To obtain agreement with experiment one requires

$$|\psi(0)|^2_{\text{exptl}} = 0.44\times10^{24}\ \text{cm}^{-3}. \quad (4)$$

From an approximate solution of the effective mass equation we find $|F^{(1)}(0)|^2 = (0.000037\pm10\%)\times10^{24}$ cm^{-3}, while an estimate of $|\psi(\mathbf{k}^{(i)},0)|^2$, if we use the tight-binding approximation, gives 370 within a factor of about 2. When substituted into (3), these values give

$$(|\psi(0)|^2)_{\text{eff. mass}} \approx 0.082\times10^{24}\ \text{cm}^{-3}. \quad (5)$$

When the ionization energy is calculated by the effective mass formalism one finds 0.028 ev, in rather serious disagreement with the observed 0.044 ev.[4] We have made rough allowance for the breakdown of the effective mass theory near the impurity atom. The resulting wave function, when the experimental binding energy is used, is rather larger in the last cell, and results in an increase of (5) by a factor of about 13. The final value is

$$(|\psi(0)|^2)_{\text{theor}} \approx 1.1\times10^{24}\ \text{cm}^{-3}. \quad (6)$$

The fact that (6) agrees with the experimental value [Eq. (4)], well within the uncertainty of the calculation, lends support to the picture that the observed resonances are associated with the usual donor states.

This work was carried out while we were guests of the Bell Telephone Laboratories, and we would like to thank the staff for their cooperation and friendliness.

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[1] R. C. Fletcher *et al.*, Phys. Rev. **94**, 1392 (1954); **95**, 844 (1954).
[2] C. Smith, Phys. Rev. **94**, 42 (1954); B. Lax and co-workers (private communication).
[3] B. Lax and co-workers (private communication).
[4] F. J. Morin *et al.*, Phys. Rev. **96**, 833 (1954).

Elastoresistance in *p*-Type Ge and Si*

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Smith has reported[1] a large elastoresistance effect in Ge and Si of both *n*- and *p*-type. In *n*-type material there is an accumulation of evidence to support the explanation that the large elastoresistance is brought about by a strain-induced transfer of electrons between nonequivalent anisotropic energy minima in the conduction band.[2] However, no such electron transfer mechanism seems capable of explaining the large elastoresistance of *p*-type material, since the spin-orbit coupling model[3] now seems firmly established, and this model predicts that the two nearly degenerate surfaces in the valence band have cubic symmetry individually.

We believe that the large elastoresistance of *p*-type specimens originates in a strain-induced mixing of the nearly degenerate bands and a consequent strong warping of the energy surfaces. We have carried through a formal theory of this process using the methods of Fuchs and Peng,[4] McClure,[5] and Shockley,[6] and assuming the spin-orbit coupling model. Our result is that to first order in the strain the energy states in the strained lattice are given by

$$E(\mathbf{p}) = (A+2B)p^2/3 \pm R + \mathbf{p}\cdot\boldsymbol{\epsilon}\cdot\mathbf{p}/m - \mathbf{p}\cdot\boldsymbol{\epsilon}\cdot\nabla_p E(\mathbf{p}) + (E_{11}+2E_{12})\epsilon/3 \pm \Delta R, \quad (1)$$

with

$$R^2 \equiv (A-B)^2p^4/9 + [C^2-(A-B)^2][p_x^2p_y^2+p_y^2p_z^2+p_z^2p_x^2]/3, \quad (2)$$

$$\Delta R = (1/R)\{CE_{44}[p_xp_y\epsilon_{xy}+p_yp_z\epsilon_{yz}+p_zp_x\epsilon_{zx}]/3 + (A-B)(E_{11}-E_{12})[(3p_x^2-p^2)(3\epsilon_{xx}-\epsilon)+(3p_y^2-p^2) \times(3\epsilon_{yy}-\epsilon)+(3p_z^2-p^2)(3\epsilon_{zz}-\epsilon)]/27\}. \quad (3)$$

The + and − signs refer to the energy surfaces for the carriers of small and large mass, respectively. In the above equations $\boldsymbol{\epsilon}$ denotes the tensor of strain, ϵ its trace. The constants E_{11}, E_{12}, and E_{44} are three unknown matrix elements which enter the theory. Their designations are chosen to emphasize that they transform like the components of a fourth rank tensor under cubic symmetry operations. The E_{ij} are expected to have the order of magnitude of electron volts, so the terms in (1) proportional to the E_{ij} are by far the largest for the states responsible for semiconduction properties.

