Masses of Light Nuclei

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Values for the atomic masses of 42 nuclei from n^1 to S^{34} are derived from data on 110 nuclear reactions and mass spectrographic doublets. A variance-covariance matrix for the results is also derived and a further 32 masses in the same region are included. These results are found to be consistent with most of the data available.

THE atomic masses of 74 elements up to Cl^{34} have been calculated by a least squares fitting of all the available data of nuclear reaction Q-values (see Table I) and mass spectroscopic doublets (see Table II). It has been found possible, after rejecting only a few results as seriously inconsistent, to obtain a set of masses consistent with both mass spectroscopic and nuclear reaction data (see Tables III and IV). Also, by using all the data available it is possible to quote lower probable errors for the final masses than previously.

For each reaction the weighted mean of the experimental results was taken as the most probable value and the error assigned to the mean was the larger of the two calculated from weighted deviations of the experimental results and from the quoted errors, i.e.,

$$\bar{x}=\sum wx/\sum w,$$

where $w=1/p^2$ and p is the probable error, and the probable error in $\bar{x}$ is the larger of

$$0.6745[\sum w(x-\bar{x})^2/(n-1)\sum w]^{\frac{1}{2}} \text{ and } \{\sum(1/p^2)\}^{-\frac{1}{2}}.$$

The first part of the calculation consisted in using the 110 equations with probable errors less than or equal to 0.040 Mev connecting the masses of 42 of the atoms which were involved in two or more of the equations. The equations were then multiplied by the integer that would bring the error of each nearest to 0.040 Mev, and then the sums of squares and products were calculated for the 42 simultaneous equations for the masses. In this way a set of final equations is obtained in which the coefficients are all integers, and such equations can be solved with a desk calculating machine by relaxation methods. The equations have in effect been weighted by the squares of integers closest to the inverse squares of their probable errors; the percentage difference caused by this change of weighting factors will be greatest for those reactions whose probable errors are near 0.050 Mev (weight 0 or 1)

TABLE I. Nuclear reaction energies not listed by Li *et al.*[a,b]

Reactions	Q-value (Mev)	Ref.	Reactions	Q-value (Mev)	Ref.
$t(t,n)He^5$	10.6	c	$F^{19}(p,\alpha)O^{16}$	8.113(30)	n
$He^5(\gamma,n)\alpha$	0.87	d	$O^{19}(\beta)F^{19}$	4.5(3)	k
	0.95(0.07)	e	$Na^{23}(p,n)Mg^{23}$	−4.88(1)	o
$Li^7(d,\alpha)He^5$	14.3	c	$Na^{25}(\beta)Mg^{25}$	3.7(3)	k
$Be^9(\alpha,2\alpha)He^5$	−2.4	c	$Mg^{24}(d,n)Al^{25}$	0.07(6)	p
$Li^6(p,\alpha)He^3$	4.024(5)	f	$Al^{26}(\beta)Mg^{26}$	4.00	k
$Li^7(p,\alpha)\alpha$	17.344(13)	g	$Mg^{24}(\alpha,p)Al^{27}$	−1.613(10)	q
$Li^7(d,p)Li^8$	−0.192(1)	f	$Al^{27}(p,n)Si^{27}$	−5.610(10)	r
$Be^8(\gamma,\alpha)\alpha$	0.0945(14)	h	$Al^{28}(\beta)Si^{28}$	4.615(35)	s
$Be^9(p,\alpha)Li^6$	2.130(10)	i		4.55(15)	k
	2.123(4)	f	$Si^{28}(d,n)P^{29}$	0.29(4)	t
$Be^9(d,\alpha)Li^7$	7.159(9)	f	$Al^{29}(\beta)Si^{29}$	2.5	k
$Be^9(p,d)Be^8$	0.558(2)	f	$P^{29}(\beta^+)Si^{29}$	4.64(7)	k
$B^{10}(p,\alpha)Be^7$	1.1470(25)	j	$Si^{29}(d,n)P^{30}$	3.27(4)	t
$B^{10}(p,He^3)Be^8$	−0.536(3)	j		3.38	u
$B^{11}(p,\alpha)Be^8$	8.579(9)	h	$Al^{27}(\alpha,n)P^{30}$	−2.93	u
$C^{10}(\beta)B^{10}$	4.37(10)	k	$P^{30}(\beta^+)Si^{30}$	4.0(1)	k
$C^{11}(\beta^+)B^{11}$	1.989(5)	k	$Si^{30}(d,n)P^{31}$	4.92(4)	t
$Be^9(\alpha,p)B^{12}$	−6.92(5)	l	$P^{31}(\gamma,n)P^{30}$	−12.35	u
$B^{11}(d,p)B^{12}$	1.140(8)	f	$S^{31}(\beta^+)P^{31}$	4.87(7)	k
$Be^9(\alpha,n)C^{12}$	5.76(4)	m	$S^{32}(\gamma,d)P^{30}$	−19.15	u
$C^{12}(d,p)C^{13}$	2.722(4)	g	$S^{32}(\gamma,n)S^{31}$	−14.8(4)	u,v
$O^{16}(d,\alpha)N^{14}$	3.119(5)	g	$Cl^{33}(\beta^+)S^{33}$	5.15(7)	k
	3.1130(25)	j	$P^{34}(\beta)S^{34}$	5.1	k
$N^{16}(\beta)O^{16}$	10.3(3)	k	$Cl^{34}(\beta^+)S^{34}$	6.1(0.3)	k
$N^{17}(\beta,n)O^{16}$	2.7(4)	k			

