The first term within the bracket arises from the nucleon self-energy, and dominates for large $\mathbf{p}$. $F^2/4\pi$ is clearly negative for $\mathbf{p}$ large enough.[9] For $\Delta=0$ it is by definition positive and equal to $G^2/4\pi$. Therefore, for some $\mathbf{p}=\mathbf{k}$ for which $\Delta<0$ $F^2/4\pi$ has a pole. A pole in $F^2/4\pi$ implies a pole in the wave function $a(\mathbf{p}u, -\mathbf{p}\alpha)$ or, in other words, an incoming or outgoing wave of momentum $\mathbf{k}$ in the coordinate-space wave function. It would be possible to interpret the pole as an inelastic scattering if $2|\mathbf{k}|<\epsilon$, but a rough numerical calculation shows this condition not to be fulfilled for laboratory energies (values of ϵ from M to $2M$). If $2|\mathbf{k}|>\epsilon$ an interpretation as inelastic scattering is not possible, because there is too much momentum for the fixed amount of energy in the system, and at least one of the scattered particles would have a spacelike energy-momentum four vector, or an imaginary mass. No reconciliation oft his nonsensical prediction with reality has yet been made, and at present it must be considered as raising serious doubt about the consistency of the Tamm-Dancoff approximation.

The author would like to express his gratitude to F. J. Dyson for suggesting this problem, for essential guidance, and for helpful discussions.

[9] R. H. Dalitz (private communication) has pointed out an error made by the author in the evaluation of these terms which radically altered their behavior. His information has prevented the possible publication of qualitatively wrong results, and is greatly appreciated.

PHYSICAL REVIEW VOLUME 96, NUMBER 3 NOVEMBER 1, 1954

Mesonic Corrections to the Quadrupole Moment of the Deuteron*

A. M. SESSLER†
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York

(Received June 10, 1954)

Mesonic corrections to the quadrupole moment of the deuteron are calculated by means of the method of Tamm and Dancoff. Only the two-nucleon and two-nucleon—one-meson amplitudes are included. The first is associated with a phenomenological wave function for the deuteron, the second yielding a correction to the quadrupole moment. A correction exists even in neutral scalar theory, and for pseudoscalar (pseudoscalar) symmetric theory the result in adiabatic approximation to second order is $\Delta Q=3.1$ percent or $\Delta Q=-0.7$ percent, depending on choice of wave functions (assuming $g^2/4\pi=10$). Contributions from multiple meson amplitudes are examined, and for a hard-core deuteron function they are shown to contribute only slightly.

I. INTRODUCTION

A PHENOMENOLOGICAL theory of the deuteron consists of choosing an arbitrary potential which is to act between two nucleons and only depends explicitly on nucleon variables, and then calculating the binding energy of the deuteron, the quadrupole moment, the n-p triplet scattering length, and the n-p triplet effective range. These quantities are then compared with experiment, and if they all agree, within experimental error, then the potential is an acceptable one.

The phenomenological theory is not complete in that it ignores the coordinates of the mesons associated with the two-nucleon system. It does not, however, completely ignore the mesons, in so far as they contribute to the potential acting between nucleons. The question arises as to how valid the phenomenological theory is, or, equivalently, how much of a correction will the inclusion of meson coordinates make in which potentials are considered acceptable.

It is not known how to obtain from meson theory a quantitatively correct answer to this question. However, as we shall see, the only one of the above mentioned quantities we need calculate with meson theory is the quadrupole moment of the deuteron. Because this is an outside quantity one should be able to obtain a very reasonable estimate of the magnitude of the effect, and in view of the rather large estimates which appear in the literature,[1,2] it was felt that a re-examination of the problem was necessary.

The results of a field-theoretic calculation of the two-nucleon system may be expressed in terms of a wave functional in Fock space. The method of Tamm[3] and Dancoff,[4] in lowest approximation, consists of equating to zero all amplitudes of the functional other than the two-nucleon (2,0), and two-nucleon—one-meson (2,1) amplitudes. We shall make this lowest approximation first, and examine its validity in Sec. IV.

We may associate the (2,0) amplitude with a phenomenological wave function. It satisfies a Schroedinger equation in which the potential term arises from the amplitudes involving mesons, but does not contain meson coordinates. Furthermore, the binding energy of the deuteron, and the n-p scattering length, and the n-p effective range may be obtained from the (2,0) amplitude alone. The quadrupole moment differs in that it

* Supported in part by the U. S. Office of Naval Research.
† National Science Foundation Post-Doctoral Fellow.

