Resonances in $Li^7(p,n)Be^7$†

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(Received July 30, 1954)

An investigation of experimental data concerning the $Li(p,n)$ reaction indicates that the resonance at 2.25-Mev proton energy is due to the influence of a spin 3, isotopic spin 1, even parity level of Be^8, 19.2 Mev above the ground state, and that the background reaction intensity is predominantly the result of a spin-1 state of odd parity, and a spin-2 state of odd parity and isotopic spin 0. Cross sections, angular distributions, and neutron polarizations calculated on this basis are in good agreement with previous experimental measurements of these quantities.

The neutron and proton widths of the isotropic spin-1 state in Be^8 are related to the neutron width of the corresponding isotopic-spin-1 state of Li^8, 2.3 Mev above the ground state, in a manner consistent with the transition probability relationships expected of a nuclear isotopic spin multiplet.

I. INTRODUCTION

MEASUREMENTS of the reaction constants of the $Li^7(p,n)Be^7$ reaction by Taschek and Hemmendinger[1] have been analyzed by Breit and Bloch,[2] who concluded that the experimental results were most likely indicative of an odd parity level in Be^8, 19.2 Mev above the ground state. The observed angular distribution was interpreted as indicating an interference between states of opposite parity and hence the reaction background was believed to be due primarily to P-wave neutrons. Breit and Bloch noted that an alternative view was also admissible; the resonance resulted from the interaction of P-wave neutrons. Since the centrifugal barrier is high for P-wave neutrons and protons, the explanation in terms of a P-wave resonance appeared to require an improbably strong resonance strength, indicating that the explanation in terms of an $L=0$ resonance was more likely.

However, in the five years which have elaspsed since this interpretation was published, a considerable body of experimental information has been accumulated which bears obliquely on this problem. In particular, the occurrence of strong or broad resonances has been shown to be common,[3] and the best value of the effective nuclear radius to use in barrier calculations appears to be rather larger[3] than that used by Breit and Bloch. If a larger radius is used in the calculations, the computed height of the barrier is lower, the resonance strength required for P neutrons is not as large, and the objections to the consideration of a P-wave resonance are weakened. It has recently been emphasized that the charge independence of nuclear forces has important consequences on nuclear reactions.[4,5] Since the Be^8 states of interest in the $Li(p,n)$ reaction have a definite isotopic spin and break up into the states Li^7+n and Be^7+p, which differ only in isotopic spin, transition probability relationships will obtain[5] which should aid considerably in interpreting the data. In view of these considerations it seemed that it would be useful to reinvestigate this problem.

II. ASSIGNMENT OF RESONANCE PARAMETERS

The lowest isotopic spin one level in Be^8, corresponding to the ground state of Li^8, appears to lie about 16.9 Mev above the ground[6] state of Be^8. Resonant scattering of neutrons by Li^7 indicates the presence of a state in Li^8, 2.3 Mev above the ground state,[7] which has a spin of 3 and even parity.[8] Charge independence of nuclear forces then necessitates the presence of an equivalent state in Be^8 which appears to be the 19.2-Mev state identified with the resonance in the $Li(p,n)$ reaction.

Arguments independent of the presence of the Li^8 state support the assignment of spin 3 and even parity to the Be^8 level. The value of the reaction cross section at resonance must be

$$\frac{(2J+1)}{(2I+1)(2S+1)}\pi\lambda\!\!\!^{-2}\frac{4\Gamma_n\Gamma_p}{(\Gamma_n+\Gamma_p)^2}, \tag{1}$$

where J is the spin of the compound state and I and S are the spins of the interacting particles, $\frac{1}{2}$ and $\frac{3}{2}$. The Γ_n and Γ_p are the widths for neutron emission and proton emission respectively, and $\lambda\!\!\!^-$ is the wavelength/2π of the incident proton. The resonant part of the $Li(p,n)$ cross section reaches a maximum value[9] of nearly 0.25 barn, at an incident proton energy of 2.25 Mev. The maximum value allowed is $(2J+1)/8\times0.375$ barns, when $\Gamma_n=\Gamma_p$. This immediately excludes states of $J=1$ and allows $J=2$ only in the somewhat improbable event

† Work supported by the U. S. Atomic Energy Commission and the Wisconsin Alumni Research Foundation.

