tron value of W is slightly higher than the other reported values. The increase in W for H_2 beyond the minimum has already been remarked, and it may be that the change from α-particle energies to minimum ionizing electrons is a part of this trend. The low value reported for 340-Mev protons is surprising in that the primary particle velocity is intermediate between the α particle and electron velocities.

Rows 9 and 10 of Table II give the expected total specific ionization for 1.7-Mev (minimum ionization) and 34-Mev electrons, respectively. These values were computed by dividing the difference in energy loss predicted by Eqs. (1) and (2) by the value of W at the minimum (Row 5 of Table II), and adding the result to the measured probable ionization.

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Polarization in Scattering by Complex Nuclei*

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Polarization effects in the elastic scattering of high-energy nucleons by complex nuclei are studied in terms of the impulse approximation. The principal aim is to reconcile the large polarizations produced by complex nuclei with the smaller effects found in nucleon-nucleon scattering. It is shown that these results are not inconsistent and can indeed be understood in terms of simple physical arguments. While, in general, our knowledge of nuclear structure is not adequate for explicit calculation of these effects even in the impulse approximation, it can be shown that for a particular class of nuclei (the deuteron and the alpha-particle nuclei) the polarization is independent of the nuclear wave function. Calculations for these nuclei have been carried out in detail, using existing nucleon-nucleon phase shifts. The resulting polarization effects are found to be large, in rough agreement with experiment, although their angular dependence is not satisfactory. It is proposed that a study of polarization in elastic scattering by deuterium and helium be used as a tool for investigation of the nucleon-nucleon interaction.

I. INTRODUCTION

EXTENSIVE experiments have been reported during the past few years concerning measurements of the azimuthal asymmetry in the double scattering of high-energy nucleons by various nuclei.[1–6] These measurements indicate the existence of quite large polarization effects in the energy region 130 to 400 Mev. The peak polarization produced in proton-proton scattering has been found to be about 40 percent in this energy region, while comparable effects are found in neutron-proton scattering. Protons of the same energy when scattered by complex nuclei seem to be polarized much more strongly, however, the major effect coming from elastic processes.[4,7] Experiments that discriminate against the inelastically scattered protons have detected polarizations as large as 80 percent.

Theoretical investigations of polarization effects in nucleon-nucleon collisions have been carried out by Goldfarb and Feldman[8] and by Swanson.[9] These calculations are based upon various assumed phenomenological potentials designed to fit existing scattering and bound-state data. A reasonably good estimate of the p-p polarization is provided by the singular tensor-force interaction, while the hard core and $\mathbf{L}\cdot\mathbf{S}$ models give, respectively, too small and too large an effect. The tensor-force model of Christian and Hart gives roughly comparable polarizations for the n-p case.

More recently, attention has been focused upon the scattering of nucleons by complex nuclei. Numerous calculations have been reported[10–15] in which the nucleon-nucleus interaction has been treated phenomenologically. The common feature of all these efforts has been the use of a complex central well con-

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[2] Marshall, Marshall, and deCarvalho, Phys. Rev. **93**, 1431 (1954).

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[5] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **93**, 1430 (1954).

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[7] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **95**, 1105 (1954).

[8] L. J. B. Goldfarb and D. Feldman, Phys. Rev. **88**, 1099 (1952).

[9] D. R. Swanson, Phys. Rev. **84**, 1068 (1951); **89**, 749 (1953).

[10] E. Fermi, Nuovo cimento **11**, 407 (1954).

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[12] W. Heckrotte, Phys. Rev. **94**, 1797 (1954).

[13] B. J. Malenka, Phys. Rev. **95**, 522 (1954).

[14] R. Sternheimer, Phys. Rev. **95**, 589 (1954).

[15] W. Heckrotte and J. V. Lepore, Phys. Rev. **95**, 1109 (1954).

structed to fit the high-energy cross sections, together with an $\mathbf{L}\cdot\mathbf{S}$ interaction whose strength must be chosen more or less *ad hoc.* The spin-orbit potential generally used is that obtained from the shell model, although there is no *a priori* justification for extrapolation to such a high energy. These calculations do predict quite large polarization effects, with maxima of about 80 to 100 percent and angular distributions that are roughly in agreement with experiment.

It is the purpose of this paper to examine the problem from a somewhat different and less phenomenological point of view. In the energy region of interest the collision times are sufficiently short compared with nuclear periods that the cooperative behavior of the entire nucleus is less important than the individual-particle aspects of the process. One is led, therefore, to attempt to describe the scattering by complex nuclei in terms of what is already known about the nucleon-nucleon interaction. From this point of view, it seems difficult to reconcile the very large polarizations produced by complex nuclei with the relatively small effects in nucleon-nucleon collisions.

