Stopping Power of Water Films*

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By a method which has been previously used with polystyrene and acetylene, the stopping powers of water films and water vapor for low-energy alpha particles have been compared. There appears to be no anomaly, the mass stopping powers being equal to within a precision of about 5 percent.

THE problem of the stopping power of water for heavy particles has been the subject of several experiments yielding conflicting results. Michl,[1] Philipp,[2] and Appleyard[3] have found stopping powers for liquids from 3 to 20 percent greater than would be expected from the Bragg law of additive stopping powers. For water their figures show a discrepancy of about 15 percent. De Carvalho and Yagoda,[4,5] and Wilkins,[6] in experiments using photographic techniques, have found ranges which indicate an absence of such an anomaly.

A method has been described[7,8] for comparing the stopping power of a thin film with that of a gas having identical chemical composition. It has recently been applied to water films and water vapor.

I. APPARATUS

Figure 1 shows the apparatus as it was used with water films. Polonium alpha particles from A are collimated by holes H and reach the ionization chamber D. (The slit S is removed when measurements with alpha particles are made.) When a film is placed at F, the beam of alpha particles can be used to find its "stopping thickness"—that is, the decrease of range of the particles upon insertion of the film.

The source of alpha particles, which has been described in reference 8, consists of a deposit of polonium covered by a thin aluminum foil, the whole being contained in a brass cartridge. Figure 3 will serve to indicate the range of the resulting radiation. The maximum range was adjusted to a convenient value (17 microns in water) by the choice of the covering foil. It appears that particles of all energies up to the maximum are present in the spectrum, but Fig. 3 will indicate that there was a preponderance at the maximum energy.

Meanwhile an optical measurement of thickness is made. Light from L is focused on A, whence it is reflected to the film F. From F it is reflected to the reflection grating G which focuses a spectrum on the photographic film P. Mercury lines are superimposed on a continuous spectrum for calibration. The "optical thickness" of the film is determined from the wavelengths of the fringes of zero intensity in such photographs as are shown in Fig. 2.

The required relationship is $t=n\lambda/(2\mu\cos\theta)$, where t is film thickness, n is order of interference, λ is a wavelength of zero intensity, μ is index of refraction, and θ is the angle of internal reflection. λ was determined by plotting the positions of the mercury lines against their wavelengths, and μ was taken to be the index of refraction for water. The presence of the small amount of detergent is assumed to have an entirely negligible effect on μ.

The values of n were assigned by choosing the set of integers which would make the quantity $n\lambda/(2\mu\cos\theta)$ the same for all fringes of the photograph. Statistical examination of the resulting values of $n\lambda/(2\mu\cos\theta)$ indicates that the optical thickness is considerably more accurate than the stopping thickness.

Such statistics do not indicate possible errors in the selection of n, and the sensitivity of the method of selection is such that an error of unity in the set of integers might occasionally be made where a set of fringes is indistinct. In most cases, however, the best

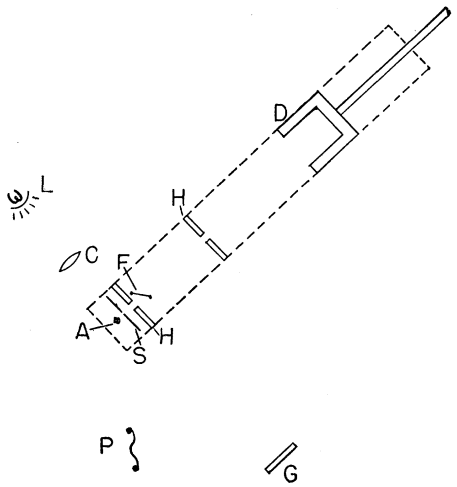

FIG. 1. The apparatus used for measurement of relative stopping power. A, source of alpha particles; H, collimating holes; D, detector; F, film; L, light source; C, condensing lens; S, removable slit; G, reflection grating; P, photographic film.