1 R. Taschek and N. Hemmendinger, Phys. Rev. **74**, 373 (1948).
2 G. Breit and I. Bloch, Phys. Rev. **74**, 397 (1948).
3 T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).
4 L. A. Radicati, Proc. Phys. Soc. (London) **A66**, 139 (1953).
5 R. K. Adair, Phys. Rev. **87**, 1041 (1952).
6 M. Gell-Mann and V. Telegdi, Phys. Rev. **91**, 169 (1953).
7 R. K. Adair, Phys. Rev. **79**, 1018 (1950).
8 P. Stelson and W. Preston, Phys. Rev. **84**, 162 (1951).
9 The cross section values of reference 1 refer to energies based on a scale where the $Li(p,n)$ threshold is 1.86 Mev. The energies have been recalculated for the purpose of this report, on the basis of the presently accepted scale where the $Li(p,n)$ threshold is 1.882 Mev.

that Γ_n is nearly equal to Γ_p. The large value of the $Li(p,n)$ cross section near threshold indicates the background part of the reaction cross section must be largely the result of S-wave neutrons. This conclusion is strongly substantiated by the high value found for the inverse $Be^7(n,p)Li^7$ reaction cross section at thermal neutron energies.[10] The variation[1] with energy at resonance of the part of the differential cross section proportional to $\cos\vartheta$ is characteristic of the interference of an S-wave background with P waves at a resonance. Since Li^7 has odd parity the state in Li^8 must then be even. Since there is evidence that α particles are not emitted appreciably from this level,[11] we can, on these grounds, further exclude the possibility that the state has spin 2 and isotopic spin 0, as such a state should have a high probability for disintegration into two alpha particles.

If it is assumed on the basis of these arguments that the Be^8 state has spin 3, even parity, and isotopic spin one, it presumably forms a charge multiplet together with the 2.3-Mev state in Li^8 and a presently undiscovered state of B^8. Since the states of the charge multiplet are in the continuum it is possible to obtain information concerning the equivalence of the wave functions of states comprising the multiplet by examining the widths for particle emission. If nuclear forces are charge-independent, the wave functions of the compound states and the wave functions of the final states will differ, respectively, only in isotopic spin space. We can write the isotopic spin wave functions of the mirror nuclei Li^7 and Be^7 as $\varphi_{\frac12}^{\frac12}$ and $\varphi_{\frac12}^{-\frac12}$, the neutron and the proton $\tau_{\frac12}^{\frac12}$ and $\tau_{\frac12}^{-\frac12}$, respectively, and the Li^8 and Be^8 levels as T_1^1 and T_1^0, where the subscript represents the total isotopic spin and the superscript stands for the third component. Only the isotopic spin part of the matrix elements for the breakup of Li^8 and Be^8 will differ. These are easily calculated:

$$\begin{aligned}&(T_1^1|\varphi_{\frac12}^{\frac12}\tau_{\frac12}^{\frac12})=1,\\&(T_1^0|\varphi_{\frac12}^{\frac12}\tau_{\frac12}^{-\frac12})=(T_1^0|\varphi_{\frac12}^{-\frac12}\tau_{\frac12}^{\frac12})=1/\sqrt{2};\\&(T_0^0|\varphi_{\frac12}^{-\frac12}\tau_{\frac12}^{\frac12})=-(T_0^0|\varphi_{\frac12}^{\frac12}\tau_{\frac12}^{-\frac12}).\end{aligned}\qquad(2)$$

