

introduction of separate energy minima, since so many other data seem to be satisfactorily accounted for on that model. The principal fact of observation remaining to be understood is the strange temperature dependence of mobilities in  $p$ -type materials. We have under way a calculation which attempts to explain this temperature dependence on the basis of the theory referred to above.

\* Details of the calculations described herein are contained in CML Technical Note No. CML-TN-P8, and are available on request from the Chicago Midway Laboratories, 6040 S. Greenwood Ave., Chicago 37, Illinois. This research was supported in whole by the U. S. Air Force.

<sup>1</sup> C. S. Smith, Phys. Rev. **94**, 42 (1954).

<sup>2</sup> F. Herman and J. Callaway, Phys. Rev. **89**, 518 (1953); B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954); Dresselhaus, Kip, and Kittel, Phys. Rev. **92**, 547 (1953); Lax, Zeiger, Dexter, and Rosenblum, Phys. Rev. **93**, 1418 (1954).

<sup>3</sup> Dresselhaus, Kip, and Kittel, Phys. Rev. **95**, 568 (1954); Dexter, Zeiger, and Lax, Phys. Rev. **95**, 557 (1954); Dexter and Lax (on  $p$ -Si), (to be published); A. Kahn, thesis, University of California, 1954 (unpublished).

<sup>4</sup> K. Fuchs and H. W. Peng, Proc. Roy. Soc. (London) **A180**, 451 (1942).

<sup>5</sup> Joel W. McClure, thesis, University of Chicago, 1954 (unpublished).

<sup>6</sup> W. Shockley, Phys. Rev. **78**, 173 (1950).

### Energies of $\pi^-$ Mesonic X-Ray $K$ Lines\*

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AS reported in a previous Letter,<sup>1</sup> we have studied the energies of the  $2p \rightarrow 1s$  transitions of the  $\pi^-$  mesonic x-rays using the method of critical absorption. The main purpose of this study is to search for possible shifts caused by a specifically nuclear interaction of the meson. A description of the experimental technique and of the method of calculation can be found in reference 1. The results are given below.

**Lithium.**—This is the lightest element whose mesonic x-rays can be detected with our equipment. Its  $2p \rightarrow 1s$  line (Table I) should fall between the  $K$  edges of Pd ( $Z=46$ , edge energy 24.35 kev)<sup>2</sup> and Ag ( $Z=47$ , edge energy 25.51 kev). Metallic foils of  $Z=42, 45, 46, 47, 48, 49$ , and 50 were used in the absorption experiments. The thickness of the foils corresponded to a transmission of 50 percent for x-rays just below the  $K$ -absorption edge. The resulting pulse-height distribution

TABLE I. Lithium and beryllium  $\pi$ -mesonic x-ray energies.

	Lithium $2p \rightarrow 1s$	Beryllium $2p \rightarrow 1s$
Klein-Gordon energy for $m_\pi = 272.5m_e$ (kev)	24.520	43.82
Vacuum polarization <sup>a</sup> (kev)	+0.096	+0.20
Finite nuclear size (for $r_0 = 1.2 \times 10^{-13}$ cm) (kev)	-0.033	-0.12
Computed energy (kev)	24.58	43.90
Experimental determination (kev)	23.22 < $E$ < 24.35	42.00 < $E$ < 43.57

<sup>a</sup> H. C. Corben and A. Mickelwait (private communication).

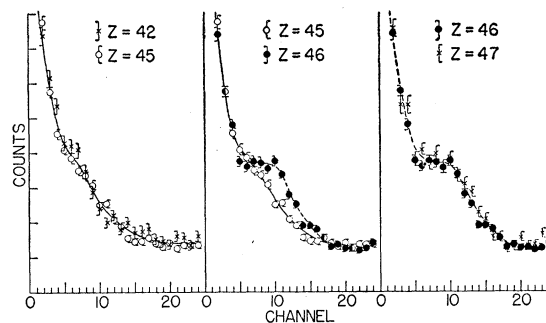


FIG. 1. Pulse-height distribution showing the transmission of the lithium  $2p \rightarrow 1s$   $\pi$ -mesonic line through absorbers  $Z=42, 45, 46$ , and 47. The errors indicated are standard statistical errors.

curves for absorbers close to the transmission discontinuity are plotted in Fig. 1 and show that the line under study lies between the edges of Rh ( $Z=45$ , edge energy 23.22 kev) and Pd. This corresponds to a shift of at least 230 ev and no more than 1360 ev toward lower energy.

