and will therefore not appear in Eq. (3.10). If expression (2.1) is used for the Coulomb scattering of the electrons Eq. (3.10) becomes

$$p_{i,j}(E_0,t_0;E,r,\theta,t)=\delta(\theta)\delta(r)\pi_{i,j}(E_0,t_0;E,t)$$
$$+\int_{t_0}^{t}dt'\int_{E}^{E_0}dE'\int_{-\infty}^{\infty}d\theta'\pi_{i,1}(E_0,t_0;E',t')$$
$$\times[\sigma_1(E',t';\theta')-\delta(\theta')\mu_1(E',t')]$$
$$\times p_{1,j}(E',t';E,r-\theta'(t-t'),\theta-\theta',t). \quad (5.2)$$

This is also a generalization of Blatt's equation[1] in that it also treats the case of an incident photon ($i=2$). It should be noted that for this case it yields an immediate solution for $p_{2,j}$ in terms of the equivalent expression for a primary electron.

It should be noted that $\pi_{i,1}$ may contain such processes as ionization loss, Compton effect, etc., and hence (5.2) is valid for any energy range for which we have the average numbers. It is obvious that this leads to a simple recursion relation for the moments which greatly simplifies previous work. See for instance Chartres and Messel.[8]

One of us (B.A.C.) wishes to thank Commonwealth Scientific and Industrial Research Organization for the grant of a studentship.

[8] B. A. Chartres and H. Messel, Proc. Phys. Soc. (London) **A67**, 158 (1954).

PHYSICAL REVIEW VOLUME 96, NUMBER 6 DECEMBER 15, 1954

Possible Triple-Scattering Experiments*

L. Wolfenstein
Carnegie Institute of Technology, Pittsburgh, Pennsylvania
(Received September 7, 1954)

Triple-scattering experiments may be used to get additional information about the spin-dependence of the scattering matrix of the second scatterer. In general two new parameters describing the scattering may be determined by means of two distinct experiments, one in which the successive scattering planes are parallel and one in which the successive scattering planes are at right angles to each other. The relation between these parameters and the scattering matrix is given for the cases of protons scattered from a spin-zero target and of proton-proton scattering. For the former case the magnitude of the left-right asymmetry in the third scattering is calculated on the basis of a phenomenological model due to Fermi in the Born approximation. Further experimental possibilities for p-p scattering are discussed.

1. GENERAL FORMULATION

Recent successful experiments[1] on the double scattering of high-energy protons make it of interest to note that further information may be obtained by means of triple-scattering experiments. Such experiments would be designed to determine how the second scatterer changes the direction and/or magnitude of the polarization of the proton; thus, the first scatterer serves simply as a polarizer and the final scatterer as an analyzer.

To describe the geometry of a triple-scattering experiment we first define for each scattering the unit vector $\mathbf{n}$,

$$\mathbf{n}=(\mathbf{k}\times\mathbf{k}')/|\mathbf{k}\times\mathbf{k}'|, \quad (1.1)$$

where $\mathbf{k}$ and $\mathbf{k}'$ are unit vectors in the incident and outgoing laboratory directions, respectively. The beam incident on the second scatterer is polarized along the direction $\mathbf{n}_1$. For a given scattering angle θ, the second scattering is completely defined by an azimuthal angle φ, here defined by

$$\cos\varphi=\mathbf{n}_1\cdot\mathbf{n}_2 \quad \sin\varphi=\mathbf{n}_1\times\mathbf{n}_2\cdot\mathbf{k}_2. \quad (1.2)$$

In the third scattering a left-right asymmetry is measured relative to a direction $\mathbf{n}_3$; since polarization along the direction of motion cannot be detected, two "settings" of the analyzer are sufficient; that is, two directions for $\mathbf{n}_3$. Therefore we need only consider the cases when $\mathbf{n}_3$ is parallel to $\mathbf{n}_2$ and when $\mathbf{n}_3$ is along the direction

$$\mathbf{s}=\mathbf{n}_2\times\mathbf{k}_2'. \quad (1.3)$$

Thus the third scattering may be chosen to determine either $\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{n}_2$ or $\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{s}$, where $\langle\boldsymbol{\sigma}\rangle_2$ is the expectation value of the spin vector after the second scattering; the corresponding asymmetries in triple scattering will be designated e_{3n} and e_{3s}.[2] The discussion throughout is nonrelativistic.