We write the whole matrix element as γ_s for $(Li^8|Li^7+n)$, γ_p for $(Be^8|Li^7+p)$, and γ_n for $(Be^8|Be^7+n)$. The reaction widths, Γ, will be equal[12] to $2k\gamma^2(F^2+G^2)^{-1}$, where k is the wave number and F and G are the usual regular and irregular free-particle radial wave functions[13] evaluated at an effective nuclear radius a. By following Christy and Latter,[14] a radius a of $1.45(A^{\frac13}+1)\times10^{-13}$ cm was used in the calculations. If one allows for the variation of level shift with energy,[15] $|\gamma_s|$ was found to be equal to $(1.6\times10^{-13}\text{ Mev cm})^{\frac12}$ from the measured width of the Li^8 level. From relations (1), $\gamma_n=\gamma_p=(0.8\times10^{-13}\text{ Mev cm})^{\frac12}$, where only the relative signs of the γ are important. A resonance energy $E=E_\lambda-\Delta=1.96$ Mev was assigned in the center-of-mass system, and the resonant part of the $Li(p,n)$ cross section was calculated by using the relation

$$\sigma_{np}=\frac{2J+1}{(2S+1)(2I+1)}\frac{\pi\lambda^2\Gamma_n\Gamma_p}{(E+\Delta-E_\lambda)^2+\Gamma^2/4}.\qquad(3)$$

The points on Fig. 1 show the measured values of the total $Li(p,n)$ cross section.[1] The difference between the dashed line and the solid line represents the resonance contribution calculated from relation 2. The excellence of the fit to the resonance cross section variation shown by the experimental points is evidence for the charge independence of nuclear forces and that $(F^2+G^2)^{-1}$ evaluated at a radius of $1.45(A^{\frac13}+1)\times10^{-13}$ cm is a good measure of barrier penetrability.

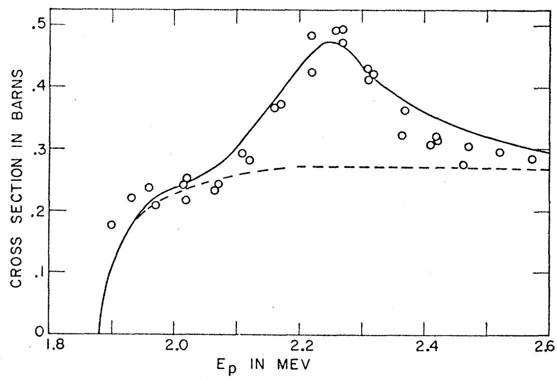

FIG. 1. Total $Li(p,n)$ reaction cross section. The points are taken from the data of reference 1. The solid curve represents the cross section calculated from the considerations of this paper. The area beneath the dashed line is the contribution of the S-wave reaction, the area between the dashed and solid curve is the resonance cross section.

III. S-WAVE INTERACTION

The part of the P-wave neutron amplitude in which the Z component of the total intrinsic spin is conserved will be coherent with, and interfere with, the neutron S-wave amplitude associated with the same spin multiplicity. The magnitude of the interference term will then be proportional to the quintet S-wave neutron amplitude, the coherent part of the P-wave amplitude, and the cosine of the phase angle between them. Since the S wave is isotropic, the interference term will have the angular dependence of the P wave, which, since the Z component of the spin is conserved, will be proportional to $\cos\vartheta$, where ϑ is the angle between the incident proton beam and the reactant neutrons measured in the center-of-mass system. As the phase and magnitude of the P-wave amplitude can be determined from the

[10] Reported by G. C. Hanna at the Birmingham Conference on Nuclear Physics, July, 1953 (unpublished).

[11] Heydenburg, Hudson, Inglis, and Whitehead, Phys. Rev. **73**, 241 (1948).

[12] E. Wigner and L. Eisenbud, Phys. Rev. **72**, 29 (1948).