[a] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951).
[b] See reference 1.
[c] Hornyak, Lauritsen, Morrison, and Fowler, Revs. Modern Phys. **22**, 364 (1950).
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[r] Kingston, Bair, Carlson, and Willard, Phys. Rev. **89**, 530 (1953).
[s] H. T. Motz, Phys. Rev. **83**, 215 (1951).
[t] Mandeville, Swann, and Chatterjee, Phys. Rev. **85**, 725 (1952).
[u] A. S. Penfold, Phys. Rev. **80**, 116 (1950).
[v] Collins, Nier, and Johnson, Phys. Rev. **86**, 408 (1952).

TABLE II. Mass spectrographic doublets.

The following doublets were used in addition to those listed in recent work.[a,b]

$(O^{16})_2-P^{31}H^1 = 82.45\pm0.12$ (10^{-4} amu)[c]
$(O^{16})_2-S^{32} = 177.82\pm0.25$[d]
$(C^{12})_4H^1-S^{33}O^{16} = 413.85\pm0.46$[e]
$(C^{12})_4(H^1)_2-S^{34}O^{16} = 529.00\pm0.40$[e]

After a preliminary check the following data were rejected as being inconsistent with the final results.

$H^1C^{12}-C^{13} = 44.10\pm0.08$	Low by $8\frac{1}{2}$ times the probable error[f]
$(C^{12})_4-S^{32}O^{16}=331.82\pm0.07$	High by 13 times the probable error[d]

The latter error was later altered to 0.28 but it was considered safer not to use this result.

[a] See reference 1.
[b] K. Ogata and H. Matsuda, Phys. Rev. **89**, 27 (1953).
[c] K. Ogata and H. Matsuda, Phys. Rev. **89**, 333 (1953).
[d] A. O. Nier, Phys. Rev. **81**, 624 (1951).
[e] Collins, Nier, and Johnson, Phys. Rev. **84**, 717 (1951).
[f] H. Ewald, Z. Naturforsch. **1**, 131 (1946).

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TABLE III. Atomic masses. (The values marked with an asterisk are unconfirmed.)

