Now by (7), $(N+1)/V>P>N/V>Q>(N-1)/V$, so that $Q=(N-\alpha)/V$, $0<\alpha<1$. These relations give rise to

$$\frac{N+1}{V}>Q+\left(\frac{N-\alpha}{V}\right)\frac{\epsilon Z_N/Z_{N-1}}{1+(1-\epsilon)Z_N/Z_{N-1}}>Q>\frac{N-1}{V}. \quad (10)$$

It follows that in the limit L, $\epsilon Z_N/Z_{N-1}\rightarrow 0$. This proves our statement.

For Bose-Einstein systems, we use[5]

$$(1+1/N)(Z_{N+1}/Z_N)>Z_N/Z_{N-1}>Z_{N+1}/Z_N. \quad (11)$$

Thus for all limiting processes for which N tends to infinity $Z_{N+1}/Z_N\sim Z_N/Z_{N-1}$, and therefore relations $a_{N+1}\sim a_N$ hold.

To establish relations of the type $x_{N+1}\sim x_N$ for Bose-Einstein systems, one can use an argument of the type given in Sec. 4 of Fraser's paper. If M be a positive integer smaller than N, one first notes that

$$R(N,M)\equiv\sum_{r=M+1}^{\infty}a_N^{-r}-\sum_{r=M+1}^{N}\frac{Z_{N-r}}{Z_N}\exp(-r\eta_j)<a_N^{-M}(a_N-1). \quad (12)$$

Hence

$$n(N,j)>b_M(1-a_N^{-M})/(a_N-1),$$

where $b_M\equiv(N-1)(N-2)\cdots(N-M+1)N^{-r+1}$. Since $x_{N+1}=(1/a_{N+1})\{1+[1/n(N,j)]\}$, it follows that

$$a_N/a_{N+1}<x_{N+1}<\frac{1}{a_{N+1}}\left[1+\frac{1}{b_M}\frac{a_N-1}{1-a_N^{-M}}\right]. \quad (13)$$

The first inequality follows immediately from (9). Let M go to infinity with N, but more slowly than $\sqrt{N}$, so that[5] $b_M\rightarrow 1$. It now follows from (13) that for all limiting processes for which $N\rightarrow\infty$, $x_{N+1}\rightarrow a_N/a_{N+1}$ whether $a_N\rightarrow 1$ or $a_N\rightarrow a>1$. By (11), $a_N/a_{N+1}=1+\gamma/N$ where $0<\gamma<1$. Hence relations $x_N\sim x_{N+1}$ hold for all limiting processes of this type, and, in particular, for the limit L. Fraser's restrictions on the nature of the single-particle energy spectrum are not required.

For $T=0$ the mean occupation numbers can be calculated exactly from (1) by evaluating $(Z_{N+1}/Z_N)\times\exp\eta(j)$ directly. The results agree with those obtained from the grand canonical ensemble.

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Use of *p-n* Junctions for Solar Energy Conversion

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The recent discovery[1] that silicon *p-n* junction photovoltaic cells possess a nearly tenfold higher efficiency for solar energy conversion than photocells previously available has reawakened interest in this type of device. Cummerow[2] has presented a theory of the photovoltaic effect in a *p-n* junction cell with monochromatic radiation incident in a direction normal to the plane of the junction and has also made as estimate of the efficiency to be expected from the use of a silicon cell in solar energy conversion.[3] Similar calculations by the writer[4] lead to conclusions substantially identical to those arrived at by Cummerow with one exception. For maximum power conversion efficiency we propose the use of a semiconductor with an energy separation in the neighborhood of 1.5–1.6 ev in preference to Cummerow's choice of a "gap energy somewhere near the photon energy characteristic of the peak of the solar spectrum, i.e., 2 ev."

The basis for this conclusion may be found in Fig. 1 which shows a plot of conversion efficiency (neglecting surface reflection losses and ohmic losses within the cell, and assuming that the depth of the junction below the illuminated surface is small compared to a diffusion length) as a function of band separation for several values of donor and acceptor densities ($N_d=N_a=10^{15}$, 10^{17}, $10^{19}/\text{cm}^3$). The plot is based upon computations performed for germanium ($\Delta E=0.7$ ev), silicon (1.0 ev), aluminum antimonide (1.6 ev) and for a hypothetical substance with a band separation of 2.0 ev. The values taken for the diffusion constants and lifetimes in germanium and silicon are the same as those employed by Cummerow[2,3] and values equal to those of silicon have been assumed for the remaining substances. Fortunately