[1] F. Villars, Phys. Rev. 86, 476 (1952).
[2] S. Deser, Phys. Rev. 92, 1542 (1953).
[3] I. Tamm, J. Phys. (U.S.S.R.) 9, 449 (1945).
[4] S. M. Dancoff, Phys. Rev. 78, 382 (1950).

receives a direct contribution from the charge density of the (2,1) amplitude, and in that it depends on the normalization of the (2,0) amplitude. This normalization does not otherwise enter in the identification of the (2,0) amplitude with the phenomenological wave function.

We shall calculate the contribution arising from the (2,1) amplitude taking into account its effect on the normalization of the (2,0) amplitude. This contribution must be regarded as a correction which should be applied to the experimental quadrupole moment before it is used as input data in a phenomenological theory. The resulting phenomenological theory may then be considered as associated with the (2,0) amplitude in a Fock space representation of the field-theoretic solution of the two-nucleon problem.

II. NEUTRAL SCALAR THEORY

We shall first, for the purpose of giving a simple exposition, carry through the calculation using neutral scalar theory. The notation is the same as that of Lévy,[5] and $\hbar=c=M=1$. The (2,1) amplitude can be obtained from the (2,0) amplitude by the following equation:

$$[E-E_{p1}-E_{p2}-\omega(k)]a(\mathbf{p}_1,\mathbf{p}_2;\mathbf{k})$$
$$=g(2\pi)^{-\frac{3}{2}}[2\omega(k)]^{-\frac{1}{2}}[a(\mathbf{p}_1+\mathbf{k},\mathbf{p}_2)+a(\mathbf{p}_1,\mathbf{p}_2+\mathbf{k})]. \quad (1)$$

The program is to associate $a(\mathbf{p}_1,\mathbf{p}_2)$ with the Fourier transform of a phenomenological wave function $\psi(\mathbf{x}_1,\mathbf{x}_2)$ and believe meson theory only to the extent of Eq. (1), which gives $a(\mathbf{p}_1,\mathbf{p}_2;\mathbf{k})$ in terms of $a(\mathbf{p}_1,\mathbf{p}_2)$. Let $a(\mathbf{x}_1,\mathbf{x}_2;\mathbf{s})$ be the Fourier transform of $a(\mathbf{p}_1,\mathbf{p}_2;\mathbf{k})$. Then $|a(\mathbf{x}_1,\mathbf{x}_2;\mathbf{s})|^2$ is the probability of locating a meson at $\mathbf{s}$, and nucleons at $\mathbf{x}_1$ and $\mathbf{x}_2$. Since we are calculating in a neutral theory, the only contribution to the quadrupole moment comes from

$$\rho(\mathbf{x}_2)=\int|a(\mathbf{x}_1,\mathbf{x}_2;\mathbf{s})|^2d\mathbf{x}_1d\mathbf{s}, \quad (2)$$

which is the charge density in the state in which there are two nucleons and one meson.

We make the adiabatic approximation of setting $E-E_{p1}-E_{p2}=0$. This yields

$$\rho(\mathbf{x}_2)=\frac{g^2}{2(2\pi)^6}[\rho_1+\rho_2+\rho_3+\rho_4], \quad (3)$$

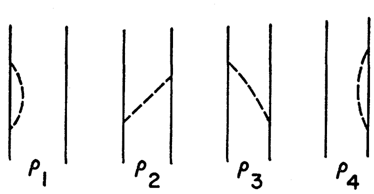
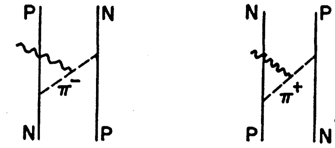


FIG. 1. Graphs contributing to the charge density in the (2,1) state.

[5] M. M. Lévy, Phys. Rev. 88, 72 (1952).