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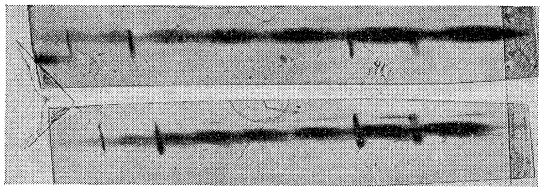

FIG. 2. Typical photographs from which film thickness was determined. Mercury lines are superimposed on the continuous spectrum.

set can be clearly determined, and the internal consistency of the data of Table I leads the authors to a feeling of confidence that no errors of this kind have been made. In this connection it is to be emphasized that there is no way for the selection of a set of values for n to be influenced by other measurements on the same film—optical or stopping.

Gas pressures have been determined by means of an oil-filled manometer. Conversion to density has been made by means of steam tables. While saturation pressure is extremely sensitive to temperature, the ratio of density to pressure is not, and there should be no difficulty from this cause in the determination of gas density.

II. FILMS

Films have been made from a 0.25 percent detergent solution. The technique was suggested by work on slow draining films published by Miles, Ross, and Shedlovsky,[9] and these investigators supplied a suitable detergent.

For water measurements a glass ring was substituted for the brass film supports used with polystyrene. The ring is one inch in diameter and made of $\frac{1}{16}$-inch glass rod. It is made with a handle which is held in a chuck at the end of a brass rod passing into the evacuated part of the apparatus through an O-ring seal. Thus the ring can be manipulated by sliding the rod in the seal.

To make a film, one pushes the ring forward to dip it in a cup of solution. Then it is withdrawn with a film across it, tilted, and advanced to a drainage position in which the edge of the ring touches the liquid surface. After draining from one to ten minutes for thinness and uniformity, the film is put in position for measurement. The resulting films were between 2.9 and 5.0 microns in thickness. (This thickness must be multiplied by 1/sin45° to get the thicknesses shown in Table I, for the beam makes a 45° angle with the film.)

Drs. Miles, Ross, and Shedlovsky are of the opinion that such a film consists of a monolayer of detergent on each surface and 0.25 percent detergent solution between. The effect of such a small amount of detergent on stopping power is considered negligible. In the absence of contrary evidence it has been assumed that the bulk density of the films is equal to that of pure water.

Visual inspection of the films by light from the mercury bulb shows that they assume a uniform wedge shape in their draining position. When they are positioned for measurements, approximately the same shape is maintained. For this reason the first collimating hole of Fig. 1 has been placed as close to the film as possible so that stopping and optical measurements sample the film at the same spot. Recognizable fringes can be photographed only by restricting the aperture at the entrance to the camera. This aperture is placed in the center of the beam of light formed by the first collimating hole and reflection from the film. Thus the optical sampling is at the center of the area through which alpha particles pass.

III. EXPERIMENTAL DATA

The range of alpha particles coming from the thick source is determined by plotting the ratio of detector current I to gas density ρ against the product of ρ and d, the separation between detector and source (Fig. 3). With both acetylene and air it has been ascertained that such a curve can be repeated at any value of source-detector separation. When a film is interposed at F, this response curve is shifted by an amount which is a measure of the "stopping thickness" of the film. With water films only one point of the displaced curve can be recorded because of the limitation that the films must stand in a saturated atmosphere.

The reference curve of Fig. 3 requires values of ρd greater than those which exist when films are measured. Because of the saturation requirement on ρ, they must be obtained by increasing d. Therefore the curve is plotted with d at its maximum by varying the gas

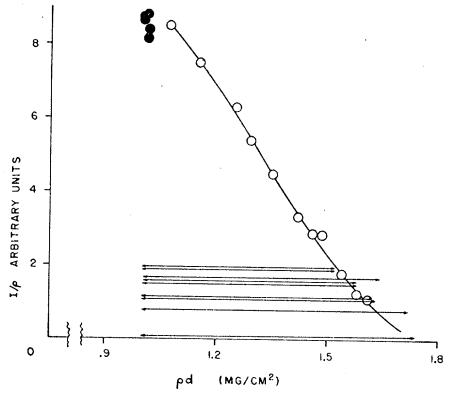

FIG. 3. Graph of a typical set of data. Coordinates are (1) detector current divided by gas density and (2) product of gas density and source-detector separation. The curve was plotted without films. The left end of each horizontal line is a point determined with a film in place, and the length of the line is the optical thickness. Solid points were read while measurements with films were being made as a check on the reliability of the curve.

