been arbitrarily normalized for each energy to give the best agreement with the theory. Perhaps the best fit is that of Fig. 1, where $s=1.65\times10^{-13}$ cm, $r_0=1.20$ ($c=6.63\times10^{-13}$ cm).[4] The charge distributions of Figs. 2 and 3, which are for slightly different values of s and r_0, show that the cross section depends rather sensitively on these parameters.

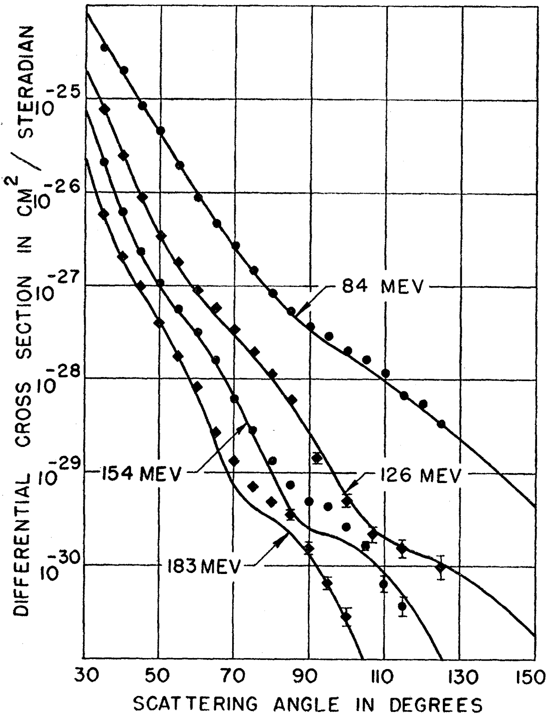

FIG. 2. Cross sections for the parameters $K=1.85$, $c=6.51$ ($r_0=1.20$, $s=1.96$).

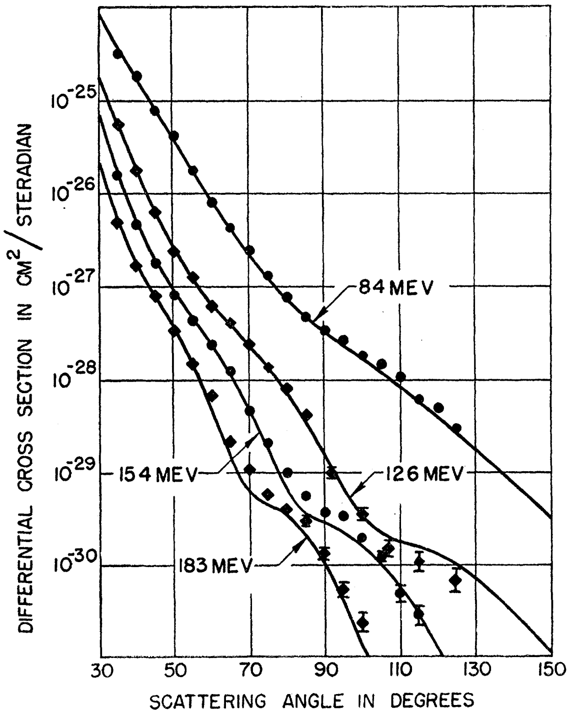

FIG. 3. Cross sections for the parameters $K=1.85$, $c=6.19$ ($r_0=1.16$, $s=1.96$).

The slight differences between the experiments and the results of Fig. 1, if significant, can be due to a number of different effects. Perhaps a more complicated charge distribution is required; preliminary calculations indicate that a decrease in the central charge density alters the cross sections in such a way as to improve the fit with the 183-Mev data.[5] There may also be appreciable contributions from the static quadrupole moment, nuclear excitations, and radiative corrections. L. I. Schiff and B. Downs[6] have considered the first of these, together with quadrupole excitations to low-lying levels, on the basis of a modified Born approximation.[3] They find that in gold these effects add at most three percent to the elastic cross section. The other effects have not yet been estimated.

For values of s and r_0 quoted above, the cross sections in Pb^{208} also agree very well with the data of Hofstadter *et al.*[2]

These calculations were made on the computer Univac at the University of California Radiation Laboratory at Livermore. We thank the authorities of this Laboratory, particularly Dr. S. Fernbach, for permission to use Univac, and Mr. H. Hanerfeld for instruction and advice in coding. We wish to thank Professor R. Hofstadter, Dr. B. Hahn, and Dr. J. A. McIntyre for discussions about their work and ours, and Professor L. I. Schiff and Professor G. C. Wick for helpful conversations. We also thank Dr. G. E. Brown for communicating his results and those of Drs. S. Brenner and L. R. B. Elton prior to their publication.

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* Supported in part by the Office of Scientific Research, Air Research and Development Command.

[1] Yennie, Ravenhall, and Wilson, Phys. Rev. **95**, 500 (1954).

[2] Hofstadter, Hahn, Knudsen, and McIntyre, Phys. Rev. **95**, 512 (1954).

[3] An elaboration of the following results will be presented later, together with some applications.

[4] This is slightly larger than the tentative value of r_0 ($r_0=1.1\pm0.1$) obtained in reference 2.

[5] Dr. G. E. Brown informs us that he and Drs. S. Brenner and L. R. B. Elton have come to a similar conclusion.

[6] L. I. Schiff, Phys. Rev. (to be published) and private communications.

Acquirement of Cosmic-Ray Energies by Electromagnetic Induction in Galaxies*

W. F. G. SWANN
Bartol Research Foundation of The Franklin Institute,
Swarthmore, Pennsylvania

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IN 1933, the present writer applied to a special case[1] the principle of acceleration of charged particles to cosmic-ray energies through electromagnetic induction.