FIG. 2. Meson current contributions to the quadrupole moment.



where

$$\rho_i(\mathbf{x}_2)=\int d\mathbf{p}_1d\mathbf{p}_2d\mathbf{p}_2'd\mathbf{k}\frac{e^{i(\mathbf{p}_2-\mathbf{p}_2')\cdot\mathbf{x}_2}}{\omega^3(k)}K_i,$$
$$K_1=a(\mathbf{p}_1+\mathbf{k},\mathbf{p}_2)a^*(\mathbf{p}_1+\mathbf{k},\mathbf{p}_2'),$$
$$K_2=a(\mathbf{p}_1+\mathbf{k},\mathbf{p}_2)a^*(\mathbf{p}_1,\mathbf{p}_2'+\mathbf{k}), \quad (4)$$
$$K_3=a(\mathbf{p}_1,\mathbf{p}_2+\mathbf{k})a^*(\mathbf{p}_1+\mathbf{k},\mathbf{p}_2'),$$
$$K_4=a(\mathbf{p}_1,\mathbf{p}_2+\mathbf{k})a^*(\mathbf{p}_1,\mathbf{p}_2'+\mathbf{k}).$$

Graphs may be drawn corresponding to ρ_1, ρ_2, ρ_3, and ρ_4. They appear in Fig. 1, from which it is clear that the logarithmically divergent terms ρ_1 and ρ_4 are self-terms and must in the spirit of noncovariant renormalization be dropped. They correspond to a meson cloud associated with each nucleon, and the finite contribution of these terms may be estimated phenomenologically.[6]

Combining the above we obtain

$$\rho(\mathbf{x}_2)=g^2(2\pi)^{-3}\int d\mathbf{y}|\psi(\mathbf{y},\mathbf{x}_2)|^2I(|\mathbf{x}_2-\mathbf{y}|), \quad (5)$$

where

$$I(|\mathbf{x}_2-\mathbf{y}|)=\int d\mathbf{k}\omega^{-3}(\mathbf{k})e^{i\mathbf{k}\cdot(\mathbf{x}_2-\mathbf{y})}. \quad (6)$$

Transforming to the center of mass, and relative coordinates $\mathbf{r}$, and evaluating $I(|\mathbf{x}_2-\mathbf{y}|)$ by the method of Lévy,[7] we obtain

$$\rho(\mathbf{r})=g^2(2\pi^2)^{-1}|\Phi(r)|^2K_0(\mu r), \quad (7)$$

where $\Phi(r)$ is the deuteron wave function in relative coordinates.

The contribution to the quadrupole moment is given by

$$\Delta Q=\Delta Q'-NQ_{\text{phen}}, \quad (8)$$

where

$$\Delta Q'=\tfrac{1}{2}\int\rho(\mathbf{r})r^2P_2(\cos\theta)d\mathbf{r},\quad N=\int\rho(\mathbf{r})d\mathbf{r}. \quad (9)$$

The first term being a direct contribution from the (2,1) state, the second term coming from a change in normalization of the (2,0) state. The two terms tend to cancel,

[6] The electron-neutron interaction measures $I=\int r^2\rho(r)$, where $\rho(r)$ is the charge density of a neutron, and can be expressed as $I<e(\hbar/\mu c)^2(\mu/M)^2$. This leads to a correction to the quadrupole moment of the order of

$$\left[\frac{\hbar}{Mc}\frac{1}{R}\right]^2,$$

where R is the radius of the deuteron. This is less than 0.3 percent, and negligible.

[7] M. M. Lévy, Phys. Rev. 88, 738 (1952), see Appendix.

TABLE I. Contributions to ΔQ for two-deuteron functions. In pseudoscalar theory $g^2/4\pi$ has been chosen equal to 10.

		Yukawa-Yukawa			Hard core		
		$\Delta Q'$	$-NQ_{\text{phen}}$	ΔQ	$\Delta Q'$	$-NQ_{\text{phen}}$	ΔQ
Scalar (2,1)		$0.467(g^2/2\pi^2)$	$-1.54(g^2/2\pi^2)$	$-1.07(g^2/2\pi^2)$ $-1.9\%\ Q_{\text{exp}}$			
Pseudo-scalar (2,1)	$s-s$	$0.03114(g^2/4\pi)$	$0.01578(g^2/4\pi)$		$0.02953(g^2/4\pi)$	$0.00183(g^2/4\pi)$	
	$s-d$	-0.01002	-0.02387		-0.01255	-0.03230	
	$d-d$	0.00245	0.00359		0.00431	0.00478	
Total				$0.0191(g^2/4\pi)$ $3.1\%\ Q_{\text{exp}}$			$-0.0044(g^2/4\pi)$ $-0.7\%\ Q_{\text{exp}}$
Pseudo-scalar (2,2)		$2.56\times10^{-4}(g^2/4\pi)^2$	$-67.7\times10^{-4}(g^2/4\pi)^2$	$-10.5\%\ Q_{\text{exp}}$	$2.82\times10^{-4}(g^2/4\pi)^2$	$-9.80\times10^{-4}(g^2/4\pi)^2$	$-1.1\%\ Q_{\text{exp}}$

