let

$$v(|k|)=\int d\mathbf{x}\rho(\mathbf{x})\exp(i\mathbf{k}\cdot\mathbf{x}),\quad(3)$$

and the electromagnetic field be given by

$$\mathfrak{H}(\mathbf{x})=\nabla\times\mathbf{A}_T(\mathbf{x}),\quad \mathbf{A}_T(\mathbf{x})=\textstyle\sum_{\mathbf{q}}\mathbf{a}_{\mathbf{q}}\exp(i\mathbf{q}\cdot\mathbf{x}),\quad(4)$$

where

$$\mathbf{q}\cdot\mathbf{a}_{\mathbf{q}}=0.$$

Then the Feynman propagators and vertex operators are as follows:

(a) Emission or absorption of a meson of momentum $\mathbf{k}$,

$$\pm i\boldsymbol{\sigma}\cdot\mathbf{k}\tau_{\lambda}v(k)\ [+\text{ for absorption, }(-)\text{ for emission}].$$

(b) Scattering of a meson from $\mathbf{k}$ to $\mathbf{k}+\mathbf{q}$ by $\mathfrak{H}(x)$,

$$2ieT_{\mu\nu}\mathbf{a}_{\mathbf{q}}\cdot\mathbf{k}.$$

(c) Propagation of a meson with momentum $\mathbf{k}$ and energy k_0,

$$1/(k_0^2-\omega_k^2),$$

where $\omega_k=(k^2+\mu^2)^{\frac{1}{2}}$.

(d) Propagation of a nucleon with energy p_0 and renormalized mass m (the nucleon always has zero momentum),

$$1/(p_0-m).$$

In (c) and (d), the masses have small negative imaginary parts.

Thus to second order the only Feynman diagram is that of Fig. 1(a). The corresponding matrix element is

$$M_a=2ie(f/\mu)^2\textstyle\sum_{\mathbf{q}}\displaystyle\int\frac{d^4k}{(2\pi)^4}\boldsymbol{\sigma}\cdot(\mathbf{k}+\mathbf{q})\tau_{\mu}v(|\mathbf{k}+\mathbf{q}|)\frac{\mathbf{a}_{\mathbf{q}}\cdot\mathbf{k}T_{\mu\nu}}{[-k_0+i\delta]}v(k)\boldsymbol{\sigma}\cdot\mathbf{k}\tau_{\nu}\times\frac{1}{[k_0^2-(\omega_{(\mathbf{k}+\mathbf{q})}-i\epsilon)^2]}\frac{1}{[k_0^2-(\omega_{\mathbf{k}}-i\epsilon)^2]},\quad(5)$$

[4] G. F. Chew and F. Low (private communication).

where $d^4k = idk_0d\mathbf{k}$. We first integrate over dk_0, and perform the indicated sum on the τ matrices to obtain

$$M_a = 2(f/\mu)^2 e\tau_3 \sum_{\mathbf{q}} \int \frac{d\mathbf{k}}{(2\pi)^3} \times \frac{v(k)\mathbf{a}_{\mathbf{q}}\cdot\mathbf{k}\boldsymbol{\sigma}\cdot\mathbf{k}\boldsymbol{\sigma}\cdot\mathbf{q}v(|\mathbf{k}-\mathbf{q}|)}{\omega_{\mathbf{k}}^2\omega_{(\mathbf{k}-\mathbf{q})}^2} \quad (6)$$

where we have made use of the following relation:

$$\mathbf{a}_{\mathbf{q}}\cdot\int \mathbf{k}f(|\mathbf{k}-\mathbf{q}|)d\mathbf{k} = \mathbf{a}_{\mathbf{q}}\cdot\mathbf{q}\int k^2dkh(q^2,k^2) = 0. \quad (7)$$

Here f is an arbitrary function and h is the result of the angular integration over $\mathbf{k}$. It is now permissible to assume that the external magnetic field is slowly varying, so that to first order in $\mathbf{q}$, Eq. (6) becomes:

$$M_a = -(e/2m)\tau_3\boldsymbol{\sigma}\cdot\mathcal{H}(x=0) \times \left\{\frac{8}{3\pi}(f^2/4\pi)\frac{m}{\mu}\left(\frac{1}{\mu}\right)\int_0^\infty \frac{dkk^4v^2(k)}{\omega_k^4}\right\}, \quad (8)$$

where we have used

$$\sum_{\mathbf{q}}(\boldsymbol{\sigma}\cdot\mathbf{a}_{\mathbf{q}})(\boldsymbol{\sigma}\cdot\mathbf{q}) = -\boldsymbol{\sigma}\cdot\mathcal{H}(x=0). \quad (9)$$