The present communication represents an extension and generalization to changing magnetic fields in galaxies. Attention is confined to problems of axial symmetry about the axis of z, i.e., to problems in which the electric and magnetic fields E and H show no variation with the θ coordinate.

If one applies Lagrange's equations to the Lagrangian function, L, given in cylindrical coordinates by

$$L=-mc^2(1-r^2\dot{\theta}^2/c^2-\dot{r}^2/c^2-\dot{z}^2/c^2)^{\frac{1}{2}}+er\dot{\theta}U_\theta/c,$$

where the scalar potential ϕ is zero in our problem, and where U_θ is the vector potential and m the rest mass of the particle, it readily follows that:

(1) If a charged particle starts to acquire kinetic energy at $t=0$, and if the magnetic field is zero at $t=0$, then the particle acquires energy continually.

(2) If, at $t=0$, the magnetic field is finite, a sufficient, but by no means necessary, condition for continual gain of energy is that $|E|\geqslant|H|$ for all positions and times.

It can be readily shown that when the magnetic field and kinetic energy T are both zero at $t=0$,

$$\frac{d}{dt}(T+mc^2)^2=e^2\frac{\partial U_\theta^2}{\partial t^2}=e^2\left(\frac{dU_\theta^2}{dt^2}-\dot{r}\frac{\partial U_\theta^2}{\partial r}-\dot{z}\frac{\partial U_\theta^2}{\partial z}\right).$$

Further, it can readily be shown that the terms $(-\dot{r}\partial U_\theta^2/\partial r)$ and $(-\dot{z}\partial U_\theta^2/\partial z)$ are always positive, so that

$$\frac{d}{dt}(T+mc^2)^2\geqslant e^2\frac{dU_\theta^2}{dt^2},$$

and consequently, since T and U_θ are zero at $t=0$,

$$(T^2+2mc^2T)^{\frac{1}{2}}\geqslant eU_\theta.$$

From this result, and neglecting $2mc^2T$ in comparison with T^2, for convenience but not of necessity, it appears that the kinetic energy T acquired by a proton which finds itself at the cylindrical radius r, where the average z component of magnetic field is $\bar{H}_z$ within that radius is such that

$$T\geqslant 150r\bar{H}_z \text{ electron volts.}$$

If $r\sim25\,000$ light years, as for Andromeda, and $\bar{H}\sim7\times10^{-6}$ gauss, $T\sim2.3\times10^{19}$ ev.

It also appears that the particle, in its motion, spirals outwards, inwards or remains on a circle, according as $\bar{H}_z>2H_z$, $\bar{H}_z<2H_z$, $\bar{H}_z=2H_z$, respectively, where $\bar{H}_z$ is the average z component within the radius r at which the z component of the magnetic field has the value H_z. If in its journey through places where $\bar{H}_z\neq2H_z$ the particle reaches a place where $\bar{H}_z=2H_z$, then it will asymptotically approach a circular orbit for that value of r.

A strict application of the foregoing to a case where the growth of the magnetic field has occurred uniformly throughout the life of the universe ($\sim5\times10^9$ years) would encounter the difficulty that since the mean life of a cosmic ray as determined by nuclear collisions is of the order 10^8 years, the rays which started to acquire energy at $t=0$ would no longer be with us. The difficulty can be surmounted by assuming that the magnetic field has a period of less than 4×10^8 years, or by considering the case of neutral particles which become ionized and start to acquire energy after the magnetic field has grown to a finite value. The latter problem requires more intricate mathematical considerations which will be dealt with in a later communication.

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* Assisted in part by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission.

[1] W. F. G. Swann, Phys. Rev. **43**, 217 (1933); J. Franklin Inst. **215**, 273 (1933). A simplified version of the original problem is given by the writer in the September, 1954 issue of the *Journal of The Franklin Institute*, and the full details associated with the *present* communication will be published in the November, 1954 issue of that Journal.

Gamma Radiation from the Reactions $Na^{23}+p$

J. O. Newton
Atomic Energy Research Establishment, Harwell, England

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THERE are three reactions which produce gamma radiation when sodium is bombarded with protons; these are $Na^{23}(p,p'\gamma)Na^{23}$, $Na^{23}(p,\alpha\gamma)Ne^{20}$, and $Na^{23}(p,\gamma)Mg^{24}$. Sodium iodide scintillation detectors have been used to study the gamma rays from these processes at most of the resonances in the range of proton bombarding energy from 0.7–1.5 Mev. The targets were of sodium bromide. Both thin-target and thick-target excitation functions were measured.

The energies of the gamma rays from the inelastic scattering process and from the $(p,\alpha\gamma)$ reactions were found to be 444±5 kev and 1.629±0.008 Mev, respectively, in agreement with other work.[1,2]

The observed capture gamma rays can be related to transitions to the ground state of Mg^{24}, to and from the well-known levels at 1.37 Mev and 4.12 Mev, and to and from the level recently reported by Hausman *et al.*[3] at 4.24 Mev (see Fig. 1). It seems likely that this level decays both to the ground state and to the 1.37-Mev level, since gamma rays of energy 4.24±0.04 Mev and 2.8±0.05 Mev, with relative intensities of about 1 and 0.2, respectively, always occurred together when this mode of decay was observed.

In addition, a gamma ray of energy 3.86±0.04 Mev was definitely observed at one resonance and less definitely at two others. It was found to be in coincidence with gamma rays of energy 1.37 Mev and 7.9 Mev. A possible interpretation is that it arises from a triple cascade transition through a level at 5.23±0.04 Mev. Such a level could be reached in the β decay of Na^{24}.