where $C_{l,l+2}$ is as in Thaler, Bengston, and Breit and $Q_{l+1}(l)$, $Q_{l+1}(l+2)$ are modified by replacing them by $Q_{l+1}(l)+C_{ll}$, $Q_{l+1}(l+2)+C_{l+2,l+2}$. The matrix

$$(\mathrm{T})=\begin{pmatrix} Q_-+C_{--}, & C_{-+} \\ C_{+-}, & Q_++C_{++} \end{pmatrix}, \tag{11}$$

can then be related to (S) by

$$(\mathrm{T})=[(S)-1]/(2i), \tag{12}$$

and (S) is a general unitary symmetric matrix expressible by means of 3 parameters. The subscripts $-$ and $+$ are used to indicate the smaller and larger of the two L. The Q as used in Thaler, Bengston, and Breit have a generic meaning in connection with the special model used by them. In calculations, however, the diagonal elements of (T) enter instead in the equations for $\alpha_1, \cdots, \alpha_5$.

Substitution of Eqs. (2), (3)$\cdots$, (7) into Eq. (1) enables one to collect coefficients of a product involving two Q's such as Q_1, Q_2^*. These combine with terms in $Q_2Q_1^*$ which can be converted under the Im sign to the order $Q_1Q_2^*$ by taking the complex conjugate and changing the sign. For uncoupled terms,

$$\mathrm{Im}(Q_2Q_1^*)=\sin\delta_2\sin\delta_1\sin(\delta_2-\delta_1)$$

then gives readily the forms needed for numerical work.

In Eq. (1) the sums over L are supposed to be taken over odd values only. The formula can be used for the calculation of $(P\sigma)_{p-n}$ provided a coefficient $\frac{1}{4}$ is inserted on the right side, thus changing the 2 in front of $\sin\theta$ to $\frac{1}{2}$ and provided the sum is made to extend over odd and even L employing all triplet states. In both cases the singlet states do not enter.

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Formula for Polarization in *p-p* Scattering for *P* and *F* Waves*

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An expression for the calculation of polarization of protons produced by single scattering taking into account P and F waves has been worked out and is given below. Coupling between $^3P_2$ and $^3F_2$ is neglected, but otherwise the most general condition describable by a set of phase shifts for states of definite orbital as well as total angular momentum is considered. The result is written in a form convenient for numerical work.

If P is the polarization, defined as twice the expectation value of the y component of the spin,[1] then

$$\begin{aligned}k^2(P\boldsymbol{\sigma})_{p-p}&=(a+b\cos^2\theta+c\cos^4\theta)\sin\theta\cos\theta\cos\varphi\\&-2\,\mathrm{Im}\{\alpha_c^*[\tfrac{3}{2}e_{10}(Q_2(P)-Q_1(P))\\&-\tfrac{1}{8}e_{30}(-8Q_2(F)-7Q_3(F)+15Q_4(F))]\\&-\alpha_c[e_{10}(Q_2(P)-Q_0(P))^*-\tfrac{3}{2}e_{30}(Q_4(F)-Q_2(F))^*]\}\\&\times\sin\theta\cos\varphi-2\,\mathrm{Im}\{\alpha_c^*[\tfrac{5}{8}e_{30}(-8Q_2(F)-7Q_3(F)\\&+15Q_4(F))]-\alpha_c[(15/2)e_{30}(Q_4(F)-Q_2(F))^*]\}\\&\times\sin\theta\cos^2\theta\cos\varphi,\end{aligned} \tag{1}$$

where $\boldsymbol{\sigma}$ is the single scattering cross section, k the wave number of the incident protons. Other quantities are: $e_{LO}=\exp(2i\sigma_{LO})$, where σ_L is the Coulomb phase shift and $\sigma_{LO}=\sigma_L-\sigma_O$; $Q_J(L)=\exp(i\delta_J{}^L)\sin\delta_J{}^L$, where $\delta_J{}^L$ is the phase shift in a state of orbital angular momentum $L\hbar$ and total angular momentum $J\hbar$;

$$\alpha_c=(\eta/4)[-\mathbf{s}^{-2}\exp(-i\eta\ln\mathbf{s}^2)+\mathbf{c}^{-2}\exp(-i\eta\ln\mathbf{c}^2)], \tag{2}$$

where $\eta=e^2/\hbar v$, $\mathbf{s}=\sin(\theta/2)$, $\mathbf{c}=\cos(\theta/2)$, θ is the scattering angle in the center-of-mass system, v the relative velocity;

$$\begin{aligned}a=9\{&(P_1,P_2)+\tfrac{2}{3}(P_0,P_2)+(7/12)(F_3,F_2)\\&-(77/12)(F_4,F_2)-(35/24)(F_4,F_3)+(F_2,P_1,31)\\&+\tfrac{2}{3}(F_2,P_0,31)-(10/3)(F_2,P_2,31)-(7/12)(F_3,P_2,31)\\&+(5/3)(F_4,P_0,31)+(5/2)(F_4,P_1,31)\\&-(23/12)(F_4,P_2,31)\};\end{aligned} \tag{3}$$

$$\begin{aligned}b=(1/16)\{&-420(F_3,F_2)+6020(F_4,F_2)+1540(F_4,F_3)\\&+1200(F_2,P_2,31)-840(F_4,P_1,31)+420(F_3,P_2,31)\\&-220(F_4,P_2,31)-560(F_4,P_0,31)]\};\end{aligned} \tag{4}$$

$$c=(25/8)\{140(F_2,F_4)+49(F_3,F_4)\}, \tag{5}$$

where

$$\begin{aligned}(L_J,L_{J'}{}')&=\sin\delta_J{}^L\sin\delta_{J'}{}^{L'}\sin(\delta_J{}^L-\delta_{J'}{}^{L'}),\\(L_J,L_{J'}{}',31)&=\sin\delta_J{}^L\sin\delta_{J'}{}^{L'}\sin(\delta_J{}^L-\delta_{J'}{}^{L'}+2\sigma_{31}),\end{aligned} \tag{6}$$

and

$$\sigma_{LL'}=\sigma_L-\sigma_{L'},\quad\text{with}\quad\sigma_L-\sigma_{L-1}=\tan^{-1}(\eta/L).$$

This result agrees with that of Goldfarb and Feldman[1] up to second order in $\cos\theta$ if coupling is neglected in their work and Coulomb terms are neglected in Eq. (1). It has been checked throughout with the zero-coupling limit of the result of Breit and Ehrman.[2]

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