IV. $(f^2/4\pi)^2$ CONTRIBUTIONS

We again obtain fourth order contributions from diagram (a) of Fig. 1, if we replace $S_F(p_0)$ by the renormalized propagator $S_{F_r}'(p_0)$, and $\pm i(\boldsymbol{\sigma}\cdot\mathbf{k})v(k)\tau_\lambda$ by $\pm i(\boldsymbol{\sigma}\cdot\mathbf{k})v(k)\tau_\lambda L_r(p_0,p_0')$. The procedure for renormalization is that of Dyson and Ward. To order f^2, S_{F_r}' and L_r are

$$S_{F_r}'(p_0) = \frac{1}{p_0-m} + \left(\frac{3}{\pi}\right)\left(\frac{f^2}{4\pi}\right)\frac{1}{\mu^2} \times \int_0^\infty \frac{dkk^4v^2(k)}{[p_0-m-\omega_k]\omega_k^3}, \quad (10)$$

$$L_r(p_0,p_0') = 1 + \left(\frac{1}{3\pi}\right)\left(\frac{f^2}{4\pi}\right)\frac{1}{\mu^2}\int_0^\infty dkk^4v^2(k) \times \left\{\frac{1}{\omega_k[\omega_k-p_0-m][\omega_k-p_0'-m]} - \frac{1}{\omega_k^3}\right\}. \quad (11)$$

In addition to this there is a contribution corresponding to diagram (b) of Fig. 1.

The calculation of these matrix elements is straight forward and employs the same procedures used in evaluating the second order term. We shall therefore just quote the results.

Let M_a^S be the f^4 contributions arising from S_{F_r}' and M_a^V be the f^4 contribution from each vertex. Then,

$$M_a^S = -\frac{e}{2m}\tau_3\boldsymbol{\sigma}\cdot\mathcal{H}(0)\left\{\frac{4}{\pi^2}\left(\frac{f^2}{4\pi}\right)^2\frac{m}{\mu}\frac{1}{\mu^3}\int_0^\infty\int \frac{dldkl^4v^2(l)}{\omega_l^3\omega_k^2(\omega_k+\omega_l)} \times k^4v^2(k)\left[\frac{1}{\omega_k}+\frac{1}{(\omega_k+\omega_l)}\right]\right\}, \quad (12)$$

$$2M_a^V = -(2/9)M_a^S. \quad (13)$$

The contribution from diagram (b) of Fig. 1 is

$$M_b = (1/9)M_a^S, \quad (14)$$

and thus the total fourth order contribution is $(8/9)M_a^S$. Hence, the anomalous moment of the proton (that of the neutron being equal and opposite) is:

order $f^2/4\pi$,

$$\mathcal{U}_2 = \left(\frac{f^2}{4\pi}\right)\left(\frac{8}{3\pi}\right)\frac{m}{\mu}\left(\frac{1}{\mu}\right)\int_0^\infty \frac{dkk^4v^2(k)}{\omega_k^4}; \quad (15)$$

order $(f^2/4\pi)^2$,

$$\mathcal{U}_4 = \left(\frac{f^2}{4\pi}\right)^2\frac{32}{9\pi^2}\frac{m}{\mu}\left(\frac{1}{\mu^3}\right)^2\left\{\int_0^\infty\int \frac{dldkl^4v^2(l)k^4v^2(k)}{\omega_l^3\omega_k^2(\omega_k+\omega_l)} \times \left[\frac{1}{\omega_k}+\frac{1}{[\omega_k+\omega_l]}\right]\right\}. \quad (16)$$

At present the meson-nucleon scattering data seems to be best fitted[1] by setting $(f^2/4\pi) = 0.058$ and taking $v^2(k)$ to be a unit step function with the step at $\omega_{\max} = 5.6\mu$. Then,

$$\mathcal{U} = \mathcal{U}_2 + \mathcal{U}_4 = 1.44, \quad (17)$$

where

$$\mathcal{U}_2 = 1.15 \quad \text{and} \quad \mathcal{U}_4 = 0.29. \quad (18)$$

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