**Beryllium.**—The absorbers used were thin-walled Lucite cells, 1 cm thick, containing an aqueous solution of a salt of the absorbing element. The concentration was such as to give a factor 4 in transmission above and below the  $K$  edge. The  $2p \rightarrow 1s$  line of Be (Table I) should fall between the edges of elements 60 (Nd, 43.57 kev) and 61 (Pm, 45.2 kev).

The curves obtained for elements 57, 58, 59, 60, and 62 are shown in Fig. 2. From this series and many other

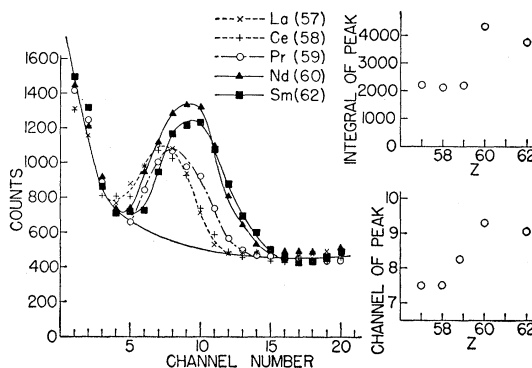


FIG. 2. Pulse-height distribution showing the transmission of the Be  $2p \rightarrow 1s$   $\pi$ -mesonic line through absorbers  $Z=57, 58, 59, 60$ , and 62. Also shown as a function of  $Z$  are the transmitted intensities and the positions of the peaks. This figure should be compared with Fig. 1 of reference 1.

similar runs it appears that the Be  $2p \rightarrow 1s$  transition definitely lies below the edge of Nd ( $Z=60$ ) and above the edge of Pr ( $Z=59, 42.00$  kev). This corresponds to an energy shift of at least 330 ev, and not more than 1900 ev toward lower energy. However, the line seems to lie closer to Pr than to Nd indicating a shift nearer to the larger value. Measurements of peak positions relative to known lines indicate a similar negative shift (see Fig. 3).

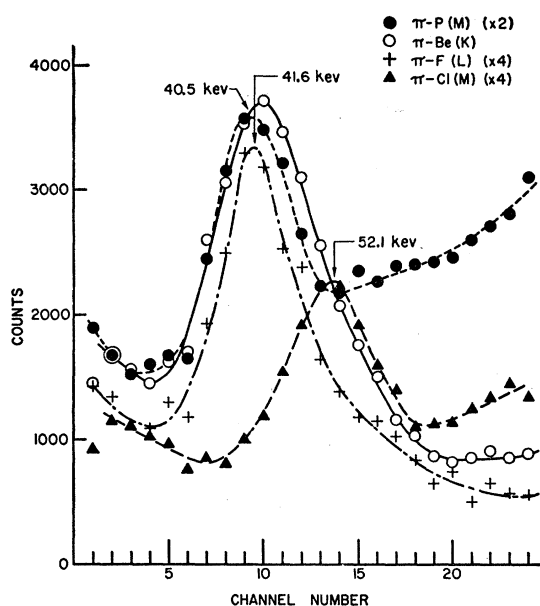


FIG. 3. Pulse-height distribution showing peak position for the Be  $2p \rightarrow 1s$   $\pi$ -mesonic line compared with other mesonic lines of known energy [P (M), 40.5 keV; F (L), 41.6 keV; Cl (M), 52.1 keV]. As a result of the comparison the Be energy is  $42.9 \pm 0.5$  keV corresponding to a negative shift of about  $1.0 \pm 0.5$  keV.

**Boron.**—The results obtained to date are not fully understood. Further studies with the separated isotopes will be made before reporting on this element.

**Carbon, nitrogen, oxygen, and fluorine.**—The  $K$ -mesonic lines of these elements cannot be studied with the critical absorption technique. We have measured their energies using the pulse-height selector and have found negative shifts varying roughly from 7 percent for carbon to about 13 percent for fluorine. The results of these measurements will be described in a later communication.

In conclusion we have observed that the  $2p \rightarrow 1s$  lines in the elements lithium to oxygen are shifted to lower energies with the possible exception of boron. This indicates an effective repulsive potential between the meson in the  $1s$  state and the nucleus.

\* Supported in part by the U. S. Atomic Energy Commission.

<sup>1</sup> Stearns, Stearns, DeBenedetti, and Leipuner, Phys. Rev. **95**, 1353 (1954); Stearns, Stearns, Leipuner, and DeBenedetti, Phys. Rev. **95**, 625(A) (1954).

<sup>2</sup> The  $K$ -absorption edge values were taken from a recent compilation of Lewis Slack, Naval Research Laboratory.