We will see, however, that if one considers the scattering of a nucleon by another nucleon bound in a nucleus, the requirement that the process be elastic imposes a constraint (in the form of a spin correlation) whose effect is to increase the resulting polarization. Section II is devoted to an exposition of this point in terms of rather simple physical arguments. In Sec. III the scattering problem is formulated in terms of the impulse approximation to make possible explicit calculations in terms of nucleon-nucleon phase shifts. These phase shifts are assumed to be known, but as far as possible no assumptions are made concerning the detailed structure of the target nucleus. In particular we find, with the aid of a few reasonable approximations, that there exists a class of nuclei for which no detailed knowledge of the nuclear wave function is required.

II. POLARIZATION BY A BOUND NUCLEON

Before proceeding to a detailed formulation of the problem of scattering of nucleons by a bound system, we find it instructive to see what may be learned from a few physical considerations. We assume, in the spirit of the impulse approximation,[16] that the total scattered wave may be obtained by summing the waves scattered by the various constituent nucleons. When the nucleon spin is ignored, the contribution from each nucleon to the transition between specified initial and final states is given by the amplitude for a free-particle collision with the same momentum transfer, multiplied by a numerical factor (the square root of the sticking factor) which is simply a measure of the probability that the nucleus finds itself in the required final state. The effect of nucleon spin can be understood in a very simple way. To describe the scattering by a particular nucleon, consider the target nucleus to be decomposed into that nucleon and a residual nucleus. Specification of the nuclear state then determines the relative orientation of the spins of these two subsystems. Although, for an unpolarized nucleus, the target nucleon presents all possible spin orientations to the incident beam, the residual nucleus provides a "memory" of the initial spin orientation of the struck particle. If we require the scattering to be elastic, the relative spin direction of the two particles must be preserved, which is impossible if the spin orientation of the struck nucleon has been changed. Such "spin-flip" events are thus suppressed in elastic scattering. We may therefore conclude that the requirement of elastic scattering is in part equivalent to the imposition of a constraint that discriminates against spin-flip scattering. The strength of this constraint depends, of course, in a rather complicated way upon the details of the nuclear state.

It now remains to be seen how the presence of such a constraint affects the polarizing power of the target nucleon. For convenience in the following discussion let us introduce the three orthogonal vectors constructed from the initial and final relative momenta $\mathbf{k}_i$ and $\mathbf{k}_f$;

$$\mathbf{n}=\mathbf{k}_i\times\mathbf{k}_f,\quad \mathbf{K}=\mathbf{k}_f-\mathbf{k}_i,\quad \mathbf{V}=\mathbf{n}\times\mathbf{K}.$$

Furthermore, let us choose our axis of spin quantization along $\mathbf{n}$, the normal to the plane of scattering. By a spin-flip scattering we mean an event in which the magnetic quantum number of the incoming nucleon changes sign.

We will first show that in a collision between two spin-$\frac{1}{2}$ particles if one particle flips its spin the other must do so also. This follows immediately from the requirement of invariance of the scattering matrix under rotations and reflections. If, for example, particle 2 flips its spin and particle 1 does not, the most general operator causing such a transition that is rotationally invariant is of the form

$$(A+B\boldsymbol{\sigma}_1\cdot\mathbf{n})(C\boldsymbol{\sigma}_2\cdot\mathbf{K}+D\boldsymbol{\sigma}_2\cdot\mathbf{V}).$$

But this operator is not invariant under reflections and must be excluded.

Consider, now, a collision in which the two nucleons are specified initially by magnetic quantum numbers m_1 and m_2, and finally m_1' and m_2'. Invariance under time reversal further requires that the transition matrix satisfy (except for a phase factor)

$$M(m_1, m_2, \mathbf{k}_i\rightarrow m_1', m_2'\mathbf{k}_f) =M(-m_1', -m_2', -\mathbf{k}_f\rightarrow -m_1, -m_2, -\mathbf{k}_i).$$

Since for nucleon-nucleon scattering $|k_i|=|k_f|$, invariance under rotations requires that this also be equal to

$$M(m_1', m_2', \mathbf{k}_i\rightarrow m_1, m_2, \mathbf{k}_f).$$

[16] G. F. Chew and G. C. Wick, Phys. Rev. 85, 636 (1952). Here we use the term "impulse approximation" in its loose sense, meaning use of all three of the approximations defined by these authors.