* Work supported in part by U. S. Atomic Energy Commission.
[1] Oxley, Cartwright, and Rouvina, Phys. Rev. **93**, 806 (1954); Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys. Rev. **93**, 1430 (1954); Marshall, Marshall, and Carvalho, Phys. Rev. **93**, 1431 (1954); J. M. Dickson and D. C. Salter, Nature **173**, 946 (1954); Kane, Stallwood, Sutton, Fields, and Fox, Phys. Rev. **95**, 1694 (1954).

[2] The asymmetry e_{3n} (or e_{3s}) is defined as $[I_3(+)-I_3(-)]/[I_3(+)+I_3(-)]$ where $I_3(\pm)$ refers to scattering such that $\mathbf{n}_3$ is parallel to $\pm\mathbf{n}_2$ (or $\pm\mathbf{s}$).

It may be shown[3] that $I_2\langle\boldsymbol{\sigma}\rangle_2$ depends at most linearly on $\langle\boldsymbol{\sigma}\rangle_1$, where I_2 is the differential scattering cross section of the second scatterer and $\langle\boldsymbol{\sigma}\rangle_1$ is the expectation value of the spin vector before the second scattering. Using this fact and noticing that $\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{n}_2$ is a scalar while $\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{s}$ is a pseudoscalar, we determine the most general dependence of these quantities on $\mathbf{k}_2$, $\mathbf{k}_2'$, and $\langle\boldsymbol{\sigma}\rangle_1$:

$$I_2\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{n}_2=I_0(P_2+D\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{n}_2) \quad (1.4a)$$

$$I_2\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{s}=I_0[A\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{k}_2+R\langle\boldsymbol{\sigma}\rangle_1\cdot(\mathbf{n}_2\times\mathbf{k}_2)]. \quad (1.4b)$$

Here P_2, D, A, and R are arbitrary functions of $\mathbf{k}_2\cdot\mathbf{k}_2'$, that is, of the scattering angle θ; for convenience we have factored out I_0, the differential cross section for an unpolarized beam, which is a function of θ alone.[4] If the beam entering the second scatterer is unpolarized it leaves with a polarization given by P_2, which is therefore just the familiar polarization function determined in double-scattering experiments. For later purposes we also write the expression for the undetectable component of $\langle\boldsymbol{\sigma}\rangle_2$:

$$I_2\langle\boldsymbol{\sigma}\rangle_2\cdot\mathbf{k}_2'=I_0[A'\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{k}_2+R'\langle\boldsymbol{\sigma}\rangle_1\cdot(\mathbf{n}_2\times\mathbf{k}_2)]. \quad (1.4c)$$

In the appendix it is shown that only three of the four parameters A, R, A', and R' are independent at any angle θ.

Substituting $P_1\mathbf{n}_1$ for $\langle\boldsymbol{\sigma}\rangle_1$ and recalling that

$$I_2=I_0(1+P_1P_2\cos\varphi) \quad (1.5)$$

as is well known from discussions of double scattering, we find for the asymmetry in the triple-scattering experiments:

$$e_{3n}=P_3(P_2+DP_1\cos\varphi)/(1+P_1P_2\cos\varphi), \quad (1.6)$$

$$e_{3s}=P_3P_1R\sin\varphi/(1+P_1P_2\cos\varphi), \quad (1.7)$$

where P_3 characterizes the third scatterer. Since $\langle\boldsymbol{\sigma}\rangle_1$ must be perpendicular to the direction of motion, $\mathbf{k}_2$, after a single scattering, the function A cannot be determined in a triple-scattering experiment. (See Sec. 4.)