### Formula for Polarization in Nucleon-Nucleon Scattering\*

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THE importance assumed by experiments<sup>1</sup> on the polarization of nucleons makes it desirable to have available a general formula for the calculation of

polarization effects. The quantity under consideration is that usually denoted by  $P$  which is related to  $\bar{s}_y$ , the mean spin component normal to scattering plane in a single scattering experiment with unpolarized particles, by  $P = 2\bar{s}_y$ . An explicit formula in terms of the scattering matrix referred to weak-field spin states is available in several theoretical papers,<sup>2</sup> for example in Eq. (15) of Goldfarb and Feldman.<sup>2</sup> In principle, these formulas solve the problem but the work involved in the transition to special forms containing phase shifts is large at times. It appeared desirable therefore to reduce the general expressions to a point representing a compromise between compactness and explicitness. For  $p-p$  scattering,

$$k^2(P\sigma)_{p-p} = 2 \sin\theta \cos\varphi \operatorname{Im}\{\alpha_1(\alpha_2 + \alpha_c)^* - (1 - \mu^2)\alpha_1\alpha_3^* + \alpha_4^*(\alpha_5 + \alpha_c)\}, \quad (1)$$

where the colatitude and azimuthal angles  $\theta$ ,  $\varphi$  are as in Swanson's paper,  $k = 2\pi$  times reciprocal of wavelength,  $\operatorname{Im}$  indicates the imaginary part of expression in braces. The six quantities  $\alpha_1, \alpha_2, \dots, \alpha_c$  are as follows:

$$\alpha_1 = -\Sigma[L(L+2)Q_{L+1}(L) - (2L+1)Q_L(L) - (L^2-1)Q_{L-1}(L)][P_L'/L(L+1)]e_{L0} - 2B[(l+1)P_{l+2}' - (l+2)P_l'], \quad (2)$$

$$\alpha_2 = \Sigma \frac{1}{2} e_{L0} [(L+2)Q_{L+1}(L) + (2L+1)Q_L(L) + (L-1)Q_{L-1}(L)]P_L - (l+1)(l+2)B(P_{l+2} + P_l), \quad (3)$$

$$\alpha_3 = \Sigma \frac{1}{2} e_{L0} [LQ_{L+1}(L) - (2L+1)Q_L(L) + (L+1)Q_{L-1}(L)][P_L''/L(L+1)] - B(P_{l+2}'' + P_l''), \quad (4)$$

$$\alpha_4 = \Sigma e_{L0} [Q_{L+1}(L) - Q_{L-1}(L)]P_L' - 2B[(l+1)P_{l+2}' - (l+2)P_l'], \quad (5)$$

$$\alpha_5 = \Sigma e_{L0} [(L+1)Q_{L+1}(L) + LQ_{L-1}(L)]P_L + 2B(l+1)(l+2)(P_{l+2} + P_l), \quad (6)$$

$$\alpha_c = \frac{1}{4}\eta[-\mathbf{s}^{-2} \exp(-i\eta \ln \mathbf{s}^2) + \mathbf{c}^{-2} \exp(-i\eta \ln \mathbf{c}^2)], \quad (7)$$

where

$$Q_J(L) = [-1 + \exp(2i\delta_J^L)]/(2i), \quad (8)$$

$$e_{L0} = \exp(2i\sigma_{L0}) = \exp[2i(\tan^{-1}(\eta/L) + \tan^{-1}(\eta/(L-1)) + \dots + \tan^{-1}\eta)]. \quad (9)$$

$\delta_J^L$  is the phase shift for total angular momentum  $J\hbar$  and orbital angular momentum  $L\hbar$ , while the other symbols have the following meaning:  $\eta = e^2/\hbar v$ ,  $v$  = relative velocity;  $\mathbf{s} = \sin\theta/2$ ,  $\mathbf{c} = \cos\theta/2$ ,  $\theta$  = scattering angle in center of mass system;  $\mu = \cos\theta$ ,  $P_L = P_L(\mu)$ ,  $P_L' = dP_L/d\mu$ ,  $P_L'' = d^2P_L/d\mu^2$ . If there is no coupling between states of different  $L$  for the same  $J$  the constant  $B=0$ . If states with  $J=l+1$  having values  $L=l$  and  $L=l+2$  are coupled to each other, then

$$B = \frac{1}{2} C_{l, l+2} (l+1)^{-\frac{1}{2}} (l+2)^{-\frac{1}{2}} \exp[i\sigma_{L,0} + i\sigma_{l+2,0}], \quad (10)$$