Restricting our attention to spin-flip scattering, we have

$$M(m_1, m_2, \mathbf{k}_i \to -m_1, -m_2, \mathbf{k}_f) = M(-m_1, -m_2, \mathbf{k}_i \to m_1, m_2, \mathbf{k}_f).$$

We now see immediately that, for random initial states, spin-flip events lead to no polarization in the final state. Because the polarization is the ratio of spin density to particle density in the final state, and because spin-flip and no-spin-flip scatterings do not interfere, suppression of spin-flip processes simply decreases the cross section while leaving the spin density unchanged, thereby increasing the polarization.

The above somewhat heuristic argument should not be considered as a rigorous proof, even granting the impulse approximation, that a system of bound nucleons always causes larger polarizations than those obtained in nucleon-nucleon scattering. Because nuclei consist of two different types of particles, the interference terms could very well drastically alter the binding effect. However, even if interference effects are ignored, there is a more fundamental gap in the argument. A complete description of the spin state of two nucleons requires not only a specification of the relative orientation of the spins, but also a relative phase. Processes in which the relative phase is changed (transitions between the singlet and triplet $m=0$ states) are inelastic and must be excluded in spite of the fact that no spins have actually been flipped. This considerably complicates the effect of binding, so that no broad assertions can be made.

A more rigorous treatment of the binding effect is given in Appendix A. It is shown there that there exists an upper bound on the polarization, which depends only upon the ratio of the spin-flip to the no-spin-flip scattering cross sections. The requirement of elastic scattering has the effect of depressing the spin-flip cross section, thereby increasing the maximum attainable polarization.

What we have shown, therefore, is not that the effect of binding is to enhance all polarization effects, but to show that it does provide a mechanism by which apparently anomalously large polarizations may be obtained.

III. CALCULATIONS AND RESULTS

To make tractable the problem of scattering by a complex nucleus, a number of simplifying assumptions are invoked. We assume the energy to be sufficiently high that the impulse approximation is valid, and that the nucleus is sufficiently light that multiple scattering may be neglected. Furthermore the internal momenta in the nucleus are neglected compared with the momentum of the incoming particle, so that the nucleon-nucleon phase shifts may be taken to be those appropriate to free-particle scattering.

With these assumptions in mind we now proceed to develop a treatment of the nucleon-deuteron scattering problem, and then generalize it to more complex targets. Let the subscript 1 denote the incident nucleon, while 2 and 3 represent the nucleons in the deuteron. For spinless particles the scattered amplitude, q, may be written[17]

$$q = I_{12}q_{12} + I_{13}q_{13}, \tag{1}$$

where q_{12} and q_{13} are the appropriate free-particle scattering amplitudes and

$$I_{1\kappa} = \int \psi_f^*(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)\psi_i(\mathbf{r}_1\mathbf{r}_2\mathbf{r}_3)\delta(\mathbf{r}_1 - \mathbf{r}_\kappa)d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3. \tag{2}$$

When the nucleon spin is included, it is also necessary to specify the initial and final spin states of the target system. If these are denoted by S_{23} and S_{23}', we must select from the free-particle scattering matrix that part which couples these two states. To enable explicit calculation of the scattered wave it is convenient to make use of the S-matrix and Racah formalisms. In reference 17, it is shown that the transition matrix for proton-deuteron scattering may be obtained by a unitary transformation upon the p-p and n-p matrices. In particular, for elastic scattering ($S_{23} = S_{23}'$),

$$q_{1\kappa}(S'M', S_{23}; SM, S_{23}) = \frac{\pi^{\frac{1}{2}}}{k} \sum_{ll'J} (2l+1)^{\frac{1}{2}} i^{l-l'} \times (lS0M|lSJM)(l'S'M-M'\,M'|l'S'JM) \times R_{1\kappa}^J(S'l'S_{23}; SlS_{23}). \tag{3}$$

The R matrix $R_{1\kappa}^J$ is defined in terms of the free particle R by

$$R_{12}^J(S'l'S_{23}'; SlS_{23}) = \sum_{S_{12}S_{12}'j} Q_{(s)}(S_{12}'S_{23}'jl'; S'J) \times Q_{(s)}(S_{12}S_{23}jl; SJ) R_{12}^j(S_{12}'l'; S_{12}l), \tag{4}$$

and similarly for R_{13}^J. Here we have introduced the notation

$$Q_{(s)}(S_{12}S_{23}jl; SJ) \equiv [(2S_{12}+1)(2S_{23}+1)(2S+1)(2j+1)]^{\frac{1}{2}} \times W(S_1S_2SS_3; S_{12}S_{23}) W(S_3Sjl; S_{12}J). \tag{5}$$

