Entropy of Radiation*

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An expression for the entropy flux of linearly polarized electromagnetic radiation of any specific intensity, K_ν, is obtained. The latter expression is then generalized to include radiation of any type of polarization.

IF one has linearly polarized light passing through a surface element, $d\sigma$, the total radiation energy passing through in time, dt, can be written as[1]

$$d\sigma dt \int K_\nu \cos\theta d\Omega d\nu = J d\sigma dt.$$

K_ν is the specific radiation intensity of frequency ν which passes through the surface in the solid angle $d\Omega$; θ is the angle between the direction of propagation and the normal to surface, and J is the net flux. In a similar manner Planck[1] has defined the specific entropy intensity, L_ν, by

$$d\sigma dt \int L_\nu \cos\theta d\Omega d\nu = H d\sigma dt,$$

where H is the net entropy flux.

Our problem is to find a relationship for L_ν in terms of K_ν. Planck has derived a functional relation, $L_\nu = \nu^2 f(K_\nu/\nu^3)$. However, no explicit expression was given for $f(K_\nu/\nu^3)$. If one knows the exact dependence on K_ν, the flux of entropy through a surface may be computed for any radiation field.

Let us consider a surface element, $d\sigma$, through which pass linearly polarized photons of momenta p_x, p_y, p_z and which lie in the momentum volume element $dp_x dp_y dp_z$. If the direction of propagation makes an angle θ with the surface normal, all photons with the above momenta which lie in the space volume element $dV = d\sigma c \cos\theta dt$ will pass through in time dt. If $N(\mathbf{r},\mathbf{p})$ is the average number of photons with position $\mathbf{r}$ and momentum $\mathbf{p}$ in a phase space element of volume h^3, then $N dp_x dp_y dp_z dV/h^3$ is the number of photons in the phase space volume under consideration. Thus the entropy of these photons (bosons)[2] is

$$dS = -k[N \ln N - (N+1) \ln(N+1)]dp_x dp_y dp_z dV/h^3.$$

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[1] M. Planck, *The Theory of Heat* (MacMillan Company, New York, 1949).
[2] J. Slater, *Introduction to Chemical Physics* (McGraw-Hill Book Company, Inc., New York, 1939).

Consequently the entropy flux for linear polarized light of any momentum distribution is

$$H = -k \int [N \ln N - (N+1) \ln(N+1)]c \cos\theta \frac{dp_x dp_y dp_z}{h^3}.$$

It can be easily seen by computing the energy flux that

$$N = c^2 K_\nu/h\nu^3.$$

In terms of K_ν we have

$$H = -\frac{k}{c^2} \int \left[\frac{c^2 K_\nu}{h\nu^3} \ln\left(\frac{c^2 K_\nu}{h\nu^3}\right) \right.$$
$$\left. - \frac{h\nu^3 + c^2 K_\nu}{h\nu^3} \ln\left(\frac{h\nu^3 + c^2 K_\nu}{h\nu^3}\right) \right] \nu^2 \cos\theta d\Omega d\nu.$$

For a general type of polarization we have two independent states of polarization with specific intensities in the principal directions (directions of maximum and minimum intensities) K_ν and K_ν'. For such a case we take the sum of the separate entropy fluxes for each polarization state to obtain the total entropy flux. The intensities in the principal directions of polarization must be used if the entropy flux is to be invariant to a rotation about the direction of propagation.

Note that information about the phase difference between two polarization states and information on the directions of the principal axes does not enter into the expression for the entropy. The photon field is described only by the occupation numbers in the momentum states and the two polarization states. The phase difference and the polarization directions have no thermodynamic significance, but lie in the domain of information theory. Both the phase difference and direction of the polarization can theoretically be measured with infinite precision since no uncertainty principle exists for these. The limits on a measurement of these quantities are solely determined by experimental apparatus, and thus these quantities would not affect the thermodynamic entropy.