[13] Bloch, Hull, Broyles, Bouricius, Freeman, and Breit, Revs. Modern Phys. **23**, 147 (1951).

[14] R. F. Christy and R. Latter, Revs. Modern Phys. **20**, 185 (1948).

[15] R. G. Thomas, Phys. Rev. **81**, 148 (1951).

resonance parameters, the experimentally observed values of the part of the differential cross section which is proportional to $\cos\vartheta$ will be a measure of the phase and magnitude of the S-wave neutron amplitude.

From the magnitude of the total background reaction cross section as well as the large values of the part of the angular distribution proportional to $\cos\vartheta$, it appears that the cross section for production of both triplet and quintet states of Be^7 and an S-wave neutron must be large in terms of the maximum allowed by the conservation laws: $(2J+1)\pi\lambda\!\!\!^{-2}/8$. It seems probable, then, that the main contribution to the reaction amplitude for each intrinsic spin state is due either to a broad nearby level of Be^8, or to the tails of a number of far away states, almost all of which have the same isotopic spin. Since, from relation (1), the reaction matrix elements of states with different isotopic spin differ in sign, reaction amplitudes for such states will on the average interfere destructively, reducing the reaction cross section. A reaction amplitude can be represented conveniently in terms of the reduced derivative matrix of Wigner and Eisenbud, and Teichmann and Wigner.[3,12,16] A reaction amplitude then takes the form

$$A=g\lambda\!\!\!^{-}_p\exp(i\delta_n+i\delta_p)\times\frac{ik_p^{\frac{1}{2}}(F_p^2+G_p^2)^{-\frac{1}{2}}R_{np}k_n^{\frac{1}{2}}(F_n^2+G_n^2)^{-\frac{1}{2}}}{1-\frac{1}{a}\sum_{j=n,p}R_{jj}\left[\frac{d\ln(F_j^2+G_j^2)^{\frac{1}{2}}}{d\ln k_ja}+ik_ja(F_j^2+G_j^2)^{-1}\right]},\quad(4)$$

where δ_n is equal to the hard sphere scattering phase shift for neutrons; $\exp 2i\delta_n=(G_n-iF_n)/(G_n+iF_n)$ and δ_p is a similar phase shift for protons;

$$\exp 2i\delta_p=\frac{G_p-iF_p}{G_p+iF_p}\,\frac{l+i\eta\cdots1+i\eta(i\eta)!}{l-i\eta\cdots1-i\eta(-i\eta)!},\quad(5)$$

where $\eta=3e^2/\hbar V$. The g is a geometric quantity depending on the orbital angular momentum, intrinsic spins of the reacting particles, total angular momentum of the state, and the channel spin. As written here, g includes the spins and angular dependence characteristic of those states. The values of g for the S-wave and P-wave amplitudes were computed[17] using the Clebsch-Gordon coefficients tabulated by Condon and Shortley.[18] The elements of the R matrix R_{jk} equal $\sum_\lambda\gamma_{j\lambda}\gamma_{k\lambda}/E_\lambda-E$, where $\gamma_{j\lambda}=(\hbar^2/2\mu)^{\frac{1}{2}}\int X_\lambda V_jdS$, where X_λ is a nuclear eigenfunction, V_j is the wave function in the channel j, and the integration over the nuclear surface includes a summation over intrinsic spins and isotopic spins. When one value of the isotopic spin T is dominant, from relation 1: $R_{nn}=R_{pp}=(-)^{T+1}R_{np}$, and the phase of the scattering amplitude will depend upon the value of T. Since the interference intensity is proportional to the cosine of the phase angle between the amplitudes of different states, the sign of the interference term will depend upon the relative isotopic spin of the two states. The dashed line in Fig. 2 shows the value of the coefficient of $\cos\vartheta$ derived by Taschek and Hemmendinger from their experimental results, and the solid line represents a calculated value of $\cos\vartheta$. For this computation the single-level approximation was used for the P-wave resonance; $R_{jk}=\gamma_j\gamma_k/(E_\lambda-E)$ with the parameters introduced in Sec. II. Far from a resonance, R will not vary strongly with energy. For the quintet S-wave amplitude, R was chosen to be 7×10^{-13} cm and the isotopic spin was taken as zero. If the quintet S-wave isotopic spin were one, the sign of the asymmetry coefficient would be reversed and in contradiction with the experimental results. The sign of R used for the S-wave resonance is that which would result from the contribution of far-off high energy levels. Actually the results are not sensitive to the magnitude of R if $R>a$. Also shown in Fig. 1 are the coefficients of $P_2(\cos\vartheta)$ determined from the measured angular distributions and calculated from the P-wave reaction parameters. The agreement is not bad when the large error involved in extracting this quantity from the experimental data is considered.