The spins of particles 1, 2, and 3 are here collectively denoted by the subscript (s). The symbol R_{12}^j refers to the scattering amplitude for the free-free collision between particles 1 and 2 in the state j. It can readily be shown that the Q coefficients satisfy

$$\sum_{jS_{12}} Q_{(s)}(S_{12}S_{23}jl; SJ) Q_{(s)}(S_{12}S_{23}'jl; S'J) = \delta_{SS'}\delta_{S_{23}S_{23}'},$$

[17] S. Tamor, Phys. Rev. **93**, 227 (1953). The notation in this paper differs from that in the above reference in that the overlap integral I is now no longer included in the definition of q_{12} and q_{13}. The reason for this change will become apparent.

and

$$\sum_{SS_{23}} Q_{(s)}(S_{12}S_{23}jl;SJ)Q_{(s)}(S_{12}'S_{23}j'l;SJ)=\delta_{S_{12}S_{12}'}\delta_{jj'}.$$

The wave scattered by each nucleon may now be expressed in terms of q as defined in Eq. (3) with the aid of BB(3.12)[18] or SW(2.2).[19] If the waves scattered by 2 and 3 are ψ_2 and ψ_3, respectively, the polarization of the scattered nucleon is

$$\mathbf{P}=\frac{1}{|S_1|}\frac{\langle I_{12}\psi_2+I_{13}\psi_3|\mathbf{S}_1|I_{12}\psi_2+I_{13}\psi_3\rangle}{\langle I_{12}\psi_2+I_{13}\psi_3|I_{12}\psi_2+I_{13}\psi_3\rangle}. \quad (6)$$

We now observe that if the initial and final states of the (23) system have definite parity, then $I_{12}=\pm I_{13}$ so that the overlap integrals cancel from Eq. (6). For elastic scattering $I_{12}=I_{13}$. But since all information concerning the spatial part of the deuteron wave function is contained in the factor I, we may conclude that, in the impulse approximation, the polarization is independent of the deuteron wave function. This means that that accuracy of our calculation depends only upon the validity of the impulse approximation (assuming the nucleon-nucleon phase shifts to be known), and not upon our choice of a deuteron wave function, for which the high-momentum components are quite uncertain.

In view of this cancellation we may inquire whether there are other nuclei which possess such a symmetry property. Consider a nucleus of spin S_{total} and fix attention upon the wave scattered by the ith nucleon. Let us factor the total nuclear wave function into the spin coordinate of the ith nucleon times a residual function of all the remaining coordinates including the spatial coordinates of particle i. These two factors then transform according to spins $\frac{1}{2}$ and $|S_{\text{tot}}\pm\frac{1}{2}|$. In general, the residual nuclear wave function contains a coherent mixture of these two spin states whose relative amplitudes and phases are determinable only from a nuclear model. However, for the special case $S_{\text{tot}}=0$, this ambiguity is removed so that we may again consider the nucleus as if it were a system of two spin-$\frac{1}{2}$ particles, and the transformation of Eq. (4) can be carried out.

The polarization is now given by Eq. (6), where we must sum over waves scattered by all nucleons. If nuclei contained only one type of nucleon all the q's would be equal and a factor $|\sum_\kappa I_{1\kappa}|^2$ would be common to both numerator and denominator. If we restrict our attention to nuclei with equal numbers of neutrons and protons, the nuclear wave function is symmetric with respect to interchange of neutron and proton coordinates if Coulomb effects are neglected. Then for every proton there is a neutron with the same sticking factor, and the overlap integrals again cancel from Eq. (6).

What this result amounts to is that for the purpose of polarization calculations all spin-zero self-mirror nuclei may be considered as deuterons of spin zero. Such nuclei are the alpha-particle nuclei. Note that in this approximation all these nuclei should polarize equally, which is what one would expect on the basis of the qualitative consideration of Sec. II.