Element		$M-A$ Mev	$M-A$ 10^{-4} amu	Li *et al.*[a] Mev	Ogata *et al.*[b] 10^{-4} amu
n		8.3653(11)	89.838	8.3638(29)	
H	1	7.5833(7)	81.440	7.5815(27)	81.45(2)
	2	13.7231(12)	147.378	13.7203(60)	147.41(3)
	3	15.8300(26)	170.004	15.8271(100)	
He	3	15.8115(26)	169.806	15.8086(100)	
	4	3.6075(13)	38.742	3.6066(140)	38.79(9)
	5	12.92(7)	138.8		
	6	19.40(3)*	208.3	19.399(36)	
Li	6	15.8563(28)	170.28	15.850(21)	
	7	16.9694(35)	182.24	16.969(24)	
	8	23.3011(37)*	250.24	23.296(28)	
Be	7	17.8323(35)	191.51	17.832(24)	
	8	7.3096(26)	78.50	7.309(27)	
	9	14.0082(29)	150.44	14.007(28)	
	10	15.5614(42)	167.12	15.560(26)	
B	9	15.0782(36)	161.93	15.076(29)	
	10	15.0038(35)	161.13	15.004(26)	161.10(10)
	11	11.9178(34)	127.99	11.909(220)	128.11(9)
	12	16.9216(61)	181.73	16.912(20)	
C	10	19.37(10)*	208.0		
	11	13.9003(46)	149.28	13.889(22)	
	12	3.5640(22)	38.275	3.542(15)	38.44(6)
	13	6.9776(26)	74.935	6.958(13)	75.05(12)
	14	7.1644(22)	76.941	7.153(10)	
N	13	9.2007(32)	98.810	9.179(13)	
	14	7.0088(21)	75.270	6.998(10)	75.50(5)
	15	4.5464(48)	48.83	4.528(11)	49.02(9)
	16	10.3(3)*	111		
	17	11.1(4)*	119		
O	15	7.251(7)	77.87	7.233(12)	
	16	Zero standard			
	17	4.217(5)	45.29	4.221(6)	
	18	4.539(8)	48.74	4.522(22)	48.83(20)
	19	8.64(30)*	92.8		
F	17	6.987(4)	75.04	6.988(5)	
	18	6.208(9)	66.67	6.193(21)	
	19	4.137(6)	44.43	4.149(14)	44.44(22)
	20	5.903(8)	63.39	5.913(16)	
Ne	19	7.393(8)	79.40	7.405(14)	
	20	−1.142(5)	−12.27	−1.139(19)	−12.28(13)
	21	0.461(6)	4.95	0.469(21)	
	22	−1.536(9)	−16.49	−1.529(23)	−16.18(24)
	23	1.640(11)*	17.61	1.646(23)	
Na	21	3.983(31)*	42.78	3.991(37)	
	22	1.305(11)	14.01	1.312(23)	
	23	−2.743(7)	−29.46	−2.742(23)	
	24	−1.329(8)	−14.28	−1.333(24)	
	25	−2.1(3)*	−23		
Mg	23	1.353(11)*	14.53	1.353(24)	
	24	−6.850(8)	−73.56	−6.864(24)	
	25	−5.828(9)	−62.59	−5.824(25)	
	26	−8.554(12)	−91.86	−8.565(27)	
	27	−6.621(14)	−71.11	−8.633(28)	
Al	25	−1.56(6)*	−16.9		
	26	−4.0(6)	−43		
	27	−9.221(8)	−99.03	−9.245(28)	−98.91(23)
	28	−8.578(9)	−92.12	−8.603(30)	
	29	−10.8(5)*	−116		
Si	27	−4.393(13)*	−47.18		
	28	−13.218(8)	−141.95	−13.253(30)	−141.75(16)
	29	−13.328(8)	−143.13	−13.362(32)	−142.95(21)
	30	−15.577(10)	−167.29	−15.609(34)	−166.93(31)
	31	−13.804(8)	−148.24	−13.837(36)	
P	29	−8.28(16)	−88.9		
	30	−11.29(6)	−121.2		
	31	−15.280(7)	−164.10	−15.317(36)	
P	32	−14.841(7)	−159.38	−14.883(38)	
	33	−16.558(19)*	−177.82	−16.606(42)	
	34	−14.8*	−159		
S	31	−10.400(70)	−111.69		
	32	−16.547(7)	−177.70	−16.590(39)	177.26(8)
	33	−16.823(12)	−180.67	−16.871(41)	180.59(37)
	34	−19.864(15)	−213.33		212.91(19)
Cl	33	−11.67(7)*	−125.3		
	34	−13.80(30)*	−148.2		

[a] Li, Whaling, Fowler, and Lauritsen, Phys. Rev. **83**, 512 (1951); and C. W. Li, Phys. Rev. **88**, 1038 (1952).
[b] K. Ogata and H. Matsuda, Phys. Rev. **89**, 27, 333 (1953).

and 0.030 Mev (weight 1 or 4), and these reactions contribute least to the final result.