Thus there exist in general two new parameters D and R that may be determined from two distinct triple-scattering experiments. The first of these consists of measuring e_{3n} with φ equal to 0 or 180°; this means that all three scattering planes are parallel. For either value of φ, one can then determine D from Eq. (1.6), assuming P_3P_2, P_3P_1, and P_1P_2 have been determined from double-scattering experiments. Alternatively one can use both values of φ; assuming that the first scattering is to the left, one then has four possible scattered intensities LL, LR, RL, and RR, where LR is the intensity when second and third scatterings are to the left and right, respectively, etc. Using Eqs. (1.6) and (1.5) and noting that e_{3n} is defined relative to $\mathbf{n}_2$ we find that

$$D=\frac{LL+RL-LR-RR}{(LL+RL+LR+RR)P_3P_1}. \quad (1.8)$$

Use of this procedure eliminates the most obvious intrinsic experimental asymmetry in the combination of second and third scatterings and uses only one result (P_3P_1) from a double-scattering experiment.

The parameter D may be considered as giving the extent to which the second scattering depolarizes an initially polarized beam. This may be seen by looking at $\langle\boldsymbol{\sigma}\rangle_2$ for the case P_1 equals unity and $\cos\varphi$ equals ±1; from Eqs. (1.4) and (1.5) we find

$$\langle\boldsymbol{\sigma}\rangle_2=\mathbf{n}_2(P_2\pm D)/(1\pm P_2). \quad (1.9)$$

It follows that a necessary and sufficient condition that there be no depolarization of a beam initially completely polarized normal to the scattering plane is that D equals unity. Looking at Eq. (1.9) one notes that if D equals P_2^2, $\langle\boldsymbol{\sigma}\rangle_2$ has the same value as if the original beam were unpolarized; however, in general D may be less than this so that D may not represent a depolarization but actually a reversal of the initial spin. Equation (1.9) together with the condition $|\langle\boldsymbol{\sigma}\rangle_2|\leq1$ yield the following limits on D:

$$-1+2|P_2|\leq D\leq1. \quad (1.10)$$

The second triple scattering experiment consists of measuring e_{3s} with φ around $\pm90°$. This means that the successive scattering planes are at right angles. For either value of φ the parameter R is determined from Eq. (1.7) using the experimental value of P_1P_3. To maximize e_{3s} the second scattering actually should be somewhat to the low cross-section side of "up" or "down" rather than at $\varphi=90°$; from Eq. (1.7) this maximum is given by

$$\cos\varphi=-P_1P_2. \quad (1.11)$$

Considering once again the case P_1 equal to unity and setting φ equal to 90°, we find from Eqs. (1.4) and (1.5):

$$\langle\boldsymbol{\sigma}\rangle_2=P_2\mathbf{n}_2+R\mathbf{s}+R'\mathbf{k}_2'. \quad (1.12)$$

It follows from Eq. (1.12) that

$$|R|\leq(1-P_2^2)^{\frac{1}{2}}. \quad (1.13)$$

2. SPIN-ZERO TARGET

If the target has zero spin and the incident particle is completely polarized, the particle after scattering will still be represented by a pure wave function and so will still be completely polarized. From our previous discussion, therefore, D is equal to unity. We may also give a physical picture of the parameter R for the case $\varphi=90°$. From Eq. (1.12) we see that the original spin

[3] L. Wolfenstein and J. Ashkin, Phys. Rev. **85**, 947 (1952). This reference will be referred to hereafter as A. Equation (5) of this reference shows in general the linear relation between the expectation value of operators before and after collision.

[4] We omit the subscript 2 on some of the scattering parameters (such as D, A, R, I_0, θ, φ) where this will cause no ambiguity. All the scattering parameters, of course, may depend on the energy.

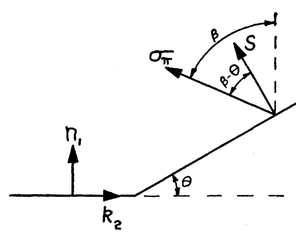

FIG. 1. Directions of vectors in the scattering plane for $\varphi=90°$.

vector $\mathbf{n}_1$ is bent out of the scattering plane so that its component along the normal $\mathbf{n}_2$ is P_2 and therefore since $\langle\boldsymbol{\sigma}\rangle_2$ must equal unity its projection in the plane has a magnitude $(1-P_2^2)^{\frac{1}{2}}$. The projected vector $\boldsymbol{\sigma}_\pi$ (Fig. 1) may be rotated by an angle β about the $\mathbf{n}_2$ axis from the direction $\mathbf{n}_1$. It follows that

$$R=(1-P_2^2)^{\frac{1}{2}}\cos(\Theta-\beta), \tag{2.1}$$

where Θ is the laboratory scattering angle.

