tillation counters showed the presence of a 0.56-Mev and a 1.06-Mev γ ray, both decaying with a half-life of 25 sec. The two γ rays are in coincidence.

(5) Since the half-life of the $i_{13/2}$ state of $_{82}Pb^{207}$ is 0.84 sec,[3] the 7.14-Mev alpha particles should not be in coincidence with γ rays. α–γ coincidence experiments with pulse-heights election confirmed this conclusion.

The absence of a 7.43-Mev α-peak in the pulse-height distribution indicates that the partial half-life of the γ transition from the 25-sec state to the 0.52-sec state is at least a hundred times as large as that for the α transition. From Weisskopf's formula[4] it follows that a transition with $L\geq 5$ is necessary to explain such a long half-life. According to shell theory, the 0.52-sec level should have a spin of 9/2. The 25-sec state then must have a spin of at least 19/2. This conclusion is also supported by α-decay considerations. Calculations of the α-decay half-lives, made using a semiempirical formula,[5] yield the result that spin changes of about 9,

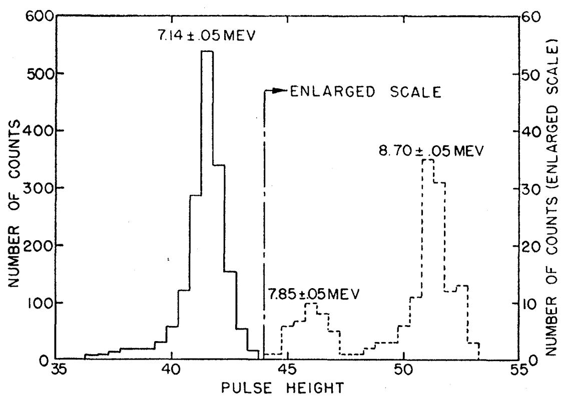

FIG. 2. Pulse-height distribution of the α particles from the 25-sec state of $_{84}Po^{211}$.

8, and 5 units accompany the 8.70, 7.85, and 7.14-Mev α decays of the 25-sec level and that the 7.43-Mev α decay occurs with a spin change of about 4 in agreement with the shell theory assignment of 9/2 for the spin of the 0.52-sec level. These results show that the spin of the 25-sec state very probably lies between 19/2 and about 23/2.

The simple single-particle model cannot explain the high spin of this excited state, since the highest reasonable spin for a low excited level of $_{84}Po^{211}$ would occur for the 127th neutron in a $j_{15/2}$ state. Spiess' suggestion that the excited state of $_{84}Po^{211}$ is due to "core isomerism"[6] is consistent with our decay scheme. By exciting the proton core to a $(h_{9/2}f_{7/2})$ configuration, and assuming the 127th neutron in the $g_{9/2}$ state,[7] the maximum spin of the 25-sec state would be 25/2.

It is somewhat surprising that the expected 25-sec α-particle group which decays to the $f_{5/2}$ state of $_{82}Pb^{207}$ could not be found. There seems to be some consistency, however, since the decay of the 0.52-sec level of $_{84}Po^{211}$ to the $f_{5/2}$ state of $_{82}Pb^{207}$ is also unexpectedly weak.[8]

According to these results the 8.70-Mev α particles decay with a partial half-life of about 360 sec from a state with a spin of at least 19/2 into a $p_{\frac{1}{2}}$ state. This is probably the largest spin change known for a nuclear decay.

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Vacuum Polarization in a Strong Coulomb Field*

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IN the interpretation of recent experiments with x-rays from μ-mesonic atoms, some reference has been made to the role of vacuum polarization both in connection with the μ-mass determination[1] and in connection with nuclear radius determinations.[2–5] The estimates made have been based on the Uehling formula,[6] which is merely the first term of an expansion in powers of αZ. Since the experiments have been carried out in elements as heavy as lead, for which $\alpha Z=0.6$, the role of higher-order corrections may be significant.

Another situation in which such effects may play an experimentally significant role is in connection with the x-ray fine-structure separation $(2P_{\frac{1}{2}}-2P_{\frac{3}{2}})$ in the heavy elements. The large value of the nuclear radius deduced by Schawlow and Townes[7] from the anomalous Z dependence of this fine structure might be brought into agreement with values determined by other methods by including electrodynamic effects. One must in this case, of course, include the effects of the mass operator as well as those of the polarization charge.

