Energy Levels of the O^{16} Nucleus

DAVID M. DENNISON
Randall Laboratory, University of Michigan, Ann Arbor, Michigan
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A discussion of the energy levels of the O^{16} nucleus is given on the basis of an α-particle model. A considerable number of excited states are calculated under the assumption that the higher order rotation-vibration interactions can be neglected. The positions of the levels depend upon four parameters, three of which relate to the potential while the fourth measures the effective radius of the nucleus. Two possible correlations between the observed and predicted levels are made. The first identification, which appears to be somewhat the more satisfactory, succeeds in correlating sixteen levels in the range from 0 to 13.25 Mev with regard to their positions, angular momenta and parities. Five levels are predicted in the range from 9.5 to 13.8 Mev which do not appear to have been observed. However, three levels have been found in this range but as yet their spins and parities have not been determined and no attempt has been made to assign them. The effective radius of the O^{16} nucleus is found to be 2.5×10^{-13} cm which is somewhat smaller than might have been expected.

AN earlier discussion[1] of the excited energy states of the O^{16} nucleus on the basis of an α-particle model yielded predictions which could be interpreted as being consistent with the two energy levels which were known at that time. During recent years a considerable number of new experimental data have been obtained, leading to the determination of so many more levels that it appears of interest to re-examine the problem. The present list of observed levels, which is given in the preceding paper by Bittner and Moffat[2] contains (including the normal state) seventeen levels whose angular momenta and parities have been meaured together with three levels of unknown momentum and parity. As will appear, the agreement between the observed and predicted levels, while by no means perfect, seems sufficiently good to suggest that the α-particle model when applied to this particular nucleus, may well contain some elements of correctness. This result is rather surprising since it is not believed that the nucleons within a nucleus have any strong tendency to form α-particle groups, as shown by the fact that, in the overwhelming majority of cases, the α-particle model fails as conspicuously as the shell model succeeds.

It is possible that the reasons for the applicability of the α-particle model in the case of O^{16} may be of the following sort. The nucleus in its normal state is undoubtedly almost spherical in form. If the spherical volume is divided into four equal parts, each part will, on the average contain two protons and two neutrons. Thus even a weak tendency on the part of the nucleons to form α particles can result in imposing a tetrahedral symmetry on the sphere and this is all that is required, at least for the qualitative applicability of the α-particle model. Tetrahedral symmetry is very close to spherical symmetry and it is for this reason that one may expect that the validity of the model may be greater for O^{16} than, for example, for C^{12} or Ne^{20}. In one sense a model consisting of four mass particles at the corners of a regular tetrahedron represents an approximation to the more complex situation of sixteen masses filling a volume which is predominantly spherical but which possesses a certain amount of tetrahedral symmetry. The advantage of employing the simpler model is that the methods of obtaining its energy levels are all well known from the theory of polyatomic molecules and the results may be written down at once. Experience in approximating the vibrations and rotations of a continuum by dividing the region into a series of discrete masses indicates that the model should predict the lower energy states reasonably well but that it will fail to account for all of the higher states.

The model to be discussed is subject to the following assumptions: (1) the O^{16} nucleus is replaced by four α particles with equilibrium positions at the corners of a regular tetrahedron, (2) the vibration rotation levels will be calculated using the molecular approximation and neglecting all higher order interactions, (3) only those levels will be considered whose wave functions are invariant under an interchange of any two α particles. (Bose-Einstein statistics.)

These assumptions are identical with those employed in the earlier paper[1] and the same notation will be retained here. A number of extensions and comments should be made.

1. The Coriolis interaction between the internal angular momentum $\zeta\hbar$ associated with the vibration ω_3 and the total angular momentum $J\hbar$ was only given for one particular case. In general, when these momenta are parallel, perpendicular, or antiparallel, $I_{n_3J}=[-2J\zeta+2\zeta^2-2\zeta]\hbar^2/2A$, $[2\zeta^2-2\zeta]\hbar^2/2A$, or $[2J\zeta+2\zeta^2]\hbar^2/2A$, respectively. For the vibrational levels[3] $\omega_3\hbar$, $(\omega_1+\omega_3)\hbar$, and $(\omega_2+\omega_3)\hbar$, $\zeta=-1/2$; while for that part of $2\omega_3\hbar$ possessing an internal angular momentum, $\zeta=+1/2$. For all of the other states listed, $\zeta=0$.

2. The motions corresponding to the three normal vibrations may be described as follows. In ω_1 (the

[1] D. M. Dennison, Phys. Rev. **57**, 454 (1940).
[2] J. W. Bittner and R. D. Moffat, preceding paper [Phys. Rev. **96**, 374 (1954)].
[3] M. Johnston and D. M. Dennison, Phys. Rev. **48**, 868 (1935). Shaffer, Nielsen, and Thomas, Phys. Rev. **56**, 1051 (1939).