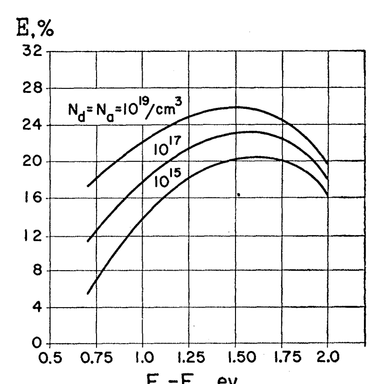

FIG. 1. Computed conversion efficiency *vs* band separation for several values of donor and acceptor density. Load resistance chosen for maximum power output.

the conversion efficiency is quite insensitive to the choice of numerical values of these parameters. The solar spectrum has been approximated by the distribution from a black body at a temperature of 5760°K with an integrated intensity at the earth's surface of 0.1 watt/cm^2.

Note that the conversion efficiency increases with the degree of doping because of an increase in the height of the potential barrier; that the band separation best matching the solar spectrum is, as noted above, 1.5–1.6 ev; and that the maximum obtainable efficiency (assuming that the saturation solubility of donors and acceptors is of the order of $10^{19}/\text{cm}^3$) is roughly twenty five percent, corresponding to a power output of about 250 watts/meter^2 in full sunlight. A slight further increase in efficiency may be realized with the use of an optical collection system which increases the radiation intensity at the photocell surface. Note in addition that silicon is clearly a better choice for the present purpose than other materials that have been proposed[2,5,6] such as germanium and cadmium sulfide ($\Delta E = 2.4$ ev) but that silicon is in turn surpassed by aluminum antimonide. The superiority of AlSb over Si with respect to conversion efficiency would prevail even if the minority carrier lifetimes were lower by several orders of magnitude in the former material. The possibility of substantially improving the efficiency of experimental units over that heretofore realized[1] appears quite promising.

It is a pleasure to acknowledge several valuable discussions with Dr. F. K. du Pré and with T. R. Kohler.

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Influence of the Geomagnetic Field on the Extensive Air Showers

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RECENTLY, Cocconi[1] published a paper on the influence of the geomagnetic field on the extensive air showers. According to this calculation the shape of the lateral spread of an extensive shower should not be circular, but elliptical, the main axis in the east-west direction being about twice as long as the other one.[2]

In July, 1954, we measured this effect on the top of Lomnický Štít (altitude 2634 m, 48° N geomagnetic latitude). We used two telescopes, each of which consisted of two trays of five argon-ethylene counters (two of them 40×500 mm, three of them 45×600 mm). The distance between these telescopes was 7 m. The separation of the trays of counters in the telescope was 800 mm. The telescopes were at the zenith angle of 45°, successively oriented towards east, west, north, and south; and fourfold coincidences were recorded. The results are as follows:

E: 45.2 ± 2.9 coincidences per hour,
W: 50.1 ± 2.6 coincidences per hour,
N: 38.3 ± 2.3 coincidences per hour,
S: 38.5 ± 2.3 coincidences per hour.

In view of the rather large statistical errors, we would not like to draw any conclusion about the precise value of the influence of the geomagnetic field on the extensive air showers, but it seems that our measurements prove the existence of such an effect. The measurements are being continued. A more complete paper, containing also a discussion of the results, will be published soon in the *Czechoslovakian Journal of Physics.*

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Coulomb Interference Effects in Proton-Proton Scattering*

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COULOMB interference effects in proton-proton scattering should be useful in discriminating between different combinations of phase shifts which equally well describe the cross section and polarization arising from purely nuclear interactions.[1] Calculations have been made of these effects for 200 Mev on the assumption only that s and p waves enter into the interaction.

In order to include the Coulomb interaction, the Møller formula for relativistic electron-electron scattering has been generalized by adding a Pauli term to the interaction Hamiltonian, to account for the proton's anomalous magnetic moment $\mu_a = 1.793$. The Coulomb scattering matrix so obtained may conveniently be written, correct through terms of order $1/\theta$,

$$M_c = M_{c+} + M_{c-}, \quad M_{c-}{}^S(\theta,\phi) = (-1)^S M_{c+}(\pi-\theta, \pi-\phi),$$

$$M_{c+} = -\frac{\eta}{2k\sin^2(\theta/2)}\left[1 - \nu i\left(\frac{\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2}{2}\right)\cdot \mathbf{n}\sin\theta\right], \quad (1)$$

$$\nu = \frac{(\epsilon - 1)}{(2\epsilon^2 - 1)}[(2\epsilon+1) + 2\epsilon(\epsilon+1)\mu_a]. \quad (2)$$