As a check on the reliability of the experimentalists' estimates of their errors, the final 42 masses were substituted back in the original 110 equations and the residual errors found. The sum of squares of these was then divided by the degrees of freedom (68) to give an estimate of the variance, 0.004093, and hence the estimated probable error was 0.043±0.003 Mev. This value is in very satisfactory agreement with the original assigned value of 0.040 Mev for the probable error in each equation.

TABLE IV. Isobaric differences. (The values marked with an asterisk are unconfirmed.)

Elements	Difference (Mev)
$H-n$	−0.7820(9)
He^3-H^3	−0.0185(1.5)
Be^7-Li^7	0.8629(16)
B^9-Be^9	1.0700(23)
$C^{10}-Be^{10}$	3.8120(1050)*
$C^{11}-B^{11}$	1.9825(29)
$N^{13}-C^{13}$	2.2231(23)
$O^{15}-N^{15}$	2.705(5)*
$F^{17}-O^{17}$	2.770(6)
$Ne^{19}-F^{19}$	3.255(6)
$Na^{21}-Ne^{21}$	3.522(30)
$Mg^{23}-Na^{23}$	4.096(8)*
$Al^{25}-Mg^{25}$	4.240(60)*
$Si^{27}-Al^{27}$	4.828(10)*
$P^{29}-Si^{29}$	5.05(16)
$S^{31}-P^{31}$	4.88(7)
$Cl^{33}-S^{33}$	5.15(7)*

The probable errors are found by inverting the (42×42) matrix of coefficients in the least-squares equations. This was done approximately (Table V) and in the result as quoted, the diagonal elements should be accurate within 1 or 2 in the last digit quoted; however, some of the smaller and less important off-diagonal terms may be seriously in error. The errors quoted in the final result are probable errors obtained by multiplying the square root of the corresponding diagonal term by 0.043 Mev. The variance-covariance matrix is obtained by multiplying each term by 0.00409 (Mev)^2. Furthermore, the probable error in any relationship involving these 42 elements may be obtained by adding the appropriate variance and covariance terms and multiplying the square root of the result by 0.043 Mev.

The remaining 32 masses are calculated from additional reaction data by using the 42 first calculated as a basis on which to work. The masses which have not been confirmed by results from a second different type of experiment are marked with an asterisk and in the meantime these may be regarded as unreliable. The He^6 mass is such a case as it is in conflict with a result for the energy of the $He^6(\beta)Li^6$ reaction. Here it was decided to accept the later data quoted by Li[1] as more reliable.

The ratio of the three sulfur masses,

$$(S^{33}-S^{32})/(S^{34}-S^{32})=0.500744(6),$$

is in conflict with a value 0.500714(3) derived from frequency measurements[2] but as our value is derived from 11 independent and consistent equations it appears that

[1] C. W. Li, Phys. Rev. **88**, 1038 (1952).
[2] S. Geschwind and R. Gunther-Mohr, Phys. Rev. **82**, 346 (1951).

TABLE V. Inverse matrix. The scale changes across the table and terms must be multiplied by 0.00409 (Mev)² to obtain the variance terms. (The scale factor for columns is given at the bottom.[a])