A theory of the elastoresistance based on the above expressions for the energy levels shows that in the notation of reference 1, the volume elastoresistance coefficient $(M_{11}+2M_{12})/3$ is small compared to M_{44}, in agreement with experiment. Of the other two principal coefficients, M_{44} is proportional to CE_{44}/kT and $(M_{11}+M_{12}/2)$ is proportional to $(A-B)(E_{11}-E_{12})/2kT$ as might be expected. Experimentally, $(M_{11}-M_{12})/2$ is small compared to M_{44}. This we must interpret as implying that $(E_{11}-E_{12})/2$ is small compared to E_{44}, since the parameters $(A-B)$ and C as determined from the cyclotron resonance experiment have the same order of magnitude.

Our formal theory does not suggest why E_{44} is relatively so large. However, we believe the physical reason is not difficult to conjecture. Essentially E_{44} is a matrix element of the strain-induced incremental electron potential for a shear which changes the angles between the cubic axes while $(E_{11}-E_{12})/2$ is a matrix element of the potential for a strain which does not change the angle. To first order in $\boldsymbol{\epsilon}$ only the former type of strain can cause a change in the nearest neighbor distances, and we suspect that that is the underlying physical reason why E_{44} is relatively so large.

We think it encouraging that the spin-orbit coupling model is capable of giving a satisfactory account of the large elastoresistance in *p*-type material without the

introduction of separate energy minima, since so many other data seem to be satisfactorily accounted for on that model. The principal fact of observation remaining to be understood is the strange temperature dependence of mobilities in p-type materials. We have under way a calculation which attempts to explain this temperature dependence on the basis of the theory referred to above.

* Details of the calculations described herein are contained in CML Technical Note No. CML-TN-P8, and are available on request from the Chicago Midway Laboratories, 6040 S. Greenwood Ave., Chicago 37, Illinois. This research was supported in whole by the U. S. Air Force.

[1] C. S. Smith, Phys. Rev. **94**, 42 (1954).
[2] F. Herman and J. Callaway, Phys. Rev. **89**, 518 (1953); B. Abeles and S. Meiboom, Phys. Rev. **95**, 31 (1954); Dresselhaus, Kip, and Kittel, Phys. Rev. **92**, 547 (1953); Lax, Zeiger, Dexter, and Rosenblum, Phys. Rev. **93**, 1418 (1954).
[3] Dresselhaus, Kip, and Kittel, Phys. Rev. **95**, 568 (1954); Dexter, Zeiger, and Lax, Phys. Rev. **95**, 557 (1954); Dexter and Lax (on p—Si), (to be published); A. Kahn, thesis, University of California, 1954 (unpublished).
[4] K. Fuchs and H. W. Peng, Proc. Roy. Soc. (London) **A180**, 451 (1942).
[5] Joel W. McClure, thesis, University of Chicago, 1954 (unpublished).
[6] W. Shockley, Phys. Rev. **78**, 173 (1950).

Energies of π^- Mesonic X-Ray K Lines*

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As reported in a previous Letter,[1] we have studied the energies of the $2p \rightarrow 1s$ transitions of the π^- mesonic x-rays using the method of critical absorption. The main purpose of this study is to search for possible shifts caused by a specifically nuclear interaction of the meson. A description of the experimental technique and of the method of calculation can be found in reference 1. The results are given below.