and would give zero if the K_0 function were absent. Since the K_0 function weights small r, the normalization term dominates and ΔQ is negative. Simply for comparison purposes, a numerical calculation was performed, the results for a reasonable value of $(g^2/4\pi)$ being listed in Table I.

III. SYMMETRIC PSEUDOSCALAR THEORY

The calculation proceeds exactly as in scalar theory, except for a few minor changes, Eq. (1) becomes

$$\begin{aligned}[E-E_{p1}-E_{p2}-\omega(k)]a(\mathbf{p}_1,\mathbf{p}_2;\mathbf{k}) &= g\Lambda_a{}^+\Lambda_b{}^+(2\pi)^{-\frac{3}{2}}[2\omega(k)]^{-\frac{1}{2}} \\ &\times[\tau_i{}^a\gamma_5{}^a a(\mathbf{p}_1+\mathbf{k},\mathbf{p}_2)+\tau_i{}^b\gamma_5{}^b a(\mathbf{p}_1,\mathbf{p}_2+\mathbf{k})]. \quad (10)\end{aligned}$$

We need still calculate only $\rho(\mathbf{k}_2)$ since the contribution of meson currents vanishes[8] (see Fig. 2). The adiabatic approximation $E-E_{p1}-E_{p2}=0$ is again made, and because of the presence of γ_5 we must use a two component reduction approximation for the deuteron function. This is equivalent to calculating with a pseudovector theory, since pair terms do not contribute to this order. We obtain

$$\rho(\mathbf{r})=-\frac{g^2}{2\pi^2}\frac{\boldsymbol{\tau}^a\cdot\boldsymbol{\tau}^b}{(2M)^2}|\Phi(\mathbf{r})|^2(\boldsymbol{\sigma}^a\cdot\nabla)(\boldsymbol{\sigma}^b\cdot\nabla)K_0(\mu r). \quad (11)$$

The deuteron function $\Phi(\mathbf{r})$ for $m=1$, may be written as

$$\Phi(\mathbf{r})=\frac{1}{\sqrt{(4\pi)}}\varphi(r)X_1+\psi(r)Y_D, \quad (12)$$

where φ and ψ are radially normalized and

$$Y_0=\left(\frac{2}{20}\right)^{\frac{1}{2}}X_1Y_{20}+\left(\frac{6}{20}\right)^{\frac{1}{2}}X_0Y_{21}+\left(\frac{12}{20}\right)^{\frac{1}{2}}X_{-1}Y_{22}. \quad (13)$$

The quadrupole moment correction is given by Eq. (8), the evaluation of which is rather involved due to the more complicated nature of $\rho(\mathbf{r})$. Separating terms corresponding to the S and D states of the deuteron, we obtain

$$\Delta Q_S'=\frac{g^2}{8\pi^2M^2\mu^3}\left(\frac{1}{5}\right)\times\int_0^\infty\varphi^2(x/\mu)x^4dx\left[K_0(x)+\frac{2K_1(x)}{x}\right],$$

$$\Delta Q_{SD}'=-\frac{g^2}{8\pi^2M^2\mu^3}\left(\frac{1}{5\sqrt{2}}\right)\int_0^\infty\varphi(x/\mu)\times\psi(x/\mu)x^4dx\left[K_0(x)+\frac{5K_1(x)}{x}\right],$$

$$\Delta Q_D'=\frac{g^2}{8\pi^2M^2\mu^3}\left(\frac{1}{20}\right)\times\int_0^\infty\psi^2(x/\mu)x^4dx\left[5K_0(x)+\frac{13K_1(x)}{x}\right], \quad (14)$$

$$N_S=\frac{g^2}{8\pi^2M^2\mu}\int_0^\infty\varphi^2(x/\mu)x^2dx\left[K_0(x)-\frac{K_1(x)}{x}\right],$$

$$N_{SD}=\frac{g^2}{8\pi^2M^2\mu}(4\sqrt{2})\int_0^\infty\varphi(x/\mu)\times\psi(x/\mu)x^2dx\left[K_0(x)+\frac{2K_1(x)}{x}\right],$$

$$N_D=-\frac{g^2}{8\pi^2M^2\mu}\times\int_0^\infty\psi^2(x/\mu)x^2dx\left[K_0(x)+\frac{5K_1(x)}{x}\right].$$