	$n-p$	p	d	α	t	Li^6	Li^7	Be^7-Li^7	Be^8	Be^9	Be^{10}	B^{10}	B^{11}	$O^{11}-B^{11}$	C^{12}	$C^{13}-C^{12}$	$N^{13}-C^{13}$	$C^{14}-N^{14}$	N^{1}
$n-p$	44	−3	−1	−1		1	3	−4	−1	2		−1		−3			−3		
p	−3	22	35	15	4	3	4		2	3	3	3	1		−1	1			2
d	−1	35	72	23	7	4	6		2	5	6	6	3			2	1		4
α	−1	15	23	87	2	8	8	−1	15	16	15	14	5		−2	1			−4
t		42	74	18	36	15	21	1	1	8	18	21	6		−2	11	−2	0.1	6
Li^6	12	33	44	84	15	41	23	−1	18	26	26	25	14	−1		7	−3		2
Li^7	28	37	58	82	21	23	65	−8	17	24	41	49	17	−2	3	7	−3		3
Be^7-Li^7	−43	2		−4	1	−1	−8	14	−1	−2	1	3	1	3		1	3		
Be^8	−11	18	23	152	1	18	17	−1	35	34	31	30	19	1	3	2			−5
Be^9	15	31	47	157	8	26	24	−2	34	43	37	34	22	−1	3	4	−2		−2
Be^{10}	2	34	60	150	18	26	41	1	31	37	92	56	23	1	4	7	−1		−1
B^{10}	−8	32	55	145	21	25	49	3	30	34	56	66	24	1	4	8	−1		
B^{11}	2	11	26	45	6	14	17	1	19	22	23	24	61		12	5	−1	0.2	4
$C^{11}-B^{11}$	−33	2	1	1		−1	−2	3	1	−1	1	1		45			2		
C^{12}	2	−6	−3	−16	−2		3		3	3	4	4	12		24	−4	2	0.2	8
$C^{13}-C^{12}$	−1	11	21	10	11	7	7	1	2	4	7	8	5		−4	20	−7	0.2	2
$N^{13}-C^{13}$	−34	3	7	−2	−2	−3	−3	3		−2	−1	−1	−1	2	2	−7	28	−0.1	
$C^{14}-N^{14}$	1			1														2.2	
N^{14}	−1	21	41	−38	6	2	3		−5	−2	−1		4		8	2		−0.2	23
N^{15}	4	6	19	37	3	4	6		10	10	11	11	14		20	1	1	0.2	10
O^{17}	1	16	42		5	4	9		1	2	6	6	3		2	2	1		3
O^{18}	4	12	42	3	5	2	6		2	3	6	10	3		5	1	1		3
$F^{18}-O^{18}$	−30	3	2			−1	−1	3	1	−1		1		1			2		
F^{19}	1	9	31	21	1	−1	−2		1	2	−1	−1			3	1	1		2
F^{20}	3	23	66	31	5	1	2		2	5	4	5	2		3	2	2		4
Ne^{20}	−5	39	71	16	8	4	8		3	6	9	10	4		3	3	1		4
Ne^{21}	−3	37	80	25	7	3	7		4	8	9	11	7		4	3	1		4
Ne^{22}	−2	62	124	53	14	9	14		9	17	20	21	25		5	5	2	0.1	8
Na^{23}	−4	12	21	27	1	1	4		5	6	6	6	4		8	1			2
Na^{24}	−2	20	57	25	4	2	6		4	6	8	9	5		10	2	1		6
Mg^{24}	−5	5	17	−24	2	−2	−1		−5	−4	−2	−2			17				11
Mg^{25}	−3	13	48		4		2		−2		4	5	2		16	1	1		10
Al^{27}	−3	−3	10	38		1	2		5	6	7	8	6		19				8
Al^{28}	13	9	40	26	3	2	6		2	6	7	8	8	−1	19	1	−1		12
Si^{28}	2	−13	−20	−11	−2	−1					1	1	4		23	−3			11
Si^{29}	3	−8		19		1	3		4	5	7	8	8		22		−1		10
Si^{30}	10	−3	10	68	−1	4	7		12	14	13	14	18	−1	20	1	−1		7
P^{31}	3	−27	−40	21	−4				7	5	6	6	8		26	−5		0.1	6
P^{32}	2	−11	−12	17	−1		1		6	6	7	8	13		34	−6	1	0.1	12
S^{32}	2	−11	−3	12	−2		1		6	6	7	7	13		36	−6	1	0.1	13
S^{33}	11	1	30	17	1	2	4		7	9	11	12	16		38	−6	1	0.1	16
S^{34}	4	−7	10	−16	−3		4		8	9	9	10	22		56	−9	4	0.4	20
		$\times 10^{-5}$			$\times 10^{-4}$														

