Charge and Isotopic-Spin Conservation in Multiple Meson Production

YEHUDAH YEIVIN
The Weizmann Institute of Science, Rehovoth, Israel

(Received June 23, 1954)

General formulas are derived for the corrections to Fermi's cross sections, which follow from the conservation of (*a*) charge, or (*b*) charge plus isotopic spin. The possible identity of some of the particles which appear after a nucleon-nucleon collision is also taken into account. It turns out that there is only a slight difference between results calculated under assumption (*a*) or (*b*).

FERMI'S theory of pion production[1] enables us to calculate the relative cross section for production of any number of pions in a nucleon-nucleon collision as a function of the collision energy. It is also possible to include production of other mesons in this theory.[2,3] In the original theory the cross sections are simply proportional to the density of the final states in momentum space. Haber-Schaim *et al.*[3] consider the different possible charges of the particles that appear after the collision, consistent with charge conservation, and also the possible identity of some of them. In a recent paper[4] Fermi considers the different charge possibilities consistent not only with charge conservation, but also with isotopic-spin conservation.

Our purpose is to give general formulas for the corrections to Fermi's cross sections which follow from the conservation of charge, or charge plus isotopic spin, and the possible identity of some of the particles.

According to the isotopic-spin formalism the proton and the neutron are treated as two different states of one and the same particle, the nucleon, the states being characterized by the quantum number $M=+\frac{1}{2}, -\frac{1}{2}$. $M=T_Z$, where $T=\frac{1}{2}$ is the isotopic spin of the nucleon. In the same way the π^+, π^0, and π^- meson are treated as three states, $M=+1, 0, -1$, of the pion, the isotopic spin of which is $T=1$.

Let $S_j(E)$ be the probabilities of two nucleons and j pions—considered as $j+2$ different, independent particles—appearing after the collision, as given by Fermi.[1] Then the following corrections should be applied to these probabilities:

(a) division by $2!j!$ owing to the identity of the nucleons and pions, respectively;

(b) multiplication by the statistical weight of the state of the two nucleons and j pions characterized by T and M, i.e., the number of their different states conserving both T and M. This number will now be derived.

The state of two nucleons may have either $T=0$ or $T=1$. Apart from the multiplicity $2T+1$ due to the electric quantum number M, there corresponds just one state to each value of T. The state of j pions may have $T=0, 1, 2, \cdots j$. In order to determine the number of states corresponding to each value of T, we note that the number of states consisting of k positive, l neutral, and m negative pions is $j!/k!l!m!$. Therefore, the number of all the states characterized by M (i.e., all states of total charge $k-m=M$) is

$$f_j(M)=\sum_{\substack{k+l+m=j\\k-m=M}}\frac{j!}{k!l!m!}$$
$$=\sum_{i=0}^{[\frac{1}{2}j-\frac{1}{2}M]}\frac{j!}{(j-M-2i)!(M+i)!i!}=\sum_i\binom{j}{i}\binom{j-i}{M+i}. \tag{1}$$

With the help of Slater's method[5] one may convince oneself that the number of states corresponding to a given T is

$$g_j(T)=f_j(T)-f_j(T+1). \tag{2}$$

We can now derive the number $G_j(T)$ of states of a given T of an assembly of the two nucleons and j pions. If the nucleons are in a state of isotopic spin 0, a state T will result if the state of the pions is T; and if the nucleons are in a state of isotopic spin 1, a state T may result for the states of the pions being either $T-1$, T, or $T+1$. Since the states of the nucleons are single, and the multiplicities of the states of the pions are $g_j(T)$, we have $G_j(T)=g_j(T-1)+2g_j(T)+g_j(T+1)$ for all values of T except $T=0$, and $G_j(0)=g_j(0)+g_j(1)$, or

$$G_j(T)=f_j(T-1)-f_j(T+1)+f_j(T)-f_j(T+2), \tag{3}$$

which, since $f_j(-M)=f_j(M)$, holds for $T=0$ as well.

Since in a collision of two nucleons the initial state may have either $T=0$ or $T=1$, the corrections are, respectively,

$$a_j(0)=\frac{1}{2j!}[f_j(0)-f_j(2)],$$
$$a_j(1)=\frac{1}{2j!}[f_j(0)-f_j(2)+f_j(1)-f_j(3)]. \tag{4}$$

In a p-p or n-n collision the initial state has $T=1$ (with $M=\pm1$), while in a pn collision it is a random mixture

[1] E. Fermi, Progr. Theoret. Phys. (Japan) **5**, 570 (1950).
[2] L. S. Kothari, Phys. Rev. **90**, 1087 (1953).
[3] Haber-Schaim, Yeivin, and Yekutieli, Phys. Rev. **94**, 184 (1954).
[4] E. Fermi, Phys. Rev. **92**, 452 (1953); **93**, 1434 (1954).
[5] E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra* (Cambridge University Press, London, 1953), second edition, p. 190.