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TABLE I. Results of measurements of 22 water films. The stopping thickness was determined from the type of data illustrated in Fig. 3. The optical thickness was obtained from photographs such as Fig. 2. The latter datum is multiplied by a factor of 1/sin45° to allow for increased thickness in the direction of the alpha beam. The order of the measurements is indicated in a vertical progression.

	Thickness (mg/cm²) Stopping	Optical
Group I		
A	...	0.54
	0.57	...
B	0.57	...
	...	0.54
	0.54	...
C	...	0.52
	0.59	...
	...	0.45
Group II		
A	...	0.578
	0.51	...
	...	0.568
	0.52	...
B	...	0.520
	0.51	...
	...	0.510
C	...	0.540
	0.49	...
D	...	0.585
	0.59	...
	...	0.558
	0.55	...
E	...	0.509
	0.52	...
	...	0.452
	0.50	...
F	...	0.411
	0.42	...
	...	0.385
	0.40	...
Group III		
A	...	0.581
	0.599	...
B	...	0.556
	0.560	...
Group III—*Continued*		
C	...	0.544
	0.567	...
D	...	0.546
	0.532	...
	...	0.510
	0.517	...
E	...	0.505
	0.505	...
	...	0.475
	0.487	...
Group IV		
A	...	0.699
	0.641	...
B	...	0.633
	0.596	...
	...	0.610
C	...	0.680
	0.601	...
	...	0.614
	0.571	...
D	...	0.537
	0.524	...
	...	0.506
	0.487	...
E	...	0.583
	0.549	...
F	...	0.595
	0.522	...
	...	0.516
	0.475	...
G	...	0.619
	0.529	...
H	...	0.584
	0.510	...
	...	0.547
	0.500	...

density over the required range of values. Then the detector is moved closer to the source, and the cup of solution is placed in the apparatus so that measurements can be made with films. Since the plotting of the curve requires pressures below saturation, the cup of solution cannot be in the apparatus while the data for the curve are taken. To make sure that the curve is still appropriate to the situation after the changes are made, data are occasionally taken without a film. These are the solid points of Fig. 3.

Each of the plotted points is corrected by subtracting the background current read with the source blocked. This allows for currents resulting from radioactive contamination in the vicinity of the source as well as any possible leakage currents.

In Fig. 3 each horizontal line represents a film. Its left end is at the point which is read with the apparatus when the film is in place. The horizontal distance from this point to the curve is the stopping thickness in milligrams per square centimeter. The length of the line represents the optical thickness in the direction of the beam.

The films invariably become thinner during measurements, and therefore the only reliable data are those in which an optical measurement is made before and after a stopping measurement, or vice versa, and the results averaged. Such data are referred to as "balanced data." In Fig. 3, half of the lines represent films for which the optical measurement preceded the stopping measurement, and for the remainder the stopping preceded the optical. Thus there is no representation of truly "balanced" data in Fig. 3. The figure is offered only as a qualitative illustration of method.

No measurements of the rate of thinning were attempted. However, the alternate observations of stopping and optical thicknesses were approximately

TABLE II. Balanced data obtained by averaging either optical or stopping measurements (see text).