Given the R matrices for nucleon-nucleon scattering, we may now calculate the polarization directly. A closed expression for the denominator of Eq. (6) is given by BB(4.5, 4.6), while the numerator is obtainable from SW(3.2). Explicit calculations were carried out for 240 Mev, using nucleon-nucleon phase shifts already used in published work. In particular, the p-p scattering phases are taken from reference 8, assuming a singular tensor force cutoff at 1.4 $\hbar/Mc$. The n-p singlet phases were taken equal to the p-p, while the triplet phases are those calculated by Swanson.[20] Actually the n-p phases were calculated at 40, 90, 200, and 285 Mev and were interpolated to 240 Mev. As a check on the concistency of the interpolated phases, the resulting S-matrix was checked for unitarity. Polarizations in scattering of protons by deuterons and by alpha-particle nuclei were calculated using all phase shifts up to $l=3$. Results are plotted in Fig. 1 together with the corresponding n-p and p-p polarizations.

Exchange terms (corresponding to pickup events in p-d scattering) have been neglected throughout. These terms are important, however, for angles larger than about 40°. Furthermore, in the p-α calculation, an accidental cancellation causes the polarization in the neighborhood of 45° to be extremely sensitive to the $l=4$ phase shifts, which have not been kept. Results are therefore plotted only for angles less than 40° in the laboratory system.

The enhancement of the polarization due to spin-flip suppression stands out clearly when the p-d and p-α curves are compared. These nuclei both are symmetric in neutrons and protons so that the interference terms enter in the same way, and the only relevent difference is the nuclear spin. The prohibition against spin-flip collisions is rather weak for spin-1 nuclei, while for spin zero it is absolute.

IV. DISCUSSION AND SUMMARY

Comparison with the Optical Model

The most important result of this calculation is that a nucleon-nucleon interaction that gives rise to small polarization effects can cause very large polarizations when the target consists of several nucleons bound together. Furthermore, this result is obtained without reference to any nuclear model. In spite of this, a comparison of the calculated angular dependence of the polarization with that predicted by the optical model

[18] J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. **24**, 258 (1952). References to this paper are designated by BB.

[19] A. Simon and T. A. Welton, Phys. Rev. **90**, 1036 (1953). References to this paper are designated by SW.

[20] D. R. Swanson, Phys. Rev. **89**, 740 (1953) and (private communication). The author is indebted to Dr. Swanson, who provided a complete tabulation of his n-p phase shifts.

reveals some striking differences. Most calculations based upon an optical model predict rather violet oscillations of the polarization in the immediate neighborhood of the diffraction minima.[11,13–15] The absence of such effects in the present calculation is entirely due to the failure of the impulse approximation. In the impulse approximation the angular dependence of the scattering cross section is governed primarily by the sticking factor, which is essentially the square of the Fourier transform of the nuclear wave function. Diffraction effects appear through the rapid variation of the sticking factor, which will in general have zeros for sufficiently short-tailed nuclear wave functions. Because of the cancellation of the sticking factor from our expression for the polarization our results pass smoothly through these zeros. At the diffraction minima, however, the corrections to the impulse approximation, in particular those arising from multiple scattering, may be expected to play a dominant role. Our results therefore apply only away from the diffraction minima and in this sense the p-α curve of Fig. 1 should be considered as an "envelope" of the correct polarization.

Since the entire approach is based upon the use of the impulse approximation and the neglect of multiple scattering, a criterion for the range of nuclei for which it is valid is easily obtained. If one calculates by this method the total scattering cross section of complex nuclei and invokes the closure approximation to sum over final states, one finds that the total cross section is equal to the sum of the cross sections of the constituent nucleons. This means that the total cross section at a given energy should vary linearly with A. For sufficiently heavy nuclei, however, multiple scattering becomes important, and the total cross section may be expected to vary more nearly as $A^{\frac{2}{3}}$. A study of the total cross sections for high-energy neutrons as a function of A[21,22] indicates that the data can be fitted by a linear dependence on A for light nuclei and an $A^{0.78}$ law for heavy nuclei. The transition between the two occurs at about $A=10$. The total carbon cross section at 280 Mev is found to differ from six times the deuterium cross section by less than 15 percent, so that even for $A=12$ multiple-scattering effects are not too important. For heavier nuclei, however, the neglect of multiple scattering may be a serious error.

While the spirit of this calculation differs from that of the optical model, there should be an intimate connection between the two approaches. In particular it is possible, at least in principle, to use the impulse approximation as a starting point for the construction of an equivalent nuclear potential which, in turn, can be used as the basis for scattering and polarization calculations with the optical model. Fernbach, Heckrotte, and Lepore[23] have investigated the general problem of the construction of nuclear potentials and have given a formal expression for the equivalent nuclear potential in terms of the nucleon-nucleon scattering amplitudes. It is very interesting to note that they are able, with the aid of an approximation quite analogous to the impulse approximation, to obtain a very simple form for the scattered wave which leads to polarizations depending only upon the nucleon-nucleon scattering amplitudes, and independent of the structure of the target nucleus. This establishes a direct correspondence between the impulse approximation and the optical model.

