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Resonance Scattering of Gamma Rays by Nuclei

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Two methods of observing the nuclear resonance scattering of gamma rays are discussed: one using gamma rays which arise from transitions to the ground state of a nucleus which is the same as the one in which the scattering is being observed, and the other using gamma rays of small energy spread and variable mean energy obtained by means of Compton scattering.

1. INTRODUCTION

INFORMATION about the widths of gamma-ray emitting states may be obtained by delayed coincidence methods if the lifetimes of the states are greater than about 10^{-10} second, for in this case the lifetime may be measured directly and hence the width may be deduced. Because of the limitations of present-day coincidence circuits, this method will not work for lifetimes much shorter than 10^{-10} second, and the widths have to be inferred either indirectly, from reactions involving particles, or directly, by measuring the resonance scattering of gamma radiation. Two methods of obtaining gamma rays of approximately the right energy for resonance scattering are: to use gamma rays which arise from a transition to the ground state of a nucleus which is the same as the nucleus in which the resonance scattering is being investigated (method A), and to use a source of variable energy gamma rays such as could be obtained by allowing monoenergetic gamma rays to undergo Compton scattering (method B). The difficulties inherent in both of these methods and ways of overcoming them are discussed below.

The cross section for resonance scattering may be written.

$$\sigma=\frac{g\lambda^2\Gamma^2}{8\pi[(E_0-E)^2+(\Gamma^2/4)]},\qquad(1)$$

where g is a statistical factor of order unity,[1] and λ, E, E_0, and Γ are, respectively, the wavelength and energy of the incident gamma rays, the resonance energy, and the width of the level. The maximum value of σ is of order 10^{-21} cm^2 for gamma rays of energy 1 Mev. Approximate expressions for Γ have been given by Weisskopf,[2] which show that the conditions for large Γ are large E_0, small multipole order, and large mass number.

2. RESONANCE SCATTERING BY METHOD A

Suppose that a nucleus N is stationary when it emits a gamma-ray of energy E_γ in flight. Since the gamma has a momentum (E_γ/c), N must have a recoil energy of $(E_\gamma^2/2Mc^2)$ where M is the mass of N. If the gamma ray is scattered by another nucleus, S, of the same species as N, then S will also recoil with an energy $(E_\gamma^2/2Mc^2)$ so that the appropriate energy E to insert in (1) is less than E_0 by an amount (E_γ^2/Mc^2). Since this last energy is of the order of tens of electron volts, whereas Γ is a very small fraction of an electron volt, the cross section for scattering of gamma rays from N will be very small, and the resonance scattering will be hidden in the background of Compton, Rayleigh, and Thomson nuclear scattering.[3] This fact has been pointed out by several writers[4,5] who have suggested that by making N move towards S at the instant of emission, E_γ will be Doppler-shifted towards E_0, and this will result in a larger value for σ than would be the case if N were stationary. Moon[4] has achieved this relative motion of N and S by rotating the source in the neighborhood of the scatterer, and choosing his geometry so that the bulk of the scattering occurs when the source is moving towards the scatterer. While his results indicate the presence of resonance scattering, his statistics are such that only a rough estimate of Γ may be made.*

Relative motion of N and S will occur if the emission of the gamma by N has been immediately preceded by the emission of either an electron or a gamma ray. The emission of the first particle will set N in motion and the energy of the gamma ray of interest will be Doppler-shifted accordingly. The simplest case will occur when the source nuclei N are in the form of a gas, so that they will move with the recoil velocity resulting from the emission of the first particle as long as they do not undergo collisions.

