PHYSICAL REVIEW VOLUME 96, NUMBER 3 NOVEMBER 1, 1954

Proton Angular Distributions from Deuteron Bombardment of Bi^{209}, Pb^{207}, and Y^{89}†*

N. S. WALL‡
Department of Physics and Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received July 9, 1954)

The proton angular distributions in the (d,p) reactions on Bi^{209}, Pb^{207}, and Y^{89} have been measured and an attempt made to interpret the results on the basis of the Butler stripping theory. The neutron added to Y^{89} is consistent with an $l_n=2$ prediction. The last neutron in Pb^{208} shows a similarity to an $l_n=1$, non-Coulomb theory angular distribution, but the Bi^{210} case is not readily capable of being interpreted with this theory. The results of these experiments are analyzed on the basis of the shell model and the systematics of the binding energy of the last neutron.

I. INTRODUCTION

THE interpretation of the binding energy of the last neutron[1,2] in terms of the shell model depends on the angular momentum of the state in which the neutron is captured. According to the theory of deuteron stripping[3–6] the angular distribution of the outgoing protons from a (d,p) reaction yields, directly, the orbital angular momentum value l_n of the captured neutron. Though this technique does not measure the total angular momentum of the resultant nucleus, it does limit it to certain values, derived from the conservation of angular momentum.[3] The present paper reports an attempt to ascertain whether the measured Q-values of (d,p) reactions can be used to determine the spin-orbit coupling in the $5g$ and $7i$ shells from the binding energies of the 50th and 51st or the 126th and 127th neutron respectively.

Pb^{208}, Bi^{210}

In the case of the Bi^{210} neutron binding energy it is believed that the protons observed do not correspond to Bi^{210} in the ground state, and therefore the binding energy as derived from the $Bi^{209}(d,p)Bi^{210}$ Q-value is too low. The main reason for this belief is the failure of a cycle involving, in addition to the Q-values of the $Bi^{209}(d,p)Bi^{210}$ and $Pb^{208}(d,p)Pb^{209}$ reaction, well established masses, decay energies, and Q-values to close.[1,2,7,8] If one assigns the entire discrepancy to the Bi^{210} Q-value, the Bi^{210} ground state should actually be about 500 kev lower than experiments to date indicate.[1,2,8] By accepting some very general results from deuteron stripping theory this discrepancy can be readily understood on the basis of the model of Bi^{210} as given by Pryce.[9]

The differential cross section $d\sigma/d\omega$ for a (d,p) stripping reaction can be written

$$\frac{d\sigma}{d\omega}\propto\frac{(2I+1)}{(2J+1)}|\langle M\rangle|^2, \qquad (1)$$

where $|\langle M\rangle|^2$ is equivalent to a matrix element between the initial and final states and J and I are the total angular momenta corresponding to these states, respectively.[3–6] This matrix element contains one term expressing the probability of finding a proton of an appropriate momentum in the deuteron and another representing the capture of the stripped neutron by the nucleus. It should be remarked that the value of the matrix element here is assumed to be independent of I. If, however, this is not the case but $|\langle M\rangle|^2$ is not too dependent on I, the conclusions below are still valid.

The model of Pryce[9] gives a ground state of Bi^{210} of minimum spin, that is the j of the neutron is antiparallel to that of the proton. On the basis of the shell model the last neutron in Bi^{210} is expected to be in a $g_{9/2}$ configuration with the proton in the $h_{9/2}$ configuration of Bi^{209}.[10] Following Pryce these two nucleons would couple to give an $I=0$ ground state for Bi^{210}, with a series of nine excited states with $I=1, 2\cdots 9$. These latter states are closely spaced in energy and are separated by several hundred kev from the $I=0$ ground state.

The spin of Ra E, Bi^{210} has recently been found to have the value $I=1$ by K. F. Smith, as quoted in a paper to be published on Ra E by E. A. Plassman and L. M. Langer. If the last neutron configuration is $h_{11/2}$ then the conclusions above are essentially the same, the statistical weight factor being 2.5 percent. If, however, the neutron configuration is $g_{9/2}$ so that a $g_{9/2}$ neutron and an $h_{9/2}$ proton couple to give a ground state of $I=1$, then this whole picture is probably incorrect.