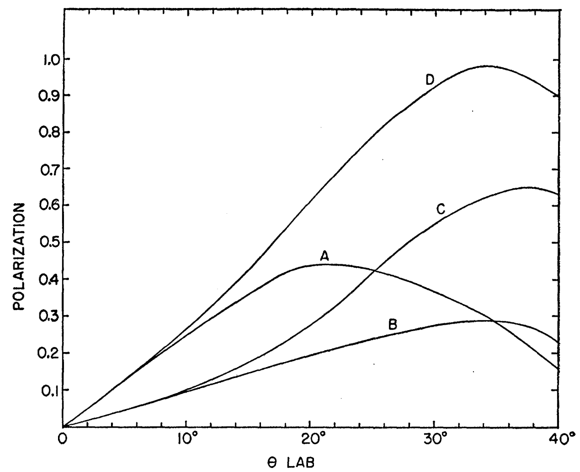

FIG. 1. Calculated polarization as a function of laboratory scattering angle. Curve A is for p-p scattering, based on reference 8, and Curve B is for n-p scattering obtained from reference 20 and interpolated to 240 Mev. Curves C and D are for p-d and p-α scattering calculated in the impulse approximation with the aid of the same phase shifts as those used for A and B.

Comparison with Experiment

Although the maximum polarization obtained by this model is in reasonably good agreement with experiment, the check of predicted angular distribution is much less satisfactory. In general the observed polarizations reach their maxima and start to fall off at considerably smaller angles than indicated in Fig. 1. It is the author's belief that this discrepancy is primarily a reflection of the poor state of our knowledge concerning the nucleon-nucleon interaction. This is particularly true for the neutron-proton interaction for which the Serber even-state interaction works at 90 Mev but is known to fail at higher energies, both for scattering cross sections[24] and polarization effects.[25] It seems, therefore, that further work along the lines indicated here will have to await an improved analysis of the n-p scattering data.

There is, however, one important qualitative feature

[21] Fox, Leith, Wouters, and MacKenzie, Phys. Rev. 80, 23 (1950).
[22] J. DeJuren, Phys. Rev. 80, 27 (1950).
[23] Fernbach, Heckrotte, and Lepore, this issue [Phys. Rev. 97, 1059 (1955)].
[24] J. De Pangher, Phys. Rev. 95, 578 (1954).
[25] Compare references 6 and 9.

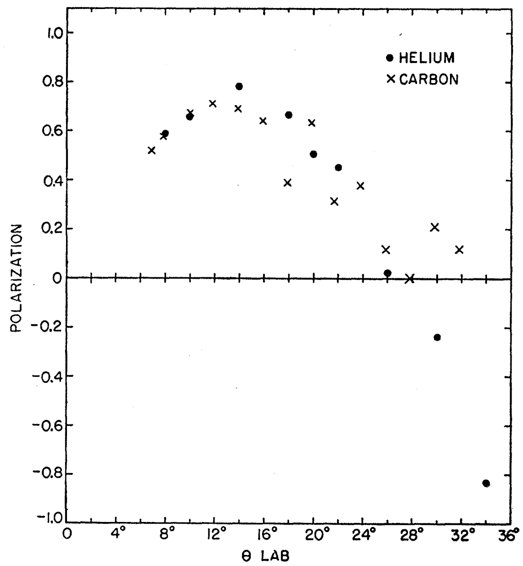

FIG. 2. Experimental results of Chamberlain *et al.* for polarization by helium and carbon. Errors are not indicated. For details consult reference 26.

of this theory that is suceptible to experimental verification and provides a crucial test of the model. This is the prediction that all nuclei of the alpha-particle type should polarize equally. The recently published data of Chamberlain *et al.*[26] on the polarization by helium and carbon bear directly upon this point. The observed polarizations for these nuclei are plotted in Fig. 2. The similarity between the helium and carbon data up to about 20° is quite striking. At angles larger than 20° the inelastic contamination in the carbon scattering increases rapidly so that detailed comparison in this region is impossible.

This encouraging check of the model lends weight to the more quantitative predictions of the theory. The very great simplicity of the polarization phenomenon suggests that experiments on polarization of nucleons in elastic scattering by nuclei may be used as a tool for investigation of the nucleon-nucleon interaction. Since we have seen that the polarizations in p-d and p-α scattering are expressible in terms of the nucleon-nucleon phase shifts alone, information regarding these processes may be considered as additional data to be fitted by any proposed model of nuclear forces. Such data are now available for the polarization by helium, while a measurement of the effect in elastic proton-deuteron scattering has been attempted at Chicago.[27] The possibility of large polarization in deuterium is indicated, but the data are too crude to permit detailed analysis.