Lithium.—This is the lightest element whose mesonic x-rays can be detected with our equipment. Its $2p \rightarrow 1s$ line (Table I) should fall between the K edges of Pd ($Z=46$, edge energy 24.35 kev)[2] and Ag ($Z=47$, edge energy 25.51 kev). Metallic foils of $Z=42$, 45, 46, 47, 48, 49, and 50 were used in the absorption experiments. The thickness of the foils corresponded to a transmission of 50 percent for x-rays just below the K-absorption edge. The resulting pulse-height distribution

TABLE I. Lithium and beryllium π-mesonic x-ray energies.

	Lithium $2p \rightarrow 1s$	Beryllium $2p \rightarrow 1s$
Klein-Gordon energy for $m_\pi = 272.5 m_e$ (kev)	24.520	43.82
Vacuum polarization[a] (kev)	+0.096	+0.20
Finite nuclear size (for $r_0 = 1.2 \times 10^{-13}$ cm) (kev)	−0.033	−0.12
Computed energy (kev)	24.58	43.90
Experimental determination (kev)	$23.22 < E < 24.35$	$42.00 < E < 43.57$

[a] H. C. Corben and A. Mickelwait (private communication).

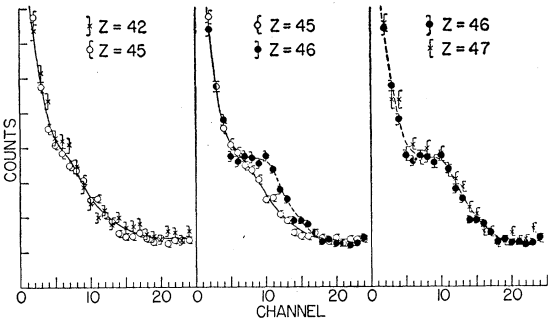

FIG. 1. Pulse-height distribution showing the transmission of the lithium $2p \rightarrow 1s$ π-mesonic line through absorbers $Z=42$, 45, 46, and 47. The errors indicated are standard statistical errors.

curves for absorbers close to the transmission discontinuity are plotted in Fig. 1 and show that the line under study lies between the edges of Rh ($Z=45$, edge energy 23.22 kev) and Pd. This corresponds to a shift of at least 230 ev and no more than 1360 ev toward lower energy.

Beryllium.—The absorbers used were thin-walled Lucite cells, 1 cm thick, containing an aqueous solution of a salt of the absorbing element. The concentration was such as to give a factor 4 in transmission above and below the K edge. The $2p \rightarrow 1s$ line of Be (Table I) should fall between the edges of elements 60 (Nd, 43.57 kev) and 61 (Pm, 45.2 kev).

The curves obtained for elements 57, 58, 59, 60, and 62 are shown in Fig. 2. From this series and many other

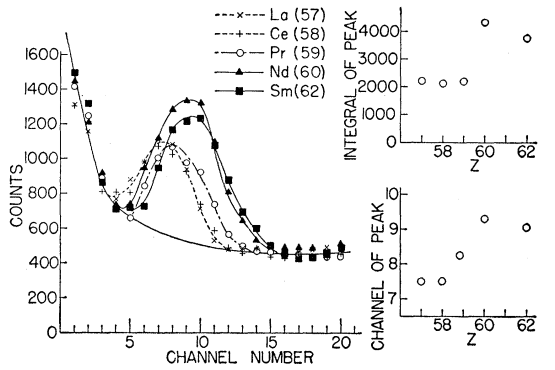

FIG. 2. Pulse-height distribution showing the transmission of the Be $2p \rightarrow 1s$ π-mesonic line through absorbers $Z=57$, 58, 59, 60, and 62. Also shown as a function of Z are the transmitted intensities and the positions of the peaks. This figure should be compared with Fig. 1 of reference 1.

similar runs it appears that the Be $2p \rightarrow 1s$ transition definitely lies below the edge of Nd ($Z=60$) and above the edge of Pr ($Z=59$, 42.00 kev). This corresponds to an energy shift of at least 330 ev, and not more than 1900 ev toward lower energy. However, the line seems to lie closer to Pr than to Nd indicating a shift nearer to the larger value. Measurements of peak positions relative to known lines indicate a similar negative shift (see Fig. 3).