If the first particle is a gamma ray of energy E_1, it is easily shown that the energy spectrum of gamma rays from the nuclei N in any direction is a constant between $E_\gamma[1-(E_1/Mc^2)]$ and $E_\gamma[1+(E_1/Mc^2)]$ and is zero everywhere else. The effective cross section $\bar{\sigma}$, for gamma rays with this energy distribution is found from (1) to be

$$\bar{\sigma}=(g\lambda^2\Gamma^2Mc^2/8\pi E_\gamma E_1)[\tan^{-1}\{2E_\gamma(E_1-E_\gamma)/\Gamma Mc^2\}+\tan^{-1}\{2E_\gamma(E_1+E_\gamma)/\Gamma Mc^2\}].\qquad(2)$$

Weisskopf's formulas[2] show that the conditions for

[1] E. Guth, Phys. Rev. **59**, 325 (1941).
[2] V. Weisskopf, Phys. Rev. **83**, 1073 (1951).
[3] P. B. Moon, Proc. Phys. Soc. (London) **A63**, 1189 (1950).
[4] P. B. Moon, Proc. Phys. Soc. (London) **A64**, 76 (1951).
[5] S. Devons, *Excited States of Nuclei* (Cambridge University Press, London, 1949).
* *Note added in proof.*—A considerable improvement in the moving source technique has been effected recently by Moon and his collaborators (Proc. Phys. Soc. (London) **A66**, 956 (1953); and **A67**, 601 (1954)).

large $\bar{\sigma}$ are, again, large resonance energy and mass number, and low multipole order. If E_1 and E_γ are of the same order of magnitude, say 1 Mev, the second term in Eq. (2) is very nearly $(\pi/2)$. If $E_1>E_\gamma$ by as little as 1 kev, the first term is also nearly $(\pi/2)$, and in this case

$$\bar{\sigma}=(g\lambda^2\Gamma Mc^2/8E_\gamma E_1). \tag{3}$$

If $E_1<E_\gamma$, again by as little as 1 kev, the two terms will nearly cancel and, by using the approximation $\tan^{-1}x=(\pi/2)-(1/x)$,

$$\bar{\sigma}=(g\lambda^2\Gamma^2M^2c^4/8\pi E_\gamma^2[E_\gamma^2-E_1^2]). \tag{4}$$

The large difference between the expressions Eq. (3) and Eq. (4) may be used as an independent method of inferring the order of emission of gamma rays in cascade. For example, if the 1.33 and 1.17 Mev gammas of Ni^{60} were emitted in that order, Eq. (3) would show $\bar{\sigma}$ to be about 3×10^{-27} cm² when $\Gamma=10^{-4}$ ev. If the order were reversed, Eq. (4) would give $\bar{\sigma}=4\times10^{-31}$ cm².

If the first particle emitted is an electron rather than a gamma ray, the spectrum of the gamma rays from the source is more complicated. Kofoed-Hansen[6] has given an expression for the momentum spectrum of nuclei recoiling after beta decay, and from this the energy spectrum of the Doppler-shifted gamma rays may be calculated. The result for a beta spectrum with maximum energy $2mc^2$ is shown in Fig. 1. The intensity is plotted against $\xi=[M(E-E_\gamma)/mE_\gamma]$, where m is the electron mass. The curve is symmetrical about $\xi=0$ and the ordinates have been chosen to make the area under the whole curve unity. A gamma ray with the same *momentum* as the maximum momentum of the electrons would give rise to a rectangular spectrum within the same limits as the spectrum following beta decay, and would thus give rise to a larger cross section if the Doppler shift was just not large enough to compensate for the loss of energy by recoils. A gamma ray with an *energy* equal to the maximum beta-ray energy would not be similarly effective.

The condition for the validity of the above considerations is that the half-life of the state being investigated should be smaller than the reciprocal of the collison frequency in the gas, i.e., smaller than (mean free path/mean velocity). Since the velocity of sound, V, in a gas is given by[7] $V=v_m(\gamma\pi/8)^{\frac{1}{2}}$, where γ is the ratio of specific heats and is of order unity, and v_m is the mean velocity of the molecules, the condition may be stated that the half-life should be less than (mean free path/velocity of sound in the gas). This condition is satisfied for the states of interest here, with lives less than 10^{-10} sec.