[26] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **96**, 807 (1954).
[27] Marshall, Marshall, Nagle, and Skolnik, Phys. Rev. **95**, 1020 (1954).

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APPENDIX A

Rigorous Treatment of the Spin-Correlation Effect

Let us consider a collision between an incident nucleon (1) and a target nucleon (2), which is considered free except insofar as its spin is coupled to that of a third nucleon (3) so that the spin S_{23} is determined. For this purpose it is convenient to describe the collision between particles 1 and 2 in terms of the transition matrix,[28]

$$M=A+B(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})+C(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot n+D(\boldsymbol{\sigma}_1-\boldsymbol{\sigma}_2)\cdot\mathbf{n}+E(\boldsymbol{\sigma}_1\cdot\mathbf{K})(\boldsymbol{\sigma}_2\cdot\mathbf{K})+F(\boldsymbol{\sigma}_1\cdot\mathbf{V})(\sigma_2\cdot\mathbf{V}). \quad \text{(A-1)}$$

When spins 2 and 3 are uncorrelated the cross section is given simply by $\frac{1}{8}$ Tr M^+M, while the expectation value of the spin of the emerging nucleon is $\frac{1}{8}$ Tr $M^+\boldsymbol{\sigma}_1\cdot M$. Imposing the spin correlation between particles 2 and 3 is accomplished by insertion of the appropriate projection operators $\frac{1}{4}(3+\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)$ for $S_{23}=1$ and $\frac{1}{4}(1-\boldsymbol{\sigma}_2\cdot\boldsymbol{\sigma}_3)$ for $S_{23}=0$. The polarization for the various cases is given by

Case 1: No Spin Correlation

$$\mathbf{P}=\mathbf{n}\frac{2\,\mathrm{Re}\{A^*(C+D)+n^2B^*(C-D)\}}{|A|^2+n^4|B|^2+2n^2(|C|^2+|D|^2)+K^4|E|^2+V^4|F|^2}; \quad \text{(A-2)}$$

Case 2: $S_{23}=1$

$$\mathbf{P}=\mathbf{n}\frac{2\,\mathrm{Re}\{A^*(C+D)+\frac{2}{3}n^2B^*(C-D)\}}{|A|^2+\frac{2}{3}n^4|B|^2+(5/3)n^2(|C|^2+|D|^2)+\frac{2}{3}n^2\,\mathrm{Re}C^*D+\frac{2}{3}K^4|E|^2+\frac{2}{3}V^4|F|^2}; \quad \text{(A-3)}$$

Case 3: $S_{23}=0$

$$\mathbf{P}=\mathbf{n}\frac{2\,\mathrm{Re}\{A^*(C+D)\}}{|A|^2+n^2|C+D|^2}. \quad \text{(A-4)}$$

Choosing, as in Sec. II, the axis of spin quantization along n, we see that only the terms E and F in (A-1) contribute to spin flip while the others are diagonal in σ_{1z}. We see, as stated in Sec. II, that the E and F terms

[28] This form of the transition matrix is seen to be equivalent to that used by L. Wolfenstein and J. Ashkin [Phys. Rev. **85**, 947 (1952)] if one notes that $\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$ is expressible as a linear combination of $(\boldsymbol{\sigma}_1\cdot\mathbf{n})(\boldsymbol{\sigma}_2\cdot\mathbf{n})$, $(\boldsymbol{\sigma}_1\cdot\mathbf{K})(\boldsymbol{\sigma}_2\cdot\mathbf{K})$, and $(\boldsymbol{\sigma}_1\cdot\mathbf{V})(\boldsymbol{\sigma}_2\cdot\mathbf{V})$.

do not contribute to the spin density [numerators in Eq. (A-2) to (A-4)] and that the spin correlation tends to decrease their contribution to the cross section [denominators in (A-2) to (A-4)]. The term B, however, while not contributing to spin flip, does cause a phase change and is suppressed by the spin correlation, thereby partly invalidating the argument of Sec. II. However, if we let the no-spin-flip cross section be σ_0 and the spin-flip cross section be σ_+ (given respectively by the A, B, C, and D terms and the E and F terms in the denominator), we see that polarization always satisfied

$$|P|\leq\frac{1}{1+(\sigma_+/\sigma_0)}. \tag{A-5}$$

Therefore, although the spin correlation effect cannot guarantee large polarization effects, it does at least increase the maximum obtainable value of $|P|$ by means of the spin-flip suppression.