The scattering amplitude may be written as a matrix M operating on the initial spinor. An arbitrary matrix may be written

$$M=g+\boldsymbol{\sigma}\cdot\mathbf{h}, \tag{2.2a}$$

and invariance arguments[5] restrict us to the form

$$M=g(\Theta)+\boldsymbol{\sigma}\cdot\mathbf{n}h(\Theta). \tag{2.2b}$$

In this notation the unpolarized differential cross section is clearly given by

$$I_0=|g|^2+|h|^2, \tag{2.3}$$

while the left-right asymmetry in the single scattering of a completely polarized beam is given by

$$P=2\,\mathrm{Re}(g^*h)/(|g|^2+|h|^2). \tag{2.4}$$

By applying Eq. (2.2b) to an arbitrary spinor, we find that β in Eq. (2.1) is given by

$$\begin{aligned}\sin\beta&=2\,\mathrm{Im}(g^*h)/I_0(1-P^2)^{\frac{1}{2}},\\ \cos\beta&=(|g|^2-|h|^2)/I_0(1-P^2)^{\frac{1}{2}}.\end{aligned} \tag{2.5}$$

A few general features of β as a function of the scattering angle may be predicted. The relatively large values of polarization P at certain angles show that the absolute values of g and h are of the same order of magnitude. Since h must contain a factor $\sin\Theta$, $g(\Theta)$ and $h(\Theta)$ should behave quite differently as functions of Θ, and so these two curves might be expected to cross somewhere near the angle of maximum polarization; at smaller angles $|g|$ is larger than $|h|$. If this is so, then it follows from Eq. (2.5) that β increases in absolute value as a function of Θ starting at 0 at 0° and passing through $\pm 90°$ somewhere near the polarization maximum. For angles around the expected diffraction minimum, β might exhibit some large variations with Θ. Further predictions can be made by employing a model proposed by Fermi and others[6] consisting of a complex potential supplemented by a real spin-orbit coupling term. In the Born approximation one finds immediately:

$$\sin\beta=(B/B_a)\{P/(1-P^2)^{\frac{1}{2}}\}, \tag{2.6}$$

where B and B_a are proportional to the magnitudes of the real and imaginary parts of the central potential, and the result is independent of the form of the spin-orbit coupling chosen. Values of β and the asymmetry e_{3s} shown in Table I have been calculated by using Eqs. (1.7), (1.11), (2.1), and (2.6), the value of 27/16 for (B/B_a) and the values for $P_2(\theta)$ given by Fermi, and by assuming $P_1=P_3=0.65$. For comparison, values of e_{3n} from Eq. (1.6) (with $D=1$) obtained with the same value of φ are also shown.

Although at each angle Θ the measurements of I_0, P, and β are quite independent, the functions $I_0(\Theta)$, $P(\Theta)$, and $\beta(\Theta)$ must be interrelated. This is clear from the possibility of a phase-shift analysis of the scattering. In principle the $(2L+1)$ complex phase shifts should be determined by the $(2L+1)$ coefficients in $I_0(\theta)$, the $2L$ coefficients[7] in $P(\theta)$, when these are expanded in powers of $\cos\theta$, and the total absorption cross section (including inelastic scattering). In practice such a phase shift analysis is hopeless for these cases, where many partial waves are involved. Furthermore, as a matter of principle, if L is the maximum orbital angular momentum to be considered it is of doubtful validity to analyze terms in the expansion of $I_0(\theta)$ much beyond the first $(L+1)$ terms since there may be significant contributions to the remaining terms from the interference of partial waves beyond the Lth with low-order partial waves. Therefore it seems likely that determination of $\beta(\theta)$ would add significant information about the scattering.[8]

TABLE I. Asymmetry e_{3s} in triple scattering for $P_1=P_3=0.65$ as a function of scattering angle θ in the second scattering, obtained by using Fermi's theory. φ is chosen to maximize e_{3s}. For comparison, the value of e_{3n} for the same value of φ is shown. The value e_{3s}' is the value of e_{3s} predicted by Fermi's theory with a change in sign of the spin-orbit interaction.