There is, of course, also some general theoretical interest in the treatment of such quantities, by techniques which either avoid power series expansions in certain quantities, usually treated as small, or which establish the convergence of these power series. Ac-

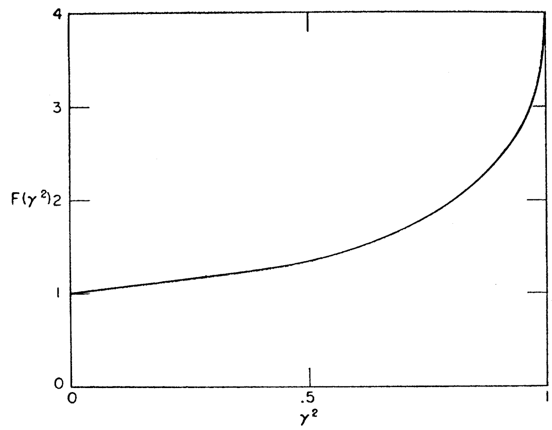

FIG. 1. The function $F(\gamma^2)$ defined in Eq. (4) plotted as a function of γ^2.

cordingly, a new attack has been made on the polarization charge problem with these objectives in view.

It is well known that the polarization charge density in a static electric field can, apart from radiative corrections, be expressed in terms of the solutions to the Dirac equation in that field. Starting from such an expression, making use of a contour integral representation of the summation over the radial eigenstates for a given angular momentum state, and making use of the fact that the solution of the Dirac equation for the case of the Coulomb field are known, one can show that the Laplace transform of $r^2\rho(r)$ is given by

$$Q(p)=\int 4\pi r^2\rho(r)e^{-pr}dr$$
$$=\frac{q}{\pi Z}\sum_{k=1}^{\infty}k\int_0^{\infty}dt\int_0^1 dz\Bigg[\bigg(\frac{1}{(1-z)(1+uz)}+\frac{1}{1-z+uz}\bigg)\frac{\gamma\cos g}{(1+t^2)^{\frac{1}{2}}}+\frac{2ts(k)\sin g}{(1+t^2)(1+uz)}+\frac{2\gamma t^2\cos g}{(1+t^2)^{\frac{3}{2}}(1+uz)}\Bigg]P^{-s(k)},\quad (1)$$

with

$$P=(1+uz)(1-z+uz)(1-z)^{-1},$$
$$g=\gamma t(1+t^2)^{-\frac{1}{2}}\ln(1-z)(1+uz)(1-z+uz)^{-1},$$
$$s(k)=(k^2-\gamma^2)^{\frac{1}{2}},\quad u=\tfrac{1}{2}p(1+t^2)^{-\frac{1}{2}},\quad \gamma=\alpha Z=|eq|/\hbar c,$$

where $\rho(r)$ is the polarization charge density induced by the potential q/r. Renormalization, divergence, and gauge difficulties are dealt with by examining the behavior of $Q(p)$ at small p and removing a term of the form $ap^{-2}+bp^{-1}+c\ln p+d$, with the coefficients chosen to yield $Q(0)=0$. It can be shown that this procedure is equivalent to regularization. It is worth noting that $Q(p)$ is virtually as useful a quantity as $\rho(r)$ itself. For example, the behavior of the polarization potential at small distances is determined by $Q(p)$ at large p, while the expectation values of the potential for Schrödinger wave functions can be expressed in terms of the derivatives of the Laplace transform of the potential multiplied by the radial variable r. This quantity is related to $Q(p)$ through

$$u(p)=\int rV(r)e^{-pr}dr=\frac{1}{p^2}\int_0^p Q(p')dp'.\quad (2)$$

Equation (1) serves as a convenient starting point for expansion in powers of γ. We have confirmed that $Q^{(1)}(p)$, the lowest-order term (linear in γ) is precisely the Uehling term, while for the third-order term we find

$$\begin{aligned}Q^{(3)}(p)&=q(\alpha\gamma^2/\pi)[-0.115(p/2)+0.166(p/2)^2-0.274(p/2)^3+0.303(p/2)^4-0.187(p/2)^4\ln p+\cdots];\quad p\ll 1\\&=-0.0209\alpha q\gamma^2\text{ for }p\to\infty.\end{aligned}\quad (3)$$

The computational problem of obtaining (3) from (1) seems to be orders of magnitude simpler than obtaining the corresponding results from Feynmann integrals. The appearance of the $p^4\ln p$ term corresponds to an asymptotic $1/r^7$ behavior at infinity for $\rho(r)$. This result can, in fact, be deduced from the Euler-Heisenberg Hamiltonian.[8] The coefficient obtained by the two methods are in agreement.

Finally we note that similar results can be obtained without expanding in powers of γ. In particular, we find

$$\begin{aligned}\delta Q&=\lim_{p\to\infty}[Q(p)-Q^{(1)}(p)]\\&=(q/\pi Z)\bigg(\frac{2}{3}\sum\frac{1}{n^3}-\frac{\pi^2}{6}+\frac{7}{9}\bigg)\gamma^3\\&\quad-(2q/\pi Z)\sum_{k=1}^{\infty}k\bigg(\sin^{-1}\frac{\gamma}{k}-\frac{\gamma}{k}-\frac{\gamma^3}{6k^3}\bigg)\\&\quad+(4q/\pi Z)\sum_{k=1,n=1}^{\infty}\bigg(\frac{nk\gamma}{[n+s(k)]^2+\gamma^2}-k\tan^{-1}\frac{\gamma}{n+s(k)}+\frac{k^2\gamma}{(n+k)^2}-\frac{3n^2-k^2}{(n+k)^4}\gamma^3\bigg)\\&=-0.0209\alpha q\gamma^2-0.0071\alpha q\gamma^4F(\gamma^2),\end{aligned}\quad (4)$$

where $F(\gamma^2)$ is plotted in Fig. 1. The small-r behavior of the polarization potential is given by

$$V(r)=\frac{2\alpha q}{3\pi r}\bigg(\ln\frac{1}{rC}-\frac{5}{6}\bigg)+\frac{\delta Q}{r},\quad (5)$$
$$C=0.577\cdots.$$