TABLE I. The energy levels of O^{16}.

n_1	n_2	n_3	J	p	$E/\hbar$	Identification (a) E_{calc}	Identification (a) E_{obs}	Identification (b) E_{calc}	Identification (b) E_{obs}
0	0	0	0	+	0	0	0+=0	0	0+=0
0	0	0	3	−	$6R$	(6.13)	3−=6.13	(6.13)	3−=6.13
0	0	0	4	+	$10R$	10.2	4+=10.36	10.2	4+=10.36
1	0	0	0	+	ω_1	(6.05)	0+=6.05	(6.05)	0+=6.05
1	0	0	3	−	ω_1+6R	12.2	3−=11.62	12.2	3−=11.62
0	1	0	2	±	ω_2+3R	(9.83)	2+=9.83	(6.9)	2+=6.9
0	1	0	4	±	ω_2+10R	17.0	···	14.0	4+=13.25
0	0	1	1	−	$\omega_3+9R/4$	(7.0)	1−=7.1	(7.1)	1−=7.1
0	0	1	2	+	$\omega_3+9R/4$	(7.0)	2+=6.9	7.1	···
0	0	1	3	−	$\omega_3+19R/4$	9.5	···	9.6	···
0	0	1	3	−	$\omega_3+27R/4$	11.6	···	11.7	···
0	0	1	4	+	$\omega_3+43R/4$	15.7	4+=13.25	15.8	···
2	0	0	0	+	$2\omega_1$	12.1	0+=11.25	12.1	0+=11.25
0	2	0	0	+	$2\omega_2$	13.5	0+=12.51	7.6	···
0	2	0	2	±	$2\omega_2+3R$	16.6	···	10.7	2+=9.83
0	0	2	0	+	$2\omega_3$	9.4	···	9.6	···
0	0	2	2	±	$2\omega_3+3R$	12.5	2+=11.51	12.7	2+=11.51
0	0	2	1	−	$2\omega_3+R/4$	9.7	1−=9.58	9.9	1−=9.58
0	0	2	2	+	$2\omega_3+17R/4$	13.7	···	13.9	···
1	1	0	2	±	$\omega_1+\omega_2+3R$	15.9	···	12.9	2−=12.51
1	0	1	1	−	$\omega_1+\omega_3+9R/4$	13.0	1−=12.43	13.1	1−=12.43
1	0	1	2	+	$\omega_1+\omega_3+9R/4$	13.0	2+=12.51	13.0	2+=12.51
0	1	1	1	±	$\omega_2+\omega_3+9R/4$	13.8	···	9.9	···
0	1	1	2	±	$\omega_2+\omega_3+9R/4$	13.8	2−=12.95	9.9	···

so-called breathing motion) $q_1=q_2=q_3=q_4=q_5=q_6$. In ω_2, which may be described as a two-dimensional torsional motion in which all particles move on the surface of a sphere, $q_1=q_2$, $q_3=q_4$, $q_5=q_6$ subject to the condition $q_1+q_3+q_5=0$. In the triple degenerate motion ω_3, $q_1=-q_2$, $q_3=-q_4$, and $q_5=-q_6$.

3. The tunnel motion, by which two of the α particles are interchanged, is more intimately connected with ω_2 rather than with ω_3 as stated in reference 1. It is expected that the tunnel energy will be larger for the state $n_2=1$ than for the normal state and larger still if $n_2=2$. A reconsideration of the tunnel energy appears to lead to the conclusion that with the present information regarding the nucleus, it is almost impossible to make a reliable estimate of its magnitude but that it is probably very small, presumably in the kev range. For this reason the tunnel energy has been omitted from the table of levels. When both of the tunnel levels are allowed by the statistics, the parity is designated ±.

4. A rather complete discussion of the symmetry species of the various vibrational and rotational wave functions is given by Jahn[4] for the methane molecule. The adaptation to the present model can be made by simply omitting the fourth methane frequency ω_4 and by setting the spin of each particle (α particle) equal to zero.

The energy expressions for the levels, together with their vibrational numbers $n_1\,n_2\,n_3$, angular momenta J, and parities are listed in the first six columns of Table I. As an abbreviation R is written in place of $\hbar/A$.

There appear to be two more or less obvious ways of correlating the predicted and observed levels. In both identifications (a) and (b), the 3− level at 6.13 Mev is interpreted as a pure rotation state, the 0+ level at 6.05 becomes the first vibrational state of ω_1, while the 1− level at 7.1 is associated with the first vibrational state of ω_3. Under identification (a) the 2+ level at 6.9 is set equal to the 2+ level of the first excited state of ω_3. The zeroth-order theory predicts that this level should coincide with the 1− level but of course small perturbations could easily account for the difference between 6.9 and 7.1. The last identification is the 2+ level at 9.83 which becomes the first excited level of ω_2. The four constants R, ω_1, ω_2, and ω_3 have now been determined and all the remaining levels are now fixed, at least, in zeroth approximation. In the first column under Identification (a) in Table I, the predicted levels are given while in the second column the observed levels are listed. Predicted levels higher than about 16 Mev are not given since this is both above the present range of observations (13.24 Mev) as well as being of dubious significance from the point of view of the validity of the theory.