θ	P_2	β	φ	e_{3s}	e_{3n}	e_{3s}'
5	0.4	47	105	0.25	0.16	0.21
10	0.51	90	109	0.06	0.21	−0.06
15	0.49	108	109	−0.02	0.20	−0.18
20	0.42	129	106	−0.11	0.17	−0.29
30	0.33	144	102	−0.14	0.13	−0.35

[5] L. Wolfenstein, Phys. Rev. **75**, 1664 (1949).

[6] E. Fermi, Nuovo cimento **11**, 407 (1954); W. Heckrotte and J. Lepore, Phys. Rev. **94**, 500 (1954); Snow, Sternheimer, and Yang, Phys. Rev. **94**, 1073 (1954); R. Sternheimer, Phys. Rev. **95**, 588 (1954). Dr. Sternheimer has kindly communicated to me a few calculations of β on the basis of the same model previously used to calculate P. These calculations seem to bear out the general remarks made above.

[7] Theorem 3(b) of reference 5. For this discussion results are expressed in terms of the c.m. angle θ rather than the lab angle Θ.

[8] This should be contrasted with the simpler case of protons of 1 to 4 Mev scattered from helium, C. L. Critchfield and D. C. Dodder, Phys. Rev. **76**, 602 (1949); M. Heusinkveld and G. Freier, Phys. Rev. **85**, 80 (1952). In this case the phase shifts are real and only s and p waves need be considered; then I_0 alone limited one to two possible sets of phase shifts and the measurement of P resolved this ambiguity.

Even if the complex phase shifts could be determined from $I_0(\theta)$ and $P(\theta)$ it is likely the determination would not be unique and measurements of $\beta(\theta)$ would help resolve ambiguities. The most common and most interesting ambiguity in analyses of this kind is the uncertainty as to the sign of the spin-orbit coupling. In the Born approximation the only effect of a change of sign of the spin-orbit interaction is to reverse the sign of h and thus change both the sign of P and of β. When many partial waves are involved in a phase-shift analysis, there should exist two very similar solutions obtained from each other by exchanging the phase shifts for $J=l+\frac{1}{2}$ and $J=l-\frac{1}{2}$, and these two will differ essentially only in the sign of P and of β. While no direct method seems to exist for determining the absolute value of the sign of P, [L. Marshall and J. Marshall have recently made a measurement of the sign of P (private communication)] it is clear from Eq. (2.1) that the triple scattering asymmetry e_{3s} depends on the sign of β. Such a measurement may thus give a key to the sign of the spin-orbit coupling. It must be noted however that for a given scattering angle Θ it is impossible to distinguish between a value β_1 and a value $(-\beta_1+2\Theta)$. In the last column of Table I values of e_{3s}' are given, which are the values predicted by Fermi's theory, with a reversal of the sign of the spin-orbit coupling.

3. PROTON-PROTON SCATTERING

The scattering in the case of proton scattering from a target with spin can again be described by a matrix of the form (2.2a), but in this case g and $\mathbf{h}$ are operators in the spin space of the target nucleus. The unpolarized differential cross section is now expressed

$$I_0=\mathrm{Tr}(gg^\dagger+\mathbf{h}\cdot\mathbf{h}^\dagger)/2(2s+1), \tag{3.1}$$

where s is the spin of the target and Tr stands for the trace of the matrix in the *composite* spin space of proton plus target. P can be calculated from M[9] and is given by

$$P=\frac{\mathrm{Tr}(g\mathbf{h}^\dagger+g^\dagger\mathbf{h})\cdot\mathbf{n}}{\mathrm{Tr}(gg^\dagger+\mathbf{h}\cdot\mathbf{h}^\dagger)}. \tag{3.2}$$