It is clear from Eq. (4) that a perturbation theory expansion of δQ converges[9] with radius of convergence $\gamma=1$. It is also interesting to note that δQ remains finite in the limit $\gamma \rightarrow 1$.

The corrections to the Uehling formula for $V(r)$ at small r are very small, even for lead, so that its use for the μ-meson work appears to be well justified.

Formulas like (4) can be obtained for all of the coefficients in the small-p expansion of $Q(p)$, although the labor involved increases with the order of the coefficient. In view of the smallness of (4), however, it is unlikely that the terms beyond the cubic are ever important.

A fuller account of the work described above, together with some applications, will be subsequently submitted for publication.

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Polarization of Arsenic Nuclei in a Silicon Semiconductor*

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A NEW mechanism capable of producing nearly 100 percent polarization of nuclear spins under moderate conditions of field and temperature has been found in the course of electron spin resonance studies.

Electron spin resonances exhibiting resolved hyperfine structure have recently been observed by Fletcher *et al.*[1] on a number of impurity-doped silicon samples. This note is concerned with a similar sample of arsenic-doped silicon; it is reported to contain 1.3×10^{17} As atoms per cm^3. The sample was generously provided by Dr. F. J. Morin of Bell Telephone Laboratories.

At a temperature of 4°K and at a frequency of about 9000 Mc/sec, the four resonances reported by Fletcher were observed. These lines correspond to each of the four possible orientations in the magnetic field of the As^{75} nucleus, which has a spin of $\frac{3}{2}$. The lines were approximately 3 oersteds broad, were spaced about 73 oersteds apart, and exhibited inhomogeneous saturation behavior.[2]

On each one of the resonances, the following observations were made: If the magnetic field was swept twice through the same resonance with a time t elapsing between the two traversals, the amplitude A_2 of the second traversal of the resonance line is decreased relative to the amplitude of the first traversal by approximately

$$A_2/A_1=1-e^{-t/T},$$

where T is about 16 seconds. Also, by sweeping through two neighboring hyperfine components sufficiently rapidly, an *enhancement* of the second line relative to the first line was obtained, this enhancement being about a factor of two larger if the first line was one of the extrema of the hyperfine multiplet.

The proposed explanation of this effect utilizes the process whereby electron spin relaxation can effect a nuclear spin reorientation through the $\mathbf{I}\cdot\mathbf{S}$ magnetic hyperfine interaction. In sweeping through a resolved hyperfine resonance line using sufficient rf power, we suppose that virtually all of the relevant As nuclei undergo a Δm_I of ±1, thus depopulating the m_I level associated with the particular line. Then, if the nuclear spin relaxation time is about 16 seconds, the observed effect results. However, in order for the net depopulation of the m_I level to occur during the electronic resonance, it is necessary to have a mechanism whereby the number of nuclear spins flipped by electron spin relaxations is greatly enhanced when *on the electronic resonance.* A mechanism for this was suggested by J. I. Kaplan and is described in the following Letter. It consists essentially of a broadening of the power spectrum of the electron relaxation, centered at the electron spin Larmor frequency, due to the shortened lifetime of an electron spin state during resonance; this produces a larger Fourier amplitude at the nuclear Larmor frequency, thereby increasing the probability of a nuclear spin flip. In the present As-doped sample, it is estimated that, for an rf amplitude of $H_1=0.010$ oersted, the time for a nuclear spin flip would be of the order of 10^{-2} second due to this mechanism, as contrasted with 16 seconds under no rf.

Manifestations of this effect are also seen in sweeping through a single line. The shape of the line shows an asymmetry and large g shift due to a progressive depopulation of the associated nuclear m_I state. The *inhomogeneous* line width and spin diffusion time of the Si^{29} nuclei enter into the quantitative analysis of this behavior.

In the present case, to polarize completely the As nuclei in a particular m_I state, one has to sweep through the resonances corresponding to the other three m_I states in a time small compared to the nuclear relaxation time. In the case of Si doped with an impurity having a spin of $\frac{1}{2}$, such as phosphorus, satisfying the resonance condition for one m_I state would completely polarize the other state. The general requirement is that sufficient rf power be used so that the nuclear spin flip time in the presence of the rf will be short compared to the nuclear relaxation time in the absence of rf. It is to be