The second column under Identification (a) lists those observed levels which can readily be accounted for. In all but one case (4+=10.36 which will be discussed later), the measured energies lie below the predicted positions by amounts which for the most part are less than 1 Mev. These displacements are in the direction that would be given by higher order interaction terms and their magnitudes are not unreasonably large. A study of Table I shows that with the aid of four parameters, sixteen levels have been correlated (although perhaps not wholly unambiguously) with regard to their positions, angular momenta and parities. Five levels are predicted in the range from 9.5 to 13.8 Mev which do not appear to have been observed. On

[4] H. A. Jahn, Proc. Roy. Soc. (London) **A168**, 469 (1938).

TABLE II. The parameters of O^{16}.

	R	ω_1	ω_2	ω_3	a	b	c
Identification (a)	1.02	6.05	6.77	4.7	3.42	−0.94	1.73
Identification (b)	1.02	6.05	3.8	4.8	1.86	−0.13	0.09

the other hand in this range, three levels (at 8.6, 11.10, and 13.09 Mev) have been found but their spins and parities are not known and hence no attempt has been made to assign them. Only one observed level, $2-=12.51$, finds no obvious counterpart in the predicted list although it may possibly be the other member of the pair $2\pm$ predicted at 12.5 Mev. It should perhaps be pointed out that the experimental method employed by the Wisconsin group of investigators would not have revealed the existence of the 2−level in the pairs $2\pm$ even if the splittings were large enough to be observed.

Identification (b) differs from (a) in only one respect: the level $2\pm$ associated with the first excited state of ω_2 is assigned $2+=6.9$. Fifteen of the observed levels are accounted for. Many of the correlations are the same as under (a) but there are some differences. Eight levels in the range from 7.1 to 13.9 Mev are predicted but not assigned. Three of these might correspond to the three observed levels of unknown spin and parity. On the whole there seems to be little reason to choose between (a) and (b), although perhaps (a) is slightly superior in that it predicts fewer low-lying levels which are not observed.

The four parameters R, ω_1, ω_2, and ω_3, in Mev for both identifications are given in Table II together with the potential constants a, b, and c described in reference 1. These latter are given in units of 10^{20} dynes/cm. Both sets of potential constants are not unreasonable. The set under (b), where b and c are both negligible in comparison with a ,would describe a potential function between the α particles which would correspond to almost pure central forces.

Although four parameters have been used in describing the energy levels, one of these, $R=\hbar/A$, is adjustable only within a very narrow range since it is closely connected with the dimensions of the nucleus. The moment of inertia A may be set equal to $2\sum Mr_0^2/5$, the formula for the moment of inertia of a uniformly dense sphere of radius r_0 and total mass $\sum M$. In this calculation the tetrahedral symmetry superimposed upon the sphere has been disregarded. A substitution of $R=1.02$ Mev yields $r_0=2.5\times10^{-13}=1.0\times10^{-13}\mathcal{A}^{\frac{1}{3}}$ cm, where $\mathcal{A}=16$, the atomic weight. This figure is not too far out of line with recent measurements[5] of nuclear radii using electron diffraction methods although it is about 10 to 15 percent smaller.

The general agreement shown in Table I between the observed and predicted levels appears to be fairly satisfactory, but there exist a number of causes for uneasiness in taking it at its face value. Among these are the following.

1. The assumption of the grouping of nucleons into α particles within a nucleus receives so little confirmation in the over-all picture of nuclear structure.

2. The two levels $3-=6.13$ and $4+=10.36$ Mev have been assigned to states of pure rotation. Their energies, on the basis of a rigid rotator should be in the ratio of 3 to 5. Thus, if the 3− level is taken to be correct, the 4+ level should lie at 10.22 Mev. If the nucleus is not rigid, the centrifugal distortion, at least for any of the usual types of restoring forces, will always tend to lower the energy. In disagreement with this prediction the observed 4+ level is slightly higher than 10.22 Mev. The next higher pure rotation level $6\pm$ would lie well outside the present range of observation at about 21 Mev.

3. The molecular approximation implies that the Hamiltonian can be developed as a power series in a parameter of smallness. This parameter may be chosen in a number of ways, one of which is to take the ratio of a rotational to a vibrational frequency, for example, R/ω_i. From Table II this is seen to be of the order of 0.2. In the case of the tetrahedral molecule of methane, $R/\omega_i\simeq0.004$. The higher order interaction terms for methane are approximately one or two percent of the zeroth-order terms, and on this basis one would expect that the agreement between the observed and calculated levels of O^{16} might be substantially worse than appears to be the case. It is, however, difficult to assess the reliability of these considerations since the forces between the particles in a molecule are essentially long range forces. Shorter-range, more abruptly varying, forces will tend to increase the rigidity of the nucleus and this in turn may substantially decrease the magnitude of the interaction terms.

I would like to express my thanks to Professor Hugh T. Richards for suggesting a re-examination of the α-particle model and to Professor T. Venkatarayudu for his help in preparing the list of energies, spins, and parities predicted by the model.

[5] Pidd, Hammer, and Raka, Phys. Rev. 92, 436 (1953).