TABLE I. Corrections a_{jk} for the probabilities $S_{jk}(E)$. Corrections are normalized to give $a_{00}=1$. (*a*) Charge conservation. (*b*) Charge-plus-isotopic-spin conservation.

	pn collisions						*pp*(*nn*) collisions					
	$k=0$		1		2		0		1		2	
j	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)	(*a*)	(*b*)
0	1.00	1.00					1.00	1.00				
1	2.00	1.50	5.00	3.00			3.00	2.00	8.00	4.00		
2	2.50	1.50	6.50	3.24	8.75	3.75	4.00	2.00	11.0	4.50	15.3	5.25
3	2.17	1.08	5.84	2.50	8.00	3.00	3.67	1.50	10.2	3.50	14.3	4.25
4	1.46	0.625	4.00	1.50	5.57	1.86	2.54	0.875	7.13	2.13	10.1	2.66
5	0.800	0.300	2.23	0.744			1.43	0.425	4.03	1.06		
6	0.371	0.124	1.04	0.312			0.671	0.177	1.91	0.449		
7	0.149	0.045					0.273	0.064				

of $T=0$ and $T=1$ (with $M=0$). We thus conclude that for *p-p* (and *n-n*) collisions the corrections are $a_j(1)$, and for *p-n* collisions they are $\frac{1}{2}[a_j(0)+a_j(1)]$.

When only charge (i.e., M) is conserved, the calculation of the statistical weights $F_j(M)$ of final states is quite simple: either

$$F_j(M)=\sum_{T=M}^{j+1} G_j(T),$$

or, directly,

$$F_j(M)=f_j(M-1)+2f_j(M)+f_j(M+1). \tag{5}$$

In this case of charge conservation only, the corrections are therefore

$$a_j=\begin{cases}(1/2j!)[f_j(0)+2f_j(1)+f_j(2)], & p\text{-}p\,(n\text{-}n),\\ (1/j!)[f_j(0)+f_j(1)], & p\text{-}n.\end{cases} \tag{6}$$

The probability for production of j pions should now be proportional to $a_jS_j(E)$, where the relevant a_j is to be used.

In order to consider production of mesons other than pions as well, we assume that these also are essentially the different states of just one particle of isotopic spin $T=1$. Let $S_{jk}(E)$ be the "uncorrected" probability for j pions and k "other mesons" to appear with the two nucleons after a collision of energy E, as given for instance by Haber-Schaim *et al.*[3] S_{jk} reduces to the S_j mentioned above for $k=0$. To find the different formulas for the corrections a_{jk} which should be applied to the S_{jk}, we consider first a final state of $j+k$ pions only. If now k pions are replaced by other mesons, correction (*a*) should be $2!j!k!$ instead of $2!(j+k)!$, while correction (*b*) remains unchanged as all mesons are equivalent with respect to isotopic spin. We thus conclude that in all cases the corrections are given by

$$a_{jk}=\binom{j+k}{k}a_{j+k,\,0}. \tag{7}$$

If mesons produced in nuclear collisions consist of more kinds of particles (of isotopic spin 1), it should now be obvious how to calculate the corrections $a_{jkl\cdots}$.

Table I gives some values of a_{jk}. With these, and the $S_{jk}(E)$ of Haber-Schaim *et al.*,[3] the average numbers of pions, n_π, and "kappons," n_κ, produced in *p-n* collisions were calculated for $3\leq E\leq 6$ assuming charge-plus-isotopic-spin conservation. E is the collision energy, in units of Mc^2, in the C system. Results of this calculation appear in Table II, where the calculation of Haber-Schaim *et al.*[3] assuming charge conservation only is also given. Comparison shows that there is hardly any difference between results under the two assumptions.

TABLE II. Average numbers of pions and "kappons" produced in proton-neutron collisions as functions of the collision energy. E is the collision energy in the C system. (*a*) Charge conservation (calculated by Haber-Schaim *et al.*[3]). (*b*) Charge-plus-isotopic-spin conservation.

E	n_π		n_κ	
(Mc^2)	(*a*)	(*b*)	(*a*)	(*b*)
3	1.26	1.17	0.109	0.115
4	2.04	1.92	0.313	0.316
5	2.64	2.52	0.511	0.511
6	3.18	3.05	0.700	0.685

The author is grateful to Dr. A. de-Shalit and Dr. G. Yekutieli for many helpful discussions.