The (d,p) experiments performed to date have not been of sufficiently high resolution to be able to dis-

---

† This work is based in part on material submitted in partial fulfillment for the Ph.D. at Massachusetts Institute of Technology.

* This work supported in part by a joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

‡ Now at the University of Rochester, Rochester 3, New York.

[1] N. S. Wall, preceding paper [Phys. Rev. **96**, 664 (1954)].

[2] J. A. Harvey, Phys. Rev. **81**, 353 (1951).

[3] S. T. Butler, Proc. Roy. Soc. (London) **A208**, 559 (1951).

[4] P. B. Daitch and J. B. French, Phys. Rev. **87**, 900 (1952).

[5] F. L. Friedman and W. Tobocman, Phys. Rev. **92**, 93 (1953).

[6] A. B. Bhatia *et al.*, Phil. Mag. **43**, 485 (1952).

[7] J. R. Huizenga *et al.*, Phys. Rev. **79**, 908 (1950).

[8] B. B. Kinsey *et al.*, Phys. Rev. **78**, 77 (1950).

[9] M. H. L. Pryce, Proc. Phys. Soc. (London) **A65**, 773 (1952).

[10] P. F. A. Klinkenberg, Revs. Modern Phys. **24**, 63 (1952).

tinguish the various components of the excited multiplet of Bi^{210}. The ground state however would have been resolved providing it had a moderate, (2 percent–3 percent), intensity relative to the unresolved multiplet. On the basis of the stripping theory, as given above, the general result that the differential cross section depends upon the statistical weight of the initial and final levels leads to the conclusion that the intensity of the ground state should be 1 percent, that of the excited multiplet for the $h_{9/2}$, $g_{9/2}$ configuration. (The excited multiplet corresponds to the $Q=1.94$-Mev proton group.) The fact that we are dealing with nuclei for which the deuteron energy used is comparable to the Coulomb barrier probably does not introduce too large an error, providing the reaction still goes by stripping. The Coulomb field will, however, introduce a small effect since the barrier penetration factor is energy dependent, and this comes into $|\langle M\rangle|^2$.

To determine the validity of the deuteron stripping under a condition of deuteron energy comparable to barrier height the $Pb^{207}(d,p)Pb^{208}$ ground-state reaction was investigated. In this case since the spins of both the initial and final nuclei are known l_n is uniquely determined. If the non-Coulomb theory is valid in this case the proton angular distribution should be predictable. Any quantitative difference between the observed and the predicted distributions can be attributed in a large part to the Coulomb effect.

Y^{90}

In the case of the $Y^{89}(d,p)Y^{90}$ ground-state (g.s.) reaction information is desired on the angular momentum of just the captured 51st neutron to see if it is a member of the same spin-orbit doublet as the 50th neutron. The 50th neutron is believed to be in a $5g_{9/2}$ level.[10] If the 51st neutron is a $5g_{7/2}$ configuration the difference in the binding energies of these two neutrons should be directly related to the spin-orbit coupling parameter.[11] It should be noted, however, that the difference in binding energies should be corrected, because the two nuclei involved have a different neutron excess.[1,2] This fact is taken into account by determining the difference Δ between the observed binding energy and the binding energy expected on the basis of the semiempirical mass formula.[12,13] The difference between $\Delta(Y^{90})$ and $\Delta(Y^{89})$ is then the magnitude of the spin-orbit coupling.

Actually the stripping analysis only gives the orbital angular momentum value of the captured particle. To the extent to which we are interpreting the shell model, however, it is valid to use the shell model prediction to determine the alignment of the spin of the captured particle relative to its orbital angular momentum.

II. EXPERIMENTAL PROCEDURES

The technique used in these experiments was very similar to that of Black.[14] 15.1-Mev deuterons from the M.I.T. Cyclotron were brought into the scattering chamber in a well collimated beam. The counter could be placed at any angle from about 5°–150° to the beam. The detector was a NaI scintillation spectrometer with an absorber, in front of it, either of polystyrene or Al thick enough to stop elastic deuterons. The angular position of the counter was determined by means of a helical potentiometer coupled to the movable arm on which the counter was mounted. This arrangement gave a resistance linearly proportional to the angular displacement of the arm. The zero angle was found by comparing angles on either side of zero degrees at which the intensity of elastically scattered deuterons was equal. In this manner angles could be measured with relative ease to an accuracy of about $\frac{1}{2}°$.