Group	Stopping thickness mg/cm²	Optical thickness mg/cm²	Relative stopping, liquid to gas (stopping thickness over optical)	Averages by groups
Part 1. (All 3-measurement sets and 4-measurement sets from Table I.)				
I	0.555	0.54	1.03	
	0.59	0.485	1.21	1.11
II	0.515	0.568	0.91	
	0.51	0.515	0.99	
	0.570	0.558	1.02	
	0.52	0.480	1.08	
	0.410	0.385	1.07	1.01
III	0.599	0.565	1.06	
	0.524	0.510	1.03	
	0.505	0.490	1.03	1.04
IV	0.596	0.622	0.96	
	0.586	0.614	0.95	
	0.506	0.506	1.00	
	0.522	0.555	0.94	
	0.505	0.547	0.92	0.95
	Average		1.010±0.015	
Part 2. (4-measurement sets from Table I selected in opposite way from above—see text.)				
II	0.51	0.573	0.89	
	0.59	0.571	1.03	
	0.510	0.452	1.13	
	0.42	0.398	1.05	1.03
III	0.532	0.528	1.01	
	0.496	0.475	1.05	1.03
IV	0.601	0.647	0.93	
	0.524	0.522	1.00	
	0.498	0.516	0.97	
	0.510	0.565	0.90	0.95
	Average		0.996±0.058	

equally spaced in time, so "balanced" data adequately meets the difficulty. A confirmation of the adequacy of "balancing" data is in the fact that data which is unbalanced in one direction shows the same magnitude of discrepancy as data unbalanced in the other.

IV. RESULTS

In Table I are listed all of the measurements made on 22 films. The four groups were measured at different times, and three different base curves were used. The measurements (optical and stopping) are listed vertically in the order in which they were made.

The data fall into three classes depending on whether a film lasted for two, three, or four measurements. (None were kept for more than four.) Three measurements are required for balanced data, and four permit balancing by leaving out either the first or last measurement.

Table II summarizes the available balanced data in Table I. Part 1 includes all three-measurement sets, and selection has been made from the four-measurement sets so as to keep equal the number of films for which there are two optical measurements and the number for which there are two stopping measurements. In part 2 are included all four-measurement sets with the omissions made in the opposite way. In other words, if a four-measurement set is included in part 1 by omitting the last measurement, it is included in 2 by omitting the first. The ratio of stopping to optical thickness is the relative mass stopping power, liquid to gas. With the averages are their standard errors, $\sum|x-\bar{x}|/n^{\frac{1}{2}}$.

It is to be expected that two-measurement sets in Table I will show a bias in favor of too low a stopping measurement. This is found to be true, the average relative stopping power from such data being 0.961 ±0.024. For comparison all available data unbalanced in the opposite way were selected from Table I. Their average was 1.043±0.017.

V. ACCURACY

Two considerations are of primary importance in estimating the accuracy of these results. The thickness gradient in the films makes the alignment of optical and stopping geometries important. The gradient can be assessed by viewing the light from the mercury bulb reflected from the film when it is in the positions for measurement and for drainage. In the area subtended by the collimating hole one can see from two to four interference fringes. With the films used this suggests a thickness variation of 10 to 20 percent across the area explored. The maximum possible error due to misalignment would occur if one determined an optical thickness at the edge of the area traversed by alpha particles. Assuming that the average stopping thickness is the same as the thickness at the center of this area, the maximum error from this cause would be half of the total variation of thickness across the area; that is, 5 to 10 percent. A reasonable value to anticipate would be not greater than 2 percent in view of the care exercised in aligning the optical system.

A correction for scattering might lower the value of stopping for the liquid by one or two percent. Exact treatment of the matter with the collimation used is difficult and has not been attempted. Particles scattered by the film so as to miss the collimating hole will be compensated for somewhat. Some will be replaced by particles scattered into the beam by the film from paths which would otherwise miss the hole. Altogether it seems unlikely that a correction of large magnitude is required, and it is of interest that any recognizable influence would indicate a discrepancy opposite in sense from that indicated by earlier experiments.

VI. CONCLUSIONS

The conclusion is reached that to the accuracy of these measurements there is no anomaly in the stopping power of liquid water for natural alpha particles. The relative stopping power, liquid to gas, is 1.00±0.05.

The interesting question which remains is: Wherein lies the discrepancy among the different experiments designed to study this problem? Yagoda and de Carvalho[5] suggested that the experiments which show an anomaly are those in which the alpha particles have had to penetrate a gas-liquid interface, so that the interface might be responsible. Since this experiment requires the penetration of two interfaces, it would seem that another explanation must be found.