N^{15}	O^{17}	O^{18}	$F^{18}-O^{18}$	F^{19}	F^{20}	Ne^{20}	Ne^{21}	Ne^{22}	Na^{23}	Na^{24}	Mg^{24}	Mg^{25}	Al^{27}	Al^{28}	Si^{28}	Si^{29}	Si^{30}	P^{31}	P^{32}	S^{32}	S^{33}	S^{34}	
			−3																				$n-p$
1	2	1		1	2	4	4	1															p
2	3	4		3	7	7	1	1		1													d
5				2	3	2		1									1						α
3	5	5		1	5	8	1	1															t
4	4	2	−1	−1	1	4		1															Li^6
6	9	6	−1	−2	2	8	1	1						1			1						Li^7
			3																				Be^7-Li^7
10	1	2	1	1	2	3		1									1	1	1	1	1	1	Be^8
10	2	3	−1	2	5	6	1	2	1	1			1	1			1		1	1	1	1	Be^9
11	6	6		−1	4	9	1	2	1	1			1	1		1	1	1	1	1	1	1	Be^{10}
11	6	10	1	−1	5	10	1	2	1	1			1	1		1	1	1	1	1	1	1	B^{10}
14	3	3			2	4	1	2					1	1		1	2	1	1	1	2	2	B^{11}
			1																				$C^{11}-B^{11}$
20	2	5		3	3	3			1	1	2	2	2	2	2	2	2	3	3	4	4	6	C^{12}
1	2	1		1	2	3													−1	−1	−1	−1	$C^{13}-C^{12}$
1	1	1	2	1	2	1																	$N^{13}-C^{13}$
																							$C^{14}-N^{14}$
10	3	3		2	4	4		1		1	1	1	1	1	1	1	1	1	1	1	2	2	N^{14}
111	3	6		4	7	6	1	1	1	1	1	2	2	2	2	2	3	3	3	3	4	5	N^{15}
3	119	6		25	23	5	1	1			1	1	1	1			1						O^{17}
6	6	310	−5	13	20	72	4	2	6	5	3	3	3	3	2	2	2	1	1	1	1	1	O^{18}
		−5	67			2																	$F^{18}-O^{18}$
4	25	13		138	127	25	4	2	3	3	3	3	3	3	1	2	4	1	1	1	1	1	F^{19}
7	23	20		127	360	34	4	2	3	3	3	3	3	3	1	2	4						F^{20}
6	5	72	2	25	34	134	8	5	9	8	6	6	5	4	2	3	3	1	1	1	1	1	Ne^{20}
8	8	40		45	40	83	19	10	10	9	7	7	6	5	3	3	4	1	1	1	1	1	Ne^{21}
12	6	20		25	20	47	10	40	5	5	4	4	4	3	2	2	2	1	1	1	1	1	Ne^{22}
9	3	60		31	30	89	10	5	27	25	17	18	15	12	6	8	9	4	3	3	3	3	Na^{23}
12	3	50		31	30	83	9	5	25	32	21	21	17	14	7	8	11	4	3	3	3	3	Na^{24}
14	9	30		30	30	58	7	4	17	21	35	31	30	24	12	14	18	7	6	6	6	6	Mg^{24}
16	9	30		30	30	61	7	4	18	21	31	45	29	23	12	14	18	8	7	7	7	7	Mg^{25}
18	8	30		29	30	53	6	4	15	17	30	29	34	26	13	16	20	8	7	7	7	7	Al^{27}
21	8	30		30	30	43	5	3	12	14	24	23	26	43	17	17	29	9	8	8	8	7	Al^{28}
25	1	20		13	15	22	3	2	6	7	12	12	13	17	31	19	17	13	11	11	11	9	Si^{28}
24	4	20		17	20	26	3	2	8	8	14	14	16	17	19	28	18	12	10	10	10	8	Si^{29}
24	9	20		40	40	35	4	2	9	11	18	18	20	29	17	18	54	9	8	8	8	7	Si^{30}
27		10		8	5	12	1	1	4	4	7	8	8	9	13	12	9	22	18	17	17	13	P^{31}
33		10		6		10	1	1	3	3	6	7	7	8	11	10	8	18	27	23	23	18	P^{32}
34		10		6		10	1	1	3	3	6	7	7	8	11	10	8	17	23	25	25	18	S^{32}
38	1	10		6		10	1	1	3	3	6	7	7	8	11	10	8	17	23	25	70	25	S^{33}
50		10		6		10	1	1	3	3	6	7	7	7	9	8	7	13	18	18	25	120	S^{34}
				$\times 10^{-4}$											$\times 10^{-3}$								

[a] The mathematical techniques used with the above matrix are described by S. S. Wilks, *Mathematical Statistics* (Princeton University Press, Princeton, 1943), especially sections 2.74 and 2.94, or any other book on multivariate analysis.

the error in the frequency value may have been underquoted.