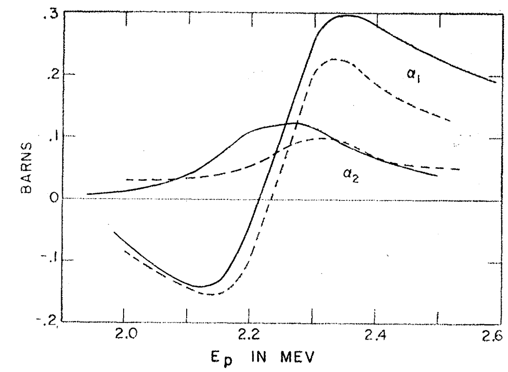

FIG. 2. Energy dependence of the coefficients α_1 and α_2 of the Legendre polynomial fit to the center of mass differential cross sections. As in reference 1 the units of the α's are adjusted so that $4\pi d\sigma/d\omega=\alpha_0P_0(\cos\vartheta)+\alpha_1P_1(\cos\vartheta)+\alpha_2P_2(\cos\vartheta)$. The dashed lines are the experimental results from reference 1; the solid lines represent the values calculated in this report.

We can obtain no information on the triplet S-wave reaction from an examination of the S-P interference term because states of different total spin do not interfere in differential cross section. Since the background reaction cross section is larger than the conservation laws permit for quintet S-wave scattering alone, and the asymmetry in the scattering is satisfactorily accounted for by quintet states, it appears that the triplet S-wave interaction is important. For the sake of definiteness the same parameters were used to calculate the triplet cross section as were used for the quintet state. The solid curve in Fig. 1 represents the total

[16] J. M. Blatt and L. C. Biedenharn, Revs. Modern Phys. 24, 258 (1952), in particular, Eqs. (5, 6).
[17] See, for example, reference 11.
[18] E. U. Condon and G. H. Shortley, *Theory of Atomic Spectra* (Cambridge University Press, Cambridge, 1935).

reaction cross section calculated from the reaction amplitudes assigned in this work. The dashed line separates the S-wave and P-wave contributions. The quintet states accounts for $\frac{5}{8}$ of the S-wave cross section and the triplet state gives the rest. While the agreement with the experimental points is quite good, especially considering that the S-wave reaction amplitudes are described by but one parameter, and considering the 12 percent uncertainty in absolute value of the experimental cross sections, the discrepancy between the experimental values and the theoretical curve appear to be significant near threshold. It is possible to calculate the $Be^7(n,p)Li^7$ thermal neutron cross section from the $Li^7(p,n)Be^7$ cross section using the principle of detailed balance. If the Be^7 thermal neutron cross section is the result of isolated definite resonances in each of the two spin states application of relations 1 and 2 shows that the maximum total cross section allowed is

$$\sigma_{np}=4\pi k_n^{-2}\frac{\Gamma_n\Gamma_p}{(\Gamma_p+\Gamma_n)^2}\approx 4\pi k_n^{-2}\frac{\Gamma_n}{\Gamma_p}$$
$$=4\pi\lambda_n\lambda_p(F_p^2+G_p^2)\approx 1.85\times 10^4 \text{ barns.} \quad (6)$$