The electronic equipment used is described in the preceding paper[1] and elsewhere.[15]

III. RESULTS

The angular distributions from the $Pb^{207}(d,p)Pb^{208}$ (g.s.), $Bi^{209}(d,p)Bi^{210}$ ($Q=1.94$ Mev group), and $Y^{89}(d,p)Y^{90}$ (g.s.) are shown in Figs. 1, 2, and 3. The

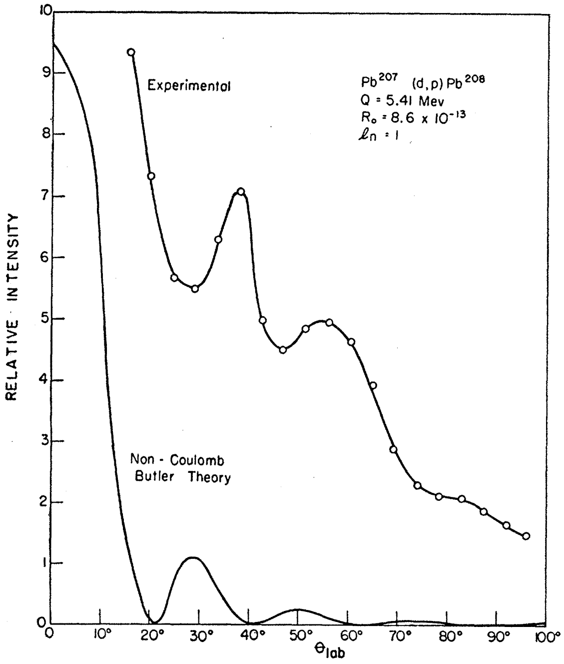

FIG. 1. The angular distribution of the protons from the ground state to ground state $Pb^{207}(d,p)Pb^{208}$ reaction.

---

[11] D. R. Inglis, Revs. Modern Phys. 25, 390 (1953).

[12] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1950).

[13] N. Metropolis and G. Reitwiesner, Atomic Energy Commission Report NP-1980, 1950 (unpublished).

[14] C. F. Black, Ph.D. Thesis, Massachusetts Institute of Technology, January, 1953 (unpublished).

[15] K. Boyer *et al.*, Rev. Sci. Instr. 22, 310 (1951).

angles are given in the laboratory system since, for the heavy nuclei studied, the laboratory and center-of-mass angles are the same within the accuracy of measurement. The probable error of the relative intensity is believed to be less than 10 percent at each point. The forward angles on the Pb^{208} curve were measured with both an Al and a polystyrene absorber since the proton resulting from (d,p) reactions in the Al, produced by deuterons scattered by the target foil, had an energy comparable to that of the Pb^{208} protons. The protons from the $C^{12}(d,p)C^{13}$ reaction in the polystyrene absorber had much lower energy[1] and therefore did not interfere. Also plotted in these figures are the angular distributions expected on the basis of the Butler expression for the angular distribution.[3] It should also be pointed out that though no absolute measurements of the Bi or Pb cross sections were made one can use Harvey's results on these two nuclei to obtain absolute values. His data gives the cross sections at 50° as 1.4 and 0.25×10^{-27} cm^2/steradian for Bi and Pb, respectively.[2] The Bi^{210} angular distribution is in agreement with that of Gove.[16]

IV. OBSERVATIONS AND CONCLUSIONS

Y^{90}

The Y^{90} g.s. angular distribution is consistent with the 51st neutron being in a $d_{5/2}$ state. From a study of radioactive decay schemes, and nuclear moments one can see that the first few neutrons over the $N=50$ shell occupy a $d_{5/2}$ state.[17] Much of this information is contained in references 10 and 18. Since the 51st neutron is therefore not in the $g_{7/2}$ member of the $5g$ doublet, the binding energy difference cannot be attributed solely to spin-orbit coupling. Any evaluation of the spin-orbit coupling parameter must therefore depend upon the position of the levels before the spin-orbit perturbation is introduced.