If both particles 2 and 3 scatter, it is readily shown that for processes in which S_{23} does not change we need merely replace the coefficients in (A-1) by the sums of the corresponding coefficients for the (12) and (13) interactions.

The above formulas are simplified if one observes that $D=0$ for identical nucleons, as is required in general if charge independence is assumed. The condition that the equality hold in (A-5) is that $A=B=C$ for Cases 1 and 2 while for Case 3 it is true if $A=C$.

APPENDIX B

Notations for the Coupled Phase Shifts

The partial wave analysis of nucleon-nucleon scattering in the presence of tensor forces is greatly complicated by the fact that orbital angular momentum is no longer a good quantum number. In particular, for triplet states of given J, the states $L=J\pm1$ are coupled together and the state $L=J$ is uncoupled. There seems, however, to be no general agreement on the precise method of describing the scattering in these coupled states. In particular, Ashkin and Wu[29] classify the states according to the quantum numbers J, l, and M_J, in which notation the phase shifts are complex because l is not a constant of the motion. Christian and Noyes[30] introduce a somewhat different set of parameters to describe the scattering, which arise quite naturally out of their method of solution of the coupled equations. Perhaps the most natural description is in the so-called "Parity Representation" of Rohrlich and Eisenstein,[31] which is used by Goldfarb and Feldman[8] and is closely related to the S-matrix formalism used here.

While all these descriptions are, of course, equivalent, to the author's knowledge the relations between them have never been set down in one place. It seemed worth while, therefore, simply to present a set of rules for transforming among these representations. These rules are given without proof, their derivation being simply an exercise in the recoupling of angular momenta, and not very illuminating.

The coupled phase shifts of Ashkin and Wu, denoted by δ_l^{JM}, are related to our S matrix by

$$S_{l'l}^J=i^{l-l'}\sum_M(SJM-M|SJl0)\times(SJM-M|SJl'0)S_{l'}^{JM},$$

where

$$S_l^{JM}=\exp(2i\delta_l^{JM}).$$

The inverse transformation is

$$S_{l'}^{JM}=\sum_l i^{l-l'}\frac{(SJM-M|SJl0)}{(SJM-M|SJl'0)}S_{l'l}^J.$$

Christian and Noyes introduce the set of parameters δ_{ll}^J, δ_{lL}^J, and a_{lL}^J where $l+L=2J$. It was shown by Christian[32] that these are related to the Ashkin and Wu phase shifts by

$$S_l^{JM}=\frac{\alpha_l^J}{\gamma_l^J}+\frac{2i\beta_l^J}{\gamma_l^J}\left(\frac{2L+1}{2l+1}\right)^{\frac{1}{2}}\frac{(SLM0|SLJM)}{(SlM0|SlJM)},$$

where

$$\alpha_l^J=\exp[i(\delta_{ll}^J-\delta_{LL}^J)]-a_{lL}^Ja_{Ll}^J\ \exp[i(\delta_{lL}^J-\delta_{Ll}^J)],$$

$$\beta_l^J=a_{lL}^J\sin(\delta_{ll}^J-\delta_{lL}^J),$$

$$\gamma_l^J=\exp[-i(\delta_{ll}^J+\delta_{LL}^J)]-a_{lL}^Ja_{Ll}^J\times\exp[-i(\delta_{lL}^J+\delta_{Ll}^J)].$$

These coefficients satisfy $\alpha_l=\alpha_L^*$, $\gamma_l=\gamma_L$, $\beta_l=\beta_L$. It is interesting to note that these quantities are related in a very simple way to the S matrix. In particular,

$$i^{l-l'}S_{l'l}^J=\frac{\alpha_l^J}{\gamma_l^J}\delta_{ll'}+\frac{2i\beta_{l'}^J}{\gamma_{l'}^J}\delta_{l+l',2J}.$$

Thus the diagonal elements of S are given by α_l^J/γ_l^J while the nonzero off-diagonal elements are $-2i\beta_l^J/\gamma_l^J$, since $\beta_l^J=0$ when $l=J$.

Finally, the explicit connection between the parity representation and the S matrix is given in BB(4.19).[18]

[29] J. Ashkin and T. Y. Wu, Phys. Rev. 73, 973 (1948).
[30] R. S. Christian and H. P. Noyes, Phys. Rev. 79, 85 (1950).
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