While the cross section calculated from the parameters of this paper is almost equal to that maximum, it is considerably lower than the value of about 5×10^4 barns reported by Hanna.[10]

So high a $Be^7(n,p)$ thermal cross section, and equally, so fast a rise of the $Li^7(p,n)$ reaction cross section near threshold cannot then be attributed solely to a resonance if the concept of charge independence of nuclear forces is to be retained. Rather, as can be seen from Eq. (3), the high cross sections must be the result of interference between states of different isotopic spins which have R-matrix elements R_{np} of the same sign. A simple example is the interference of two states of different isotopic spins, one lying above, one below threshold. The $T=1$ state which appears to contribute may be related to the Li^8 state, reported by Levine *et al.*[19]

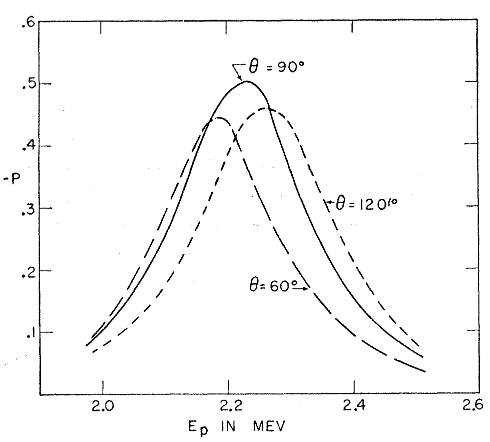

FIG. 3. Values of the polarization P of neutrons produced by the $Li(p,n)$ reaction as calculated from the parameters of this paper.

IV. NEUTRON POLARIZATION

The existence of the isolated spin-3 even-parity state of Be^8 is an effect of the strong spin-orbit coupling found in nuclei. In general a spin-orbit force will result in a polarization of the products of a nuclear reaction. Since effects of the spin-orbit energy are determined kinematically from the parameters of the resonance, without the necessity of obtaining further information concerning the source of this energy, it is possible to calculate the polarization of the neutrons from the $Li(p,n)$ reaction. The polarization of neutrons in the X direction, where Z is the direction of the incident protons, will be equal to $(A^*|\sigma_x|A)/(A^*|A)$, where A represents the total reaction amplitude, including spins, and σ_x is the usual spin operator. The calculation, which was straightforward, was performed by the usual addition and resolution of angular momentum vectors,[16] using for the P-wave and quintet S-wave amplitudes the values discussed previously. Although the σ_x operator will mix states of different total spin, triplet and quintet states do not interfere in this case[18] because the initial Li^7+p triplet and quintet states are incoherent and total spin is conserved in the approximation used; that is, neglect of D and F waves.[20] Therefore, the polarization depends only upon the quintet amplitudes and is likely to be fairly reliable. Figure 3 shows the polarization of neutrons emitted at angles of 60°, 90° and 120° in the center-of-mass system as a function of neutron energy. To be properly compared with experimental data the curves should be shifted to higher energy by about 25 kev, which is the energy discrepancy between the experimental and measured angular distributions at resonance as shown in Fig. 3. A measurement[21] of the polarization at an angle of 66° in the center-of-mass system at a proton energy of 2.23 Mev gave a result of $P=-0.53\pm 0.06$ as compared to a value of about -0.45 taken from the parameters of this paper. Considering the approximations made in the calculations the agreement seems satisfactory.

It might be expected that the protons scattered at these energies will also be partially polarized.

[19] Levine, Bender, McGruer, and Vogelsang, Phys. Rev. **95**, 640 (1954).

[20] I wish to thank Dr. C. N. Yang, who clarified these considerations for me.

[21] Adair, Darden, and Fields, Phys. Rev. **96**, 503 (1954).