When the atoms of the source are in solid and not gaseous form, the situation is more difficult to analyze in view of the paucity of exact knowledge of the elastic forces binding atoms in a crystal lattice. Some progress may be made by considering a linear Blackman model of the crystal lattice, i.e., a set of identical mass points, mass m, with a constant spacing s, joined together with identical springs with force constant k. The equations of motion of the chain may be readily solved with the initial conditions that the displacements of all the particles is zero and that the initial velocities of all but the nth are zero, the nth particle having the initial velocity v_0. This is a model of the system of interest in which an atom is set in motion by the emission of the first particle. The velocity of the $(n\pm r)$th particle is given as a function of time t by

$$v_{n\pm r}(t)=v_0J_{2r}(2\alpha t), \tag{5}$$

where $\alpha=(k/m)^{\frac{1}{2}}$. The velocity of the nth particle is thus $v_0J_0(2\alpha t)$, which falls to zero for the first time when $t=(1.2/\alpha)$. $J_p(x)$ reaches its maximum when $x\approx p$, hence the velocity of the $(n\pm r)$th particle will be a maximum when $t=(r/\alpha)$, and, since it is displaced a distance rs from the nth particle, the velocity of propagation of the disturbance will be $V_s=\alpha s$. Thus the velocity of the nth particle will be zero for the first time when $t=(1.2s/V_s)$. Since V_s is obviously the velocity of sound in the solid, the results of this section will still be valid if the half-life of the state under investigation is small compared to (interatomic distance/velocity of sound). In view of the agreement between the form of the conditions of validity for solids and gases, it is apparent that this criterion will also hold for liquids, i.e., half-life short compared to (interatomic distance or mean free path in the liquid/velocity of sound in the liquid). If these conditions are not fulfilled, then the velocity of the atom after the emission of the first particle will diminish because of dissipation of energy in liquids and solids and will be modified and generally diminished by collisions in gases. The Doppler shift and hence $\bar{\sigma}$ will then be smaller than the value given by Eq. (2).

In all of the above considerations it has been assumed that the distribution of recoil velocities after the emission of the first particle is isotropic, so that there will be as many nuclei recoiling away from the scatterer as are moving towards it, and hence as many of the

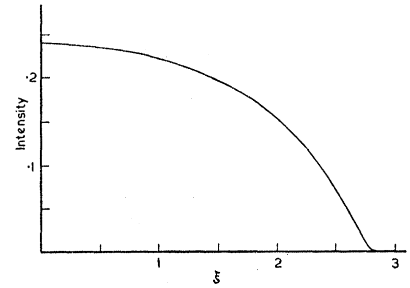

FIG. 1. The intensity of gamma rays of energy E emitted by nuclei recoiling after beta decay with a maximum energy $2mc^2$, plotted against $\xi=[M(E-E_\gamma)/mE_\gamma]$.

[6] O. Kofoed-Hansen, Phys. Rev. **74**, 1785 (1948).
[7] A. B. Wood, *Text-book of Sound* (G. Bell & Sons, London, 1944).

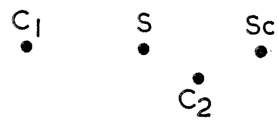

FIG. 2. Schematic arrangement of coincidence method for increasing the effective cross section for resonance scattering in the scatterer Sc of gamma rays from the source S. C_1 and C_2 are counters in coincidence.

gamma rays of interest will have their energy Doppler-shifted away from the resonance energy as towards it. However, gamma rays which have had their energy Doppler-shifted towards E_0 may be selected by means of the coincidence circuit shown in Fig. 2. C_1 and C_2 are detectors, S is the source, and Sc is the scatterer. C_2 is shielded from direct radiation from S. If a coincidence between C_1 and C_2 is registered, one will know that the gamma ray which reached C_2 from S via Sc was emitted by an atom moving towards Sc, since the first particle went in the direction of C_1. This will result in a much larger cross section than would be obtained using unselected gamma rays. Experiments on these lines are in progress in this laboratory. An obvious, but not readily applicable, extension using three scintillation counters is to make one of the phosphors the scatterer and to register coincidences between the other two but not triple coincidences. For large scattering angles this will eliminate the Compton scattering completely and give the advantage of selected gamma rays mentioned above.