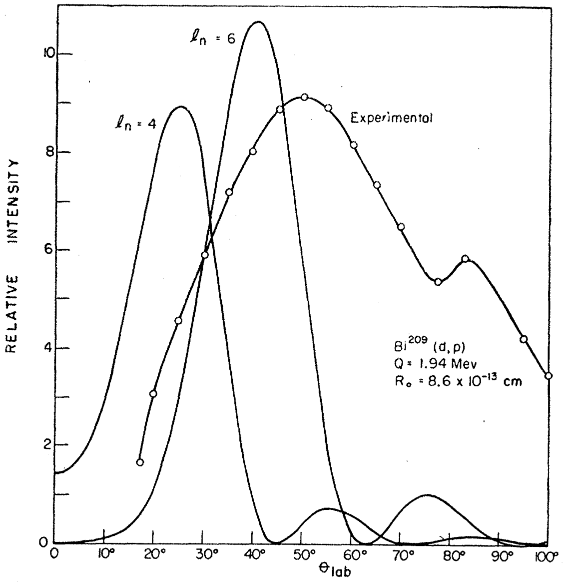

FIG. 2. The angular distribution of the $Q=1.94$ Mev proton group from the reaction $Bi^{209}(d,p)Bi^{210}$.

The angular distribution of the proton group corresponding to the first excited state of Y^{90} was also measured. As one can see in Fig. 2 of reference 1, the observed width of this proton group is slightly larger than that of the ground state indicating a possible additional unresolved level. Furthermore at small angles the C^{13} proton groups interfere with the Y^{90*} proton groups to a certain degree making the determination of the intensity of the Y^{90*} somewhat inaccurate. The angular distribution which is observed, however, qualitatively resembles that of an $l_n=0$ Butler distribution. It should be mentioned that Holt and Marsham[19] obtain fair agreement with experiment for the angular distribution of the protons corresponding to the first excited state of Sr^{89}, in the reaction $Sr^{88}(d,p)Sr^{89}$, by assuming a mixture of $l_n=0$ and $l_n=3$. On the basis of the shell model it would be expected that these two nuclei have the same neutron configurations.

Pb^{208} and Bi^{210}

The Pb^{208} g.s. angular distribution has relative maxima and minima which correspond, at least in the location of these maxima and minima, to those of an $l_n=1$ distribution neglecting Coulomb forces. It is believed that the effect of the Coulomb field will be to

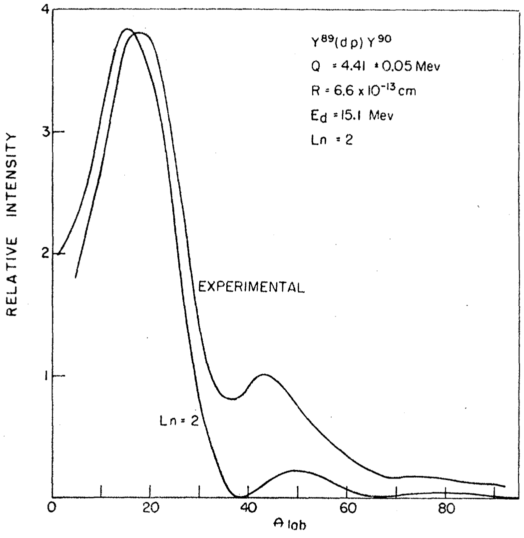

FIG. 3. The angular distribution of the ground state to ground state $Y^{89}(d,p)Y^{90}$ reaction.

---

[16] H. E. Gove, Phys. Rev. **81**, 364 (1951).

[17] The $d_{5/2}$ assignment for odd-odd nuclei follows if one assumes the shell model predictions for the proton and the (2,−) ground state configuration.

[18] J. M. Hollander *et al.*, Revs. Modern Phys. **25**, 469 (1953).

[19] J. R. Holt and T. N. Marsham, Proc. Phys. Soc. (London) **A66**, 565 (1953).

"fill in" the minima.[20] If this is the case, there seems to be a reasonable agreement between theory and experiment.