A pleasant feature of this calculation is that so few of the experimental results examined are in conflict with the final values. It has been necessary to reject only a few results as being seriously inconsistent.

The corrections to experimental results were further classified according to type of experiment and it was found that the numerical values for the $d-p$ and $d-\alpha$ reaction Q values were in general slightly low and the probable errors in the $d-\alpha$ and mass spectrographic results appeared to be underquoted, but in no case were these deviations significant. Furthermore, a crude statistical test on Li's and Ogata's results seems to indicate that the agreement between these values is better than can normally be expected, so there is little point in using these results separately. It would however be useful for individual experimenters to compare their own results with the values quoted in Table III.

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Decays of Ta^{182} and Ta^{183}†*

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The β^- decays of Ta^{182} and Ta^{183} into excited states of W^{182} and W^{183} have been studied using a curved crystal gamma-ray diffraction spectrometer and a homogeneous field, ring focusing beta-ray spectrometer. In each case de-excitation of the daughter nucleus gives rise to complex gamma-ray and conversion electron spectra. Energies and relative intensities of gamma rays and conversion lines arising from 27 transitions in W^{182} and 29 transitions in W^{183} are presented. Internal conversion coefficients and multipolarities have been deduced for most of the transitions and together with the gamma-ray energies form the basis of decay schemes proposed for both W^{182} and W^{183}. The two decays are reported together because of the close experimental relationship which existed between them as a consequence of the method used for their production, namely, simultaneous production of Ta^{182} by single neutron capture and Ta^{183} by double neutron capture from stable Ta^{181}. A corollary result is the value 1.3×10^4 barns for the thermal neutron cross section of Ta^{182}.

An interpretation of these results on W^{182} in terms of collective rotational motion has been given by A. Bohr and collaborators [Kgl. Danske Videnscab. Selskab, Mat.-fys. Medd. **29**, No. 9 (1955)].

INTRODUCTION

IN regard to the decay of Ta^{182} this work is complementary to several earlier investigations,[1,2] its principal contributions being the investigation of a number of new gamma lines and the multipole assignments for most of the transitions, making possible the establishment of an energy level diagram for W^{182}. For Ta^{183} the results presented here provide the first detailed study of the decay of this isotope. They are in essential agreement with preliminary results[3] obtained earlier in this laboratory.

The Ta^{183} used in this experiment was produced from stable Ta^{181} by double neutron capture. This process was practical because of the unusually large thermal neutron cross section[4] of Ta^{182} together with the high neutron flux available in the Materials Testing Reactor at Arco where the irradiation was performed.[5] At the same time, of course, Ta^{182} was produced. The ratio of the populations, N_3 and N_2 of Ta^{183} and Ta^{182} which results after irradiation in a neutron flux, ψ, for a time, t, is (assuming the Ta^{181} population to be constant):

$$\frac{N_3}{N_2}=\frac{\sigma_2\psi}{\lambda_3}\left\{\frac{1-e^{-\lambda_3 t}-\lambda_3 t e^{-\frac{1}{2}(\lambda_3+\lambda_2+\sigma_2\psi)t}\left[\dfrac{\sinh\frac{1}{2}(\lambda_3-\lambda_2-\sigma_2\psi)t}{\frac{1}{2}(\lambda_3-\lambda_2-\sigma_2\psi)t}\right]}{1-e^{-(\lambda_2+\sigma_2\psi)t}}\right\}\rightarrow\sigma_2\psi/\lambda_3 \quad \text{as} \quad t\rightarrow\infty, \tag{1}$$

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[1] Muller, Hoyt, Klein, and DuMond, Phys. Rev. **88**, 775 (1952).

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[3] DuMond, Hoyt, Marmier, and Murray, Phys. Rev. **92**, 202 (1953).

[4] See Appendix I.

[5] We gratefully acknowledge the efforts of Dr. W. B. Lewis in arranging for irradiations in the Materials Testing Reactor, Arco, Idaho, under the auspices of the Phillips Petroleum Company.