3. RESONANCE SCATTERING BY METHOD *B*

Suppose that it is possible to obtain gamma rays with an energy spectrum which is constant and equal to $(1/E_m)$ in the range $(E_0'\pm\frac{1}{2}E_m)$, and zero everywhere else, where E_0' is the resonance energy corrected for recoil. The effective cross section $\bar{\sigma}$ is found from (1) to be

$$\bar{\sigma}=(g\lambda^2\Gamma/2\pi E_m)\tan^{-1}(E_m/\Gamma), \tag{6}$$

and, if E_m is as small as a few ev, $\tan^{-1}(E_m/\Gamma)\approx(\pi/2)\cdot$ Hence (8) may be approximated to by

$$\bar{\sigma}=(g\lambda^2\Gamma/4E_m). \tag{7}$$

$\bar{\sigma}$ will still have this value if the center of the spectrum is displaced by nearly $\pm(E_m/2)$ as long as E_m is large compared to Γ. Hence, as the center of the spectrum is moved from outside this range through it and outside it again, $\bar{\sigma}$ will change from a small to a large value and back again, "small" and "large" depending on the value of E_m. In the case of the 0.487-Mev level in Li^7, which has a width of about 0.01 ev,[8] $\bar{\sigma}=1.8\times10^{-26}$ cm^2, or about 2 percent of the Compton cross section in Li if $E_m\approx 9$ kev.

A gamma-ray beam with a spread of a few kev and a mean energy which may be varied continuously may be obtained by the use of the Compton effect in a primary scatterer. If gamma rays with an energy E_p are scattered by an angle θ, the energy of the scattered radiation E is given by the Compton formula

$$E=E_p[1+\{E_p(1-\cos\theta)/mc^2\}]^{-1}; \tag{8}$$

and the spread in energies ΔE due to a spread of scattering angles $\Delta\theta$ is

$$\Delta E=\frac{(E_p{}^2/mc^2)\sin\theta\Delta\theta}{[1+\{E_p(1-\cos\theta)/mc^2\}]^2}, \tag{9}$$

from which it is seen that, for a given $\Delta\theta$, ΔE is smallest when $\theta\approx 0$ or π. For a given E_p, the smallest value of ΔE will occur for $\theta\approx\pi$, but this region suffers from the disadvantages that the differential Compton cross section is smallest here, and, more important, that the maximum energy in this region is $(mc^2/2)$, no matter how high E_p. Thus for most purposes the region of small θ will be important.

If annihilation radiation was used as a source for the investigation of the resonance scattering from the 0.487-Mev Li^7 level, the right energy of gamma rays from the primary scatterer would be obtained for a scattering angle of about 21°. Equation (9) shows that $\Delta\theta$ should be 3.3° in order to obtain $\Delta E=9$ kev, which gives $\bar{\sigma}=1.8\times10^{-26}$ cm^2. If a more favorable source were used, with E_p less than 0.51 Mev, the same ΔE would be obtained with an even larger value of $\Delta\theta$. The geometry of the experiment could be chosen without great difficulty to give such a value of $\Delta\theta$. It would be determined by the finite extension of source, primary scatterer, and lithium scatterer, and would give a cross section somewhat larger than the value mentioned since the energy distribution of the gamma rays would then be peaked about the average value, instead of having the rectangular shape assumed in the derivation of Eq. (7).

The main difficulty is to obtain a large enough intensity of Compton gammas, scattered at just the right angle. This may be done by using a scatterer in the form of part of a surface of revolution, the axis being the line joining source and scatterer, and the profile being a circle. Gamma rays from all parts of the surface will then be scattered through the same angle. Strong sources, careful screening, and possibly the use of coincidence methods to reduce background will be necessary since the counting rates obtainable in double scattering experiments of this kind will not be high.

4. CONCLUSIONS

The methods outlined should prove useful in the study of high-energy, low-multipole order gamma-ray transitions. Because of the short lives of the states involved and the low internal conversion coefficients, etc., these transitions are not readily accessible by the methods which have proved so fruitful in the investigation of lower-energy, higher-multipole order transitions.

[8] J. M. Blatt, and V. Weisskopf, *Theoretical Nuclear Physics* (John Wiley and Sons, Inc., New York, 1952).