On the other hand, in the case of Bi^{210} there does not seem to be much similarity between the experimental angular distribution and the distribution expected for either $l_n=4$ or $l_n=6$, the values expected on the basis of the shell model. The fact that the distribution differs from the Pb^{208} g.s. distribution in having a minimum at small angles would lead one to conclude that $l_n \geq 2$. Consistent with the interpretation of a high l_n value for the last neutron in the Bi^{210} is the observation that no proton group occurs with an intensity of 1/30 the intensity of the $Q=1.94$ Mev state at an angle of 45°.[1] The nonsimilarity between the predicted and observed distributions may be because approximations of the stripping theory with respect to the Coulomb interaction are worse for larger angular momentum transfers or that for large l_n the reaction may not be Butler stripping. One unfortunately is therefore unable to assign an l_n value to the last neutron in Bi^{210}.

Odd-Even Effect

An explanation, as offered above for the low-intensity Bi^{210} g.s., would be consistent with Harvey's[2] observation of low (d,p) cross sections for odd $N-$even Z target nuclei relative to isotopes of the same element and even N, if the matrix elements for nearby isotopes were similar. For example, Harvey gives, at an angle of 20°,

$$\frac{d\sigma}{d\omega}(\mathrm{Zr}^{92}) \Big/ \frac{d\sigma}{d\omega}(\mathrm{Zr}^{91}) = \frac{1}{40}.$$

If in both cases the last neutron is in a $d_{5/2}$ state, the relative differential cross section would be

$$\left[\frac{2I(\mathrm{Zr}^{92})+1}{2J(\mathrm{Zr}^{91})+1}\right]\left[\frac{2J(\mathrm{Zr}^{90})+1}{2I(\mathrm{Zr}^{91})+1}\right] = \frac{1}{[2J_i+1]^2} = \frac{1}{36},$$

where J_i is the total angular momentum of the resultant nucleus for the reaction involving the lighter target nucleus.

In the case of the Ti isotopes, however, where the last neutron is probably in a $f_{7/2}$ state, the expected ratio is $1/[2J_i+1]^2=1/64$, whereas the observed ratio is only about 0.10.

However, many of the nuclei given by Harvey have relative differential cross sections in agreement with the supposition that the matrix elements for nearby nuclei are similar to within a factor of 2 or 3. It may be that for those nuclei where there is serious disagreement, the l_n in each case is different. Factors other than the statistical weight would then be expected to cause such a difference.

---

[20] W. Tobocman (private communication).

PHYSICAL REVIEW VOLUME 96, NUMBER 3 NOVEMBER 1, 1954

Determination of the Ranges and Straggling of Low-Energy Alpha Particles in a Cloud Chamber

S. Barile, R. Webeler, and G. Allen
Lewis Flight Propulsion Laboratory, National Advisory Committee for Aeronautics, Cleveland, Ohio

(Received June 14, 1954)

Ranges of 105 alpha particles with energies of 0.48, 0.545, and 0.615 Mev as fixed by a velocity selector have been measured by the use of a low-pressure cloud chamber. From the results of these measurements the average ranges R_{Av} and the straggling coefficients in air at 15° and 760 mm were calculated for these three energies. The values of R_{Av} were found to be respectively 0.299, 0.327 and 0.354 cm; and the values of the standard deviation were respectively 0.011, 0.0135, and 0.010 cm. After a correction to take into account the difference in definition, the R_{Av}'s are two to three percent higher than those following from a range-energy curve given by Bethe.

INTRODUCTION

The theoretical expression obtained by Bethe for the energy loss of charged particles in a gas gives, if the effects of the binding of the K-shell electrons of the atoms of the gas are taken into account, quite accurate results for alpha particles down to about 5 Mev.[1] For energies below this, its usefulness is limited because of (1) the difficulty of calculating the L-shell binding correction if the atoms of the gas are of low atomic number, (2) the breakdown of the Born approximation upon which Bethe's derivation is based, and finally, (3) the occurrence below about 1 Mev of an additional complication in the stopping process itself, namely, charge exchange, for which as yet no satisfactory theoretical treatment exists.[2] Hence, the range-

---

[1] M. S. Livingston and H. A. Bethe, Revs. Modern Phys. 9, 261 (1937).

[2] Further details and references are to be found in the review article by H. A. Bethe and J. Ashkin in *Experimental Nuclear Physics* (John Wiley and Sons, Inc., New York, 1953), Vol. I, Part II.