The evaluation may not be performed using zero-range functions due to the singularity in N_S. Any function corresponding to a nonzero-range force gives a finite result. We have carried through the numerical work

[8] H. Miyazawa, Progr. Theoret. Phys. (Japan) 7, 207 (1952).

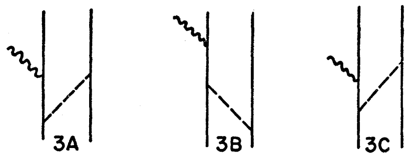


FIG. 3. Typical diagrams in g^2 order.

using two functions.[9] The first is a solution of a central and tensor Yukawa well potential, and the second is from a similar potential with a hard core at $r=0.375\hbar/\mu c$. Both functions fit the triplet data. The results are given in Table I.

We have not calculated the self-terms by means of a renormalization program. It seems probable that these terms may be sufficiently accurately estimated phenomenologically.[6]

IV. TWO-MESON AMPLITUDE TERMS

In pseudoscalar theory one might think the fourth-order terms are much larger than the previously calculated terms. Because the quadrupole moment is an outside effect and the multiple-meson effects are concentrated at small distances, this is not so. The leading terms are the two-pair terms,[7] and we shall calculate the charge density associated with the (2,2) amplitude. A straightforward calculation yields

$$\rho(\mathbf{r})=g^4(4M)^{-2}(2\pi)^{-6}\tau_\lambda{}^a\tau_\mu{}^a[\tau_\lambda{}^b\tau_\mu{}^b+\tau_\mu{}^b\tau_\lambda{}^b]|\Phi(r)|^2J(r), \quad (15)$$

where

$$J(r)=\int\int\frac{d\mathbf{k}d\mathbf{q}e^{i(\mathbf{k}+\mathbf{q})\cdot\mathbf{r}}}{\omega(k)\omega(q)[\omega(k)+\omega(q)]^2}. \quad (16)$$

$J(r)$ may be easily evaluated,[7] and the results are to be found in Table I. Because of the short range and the singularity of the operators, the results are very sensitive to the inner regions of the wave function. Very little is known about the deuteron function in this region, but a hard core type of behavior seems probable. If we assume a hard core, none of the other multiple-meson amplitudes contribute appreciably.

[9] Dr. M. Kalos has been kind enough to supply us with these functions.

V. CONCLUSIONS

Using pseudoscalar theory, and including the (2,0) and (2,1) amplitudes, we find the nonphenomenologic contribution to the quadrupole moment is 3.1 percent and -0.7 percent for Yukawa well and hard-core functions, respectively (this assumes $g^2/4\pi=10$). The plus sign is such as to imply a decrease in the percentage of D state of a phenomenological function. With a hard-core function, modification due to multiple-meson amplitudes is small, and one is led to believe that the whole effect is probably of the order of one percent. With Yukawa functions, which is a rather extreme case, multiple-meson amplitudes contribute appreciably, and a definite statement cannot be made.

One should compare this with the four-dimensional calculations of Villars[1] and Deser.[2] Deser breaks the interaction between the nucleons into an instantaneous and a retarded part. He associates the solution obtained with only the instantaneous interaction, with a phenomenological theory. He then calculates the effect of the retarded part and considers this as the correction to the quadrupole moment. Actually most of this is included in a phenomenological theory, for if we take the one four-dimensional diagram Fig. 3A and break it up into time-ordered diagrams, then clearly diagrams where no meson is in flight during the interaction with the photon, like Fig. 3B, are included in the phenomenological theory, while only Fig. 3C should be considered a true correction to the quadrupole moment. In addition to this, differences in the numbers are due to Deser's use of zero-range functions which, being singular at the origin, give a much larger contribution. Neither Deser nor Villars has included the change in normalization of the wave function, which tends to decrease the effect.

I wish to thank Professor H. A. Bethe, Dr. N. Austern, Dr. A. Klein, and Dr. R. H. Dalitz for many enlightening conversations.