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Amplitudes in Nucleon-Nucleon Scattering*

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When scattering anomalies involving many phase shifts are to be studied, it appears desirable to treat the amplitudes directly before combining them into differential cross sections. Amplitudes suitable for the study of elastic collisions of charged and uncharged Fermi-Dirac particles of spin $\frac{1}{2}$, taking account of possible identity are, therefore, given in forms convenient for computation. The case of coupling between states of the same total but different orbital angular momentum is not discussed. Formulas using the spin functions usually denoted by χ_m are supplemented by forms based on spin functions which transform like the components of an ordinary space vector, the latter allowing more compact expressions in some cases.

I. INTRODUCTION

THE calculation of scattering of protons by protons and neutrons by protons has been the subject of many investigations. Recent experimental work in the region of several hundred Mev has made it desirable to be able to deal with scattering anomalies caused by many phase shifts. The calculations have been systematized therefore to a greater extent than has been done previously. The present paper is restricted to a nonrelativistic treatment and the introduction of coupling between states with the same total angular momentum $J\hbar$ but different orbital angular momenta $L\hbar$ is postponed to a succeeding and closely related one.

The treatment presupposes that either all collisions are elastic or else that the cross sections for inelastic collisions are so small that their damping effect may be neglected. In the approximations of this paper, therefore, the phase shifts may be taken to be real.[1] Some of the mathematical forms worked with are very similar to those of Ashkin and Wu[2] for complex phase shifts. Both in the present as well as the succeeding paper it has been found convenient to make use of the fact that the triplet spin behaves under rotations like an ordinary space vector.[3] The corresponding spin functions are denoted by ξ_1, ξ_2, ξ_3. Many formulas are more convenient in terms of amplitudes referred to these variables.

The set of angular and spin functions used is presented in Eqs. (1) to (1.4). The amplitudes are introduced in Eqs. (2), (2.1). Formulas for amplitudes for nonidentical particles and referred to magnetic quantum numbers are available in Eqs. (2.2) through (2.9). Effects of antisymmetry are introduced in Eqs. (3) through (3.3) and the modified results are collected in Eqs. (4) through (4.2). The relation to differential cross sections for unpolarized particles is as in Eqs. (5.1), (6.1). The ξ_1, ξ_2, ξ_3 modifications start with Eq. (7) with the scattering matrix S^ξ as in Eq. (7.5), cross sections as in Eqs. (8), (8.1).

II. NOTATION

α, β = nucleon spin function for states with magnetic quantum number $\frac{1}{2}$, $-\frac{1}{2}$, respectively.

$\chi_1, \chi_0, \chi_{-1}$ = triplet spin functions for two nucleons; $\chi_1=\alpha_1\alpha_2$, $\chi_0=(\alpha_1\beta_2+\alpha_2\beta_1)/2^{\frac{1}{2}}$, $\chi_{-1}=\beta_1\beta_2$.

$\chi_0^0=(\alpha_1\beta_2-\alpha_2\beta_1)/2^{\frac{1}{2}}$ singlet spin function for two nucleons.

r = distance between nucleons.

v = relative velocity.

M = nucleon mass.

$k=Mv/(2\hbar)$.

$$Y_{Lm}=\frac{(-)^m}{2^L L!}\left[\frac{(2L+1)(L-m)!}{4\pi(L+m)!}\right]^{\frac{1}{2}} \times e^{im\varphi}\sin^m\theta\left(\frac{d}{d\cos\theta}\right)^{L+m}(\cos^2\theta-1)^L.$$

θ = colatitude angle in polar coordinates = scattering angle.

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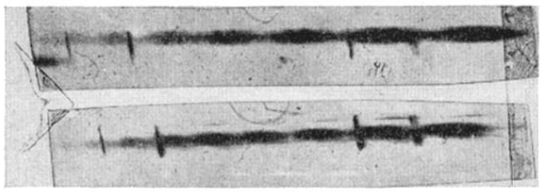

FIG. 2. Typical photographs from which film thickness was determined. Mercury lines are superimposed on the continuous spectrum.