The expectation value of the spin after the second scattering[10] may be found from Eq. (5) of A:

$$\begin{aligned}I_2\langle\boldsymbol{\sigma}\rangle_2=&\{\mathrm{Tr}(M\boldsymbol{\sigma}\cdot\langle\boldsymbol{\sigma}\rangle_1M^\dagger\boldsymbol{\sigma})+\mathrm{Tr}(MM^\dagger\boldsymbol{\sigma})\}/2(2s+1)\\ =&\{\mathrm{Tr}[(gg^\dagger-\mathbf{h}\cdot\mathbf{h}^\dagger)\langle\boldsymbol{\sigma}\rangle_1\\ &+i(g^\dagger\mathbf{h}-g\mathbf{h}^\dagger)\times\langle\boldsymbol{\sigma}\rangle_1+\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{h}\mathbf{h}^\dagger+\mathbf{h}\mathbf{h}^\dagger\cdot\langle\boldsymbol{\sigma}\rangle_1]\\ &+PI_0\mathbf{n}\}/2(2s+1).\end{aligned} \tag{3.3}$$

For the case of proton-proton scattering the most general form of M is given by Eq. (9) of A with $D=0$, and this may be rewritten in the form:

$$\begin{aligned}M=&BS+C(\boldsymbol{\sigma}+\boldsymbol{\sigma}_t)\cdot\mathbf{n}+\tfrac{1}{2}G(\boldsymbol{\sigma}\cdot\mathbf{K}\boldsymbol{\sigma}_t\cdot\mathbf{K}+\boldsymbol{\sigma}\cdot\mathbf{P}\boldsymbol{\sigma}_t\cdot\mathbf{P})T\\ &+\tfrac{1}{2}H(\boldsymbol{\sigma}\cdot\mathbf{K}\boldsymbol{\sigma}_t\cdot\mathbf{K}-\boldsymbol{\sigma}\cdot\mathbf{P}\boldsymbol{\sigma}_t\cdot\mathbf{P})T+N\boldsymbol{\sigma}\cdot\mathbf{n}\boldsymbol{\sigma}_t\cdot\mathbf{n}T,\end{aligned} \tag{3.4}$$

where $\mathbf{K}$ and $\mathbf{P}$ are unit vectors in the directions $(\mathbf{p}'-\mathbf{p})$ and $(\mathbf{p}'+\mathbf{p})$, respectively, $\mathbf{p}'$ and $\mathbf{p}$ are outgoing and incident momenta in the c.m. system, and S and T are singlet and triplet projection operators, respectively. From parity considerations it follows that B and H are even functions of $\cos\theta$, G and N are odd functions, and C is an even function times $\sin\theta$. Comparing Eq. (3.4) with Eq. (2.2a) we find the operators g and $\mathbf{h}$ and then using Eqs. (3.1) through (3.3) we obtain

$$\begin{aligned}I_0=&\tfrac{1}{4}|B|^2+2|C|^2+\tfrac{1}{4}|G-N|^2\\ &+\tfrac{1}{2}|N|^2+\tfrac{1}{2}|H|^2,\end{aligned} \tag{3.5a}$$

$$I_0P=2\,\mathrm{Re}(C^*N), \tag{3.5b}$$

$$I_0(1-D)=\tfrac{1}{4}|G-N-B|^2+|H|^2, \tag{3.5c}$$

$$\begin{aligned}I_0R=&\tfrac{1}{2}\,\mathrm{Re}[(G-N)^*(N+H)+B^*(N-H)]\\ &\times\cos(\theta/2)+\mathrm{Im}[C^*(G-N+B)]\sin(\theta/2).\end{aligned} \tag{3.5d}$$

In finding the expression for R in terms of the c.m. angle θ we have used the fact that the vector $\mathbf{s}$ defined in the lab system by Eq. (1.3) becomes the vector $\mathbf{K}$ in the present case.

The value of D for $\theta=90°$ may be particularly revealing; there is then no interference term in D and the three possible terms B, C, and H give by themselves values of D of 0, 1, and -1, respectively. Experiments[11] suggest a value of D at 90° of about 0.7, from which it would follow that at least 70 percent of the 90° cross section is due to the C term and at most 15 percent is due to the H term; thus at most 30 percent of the 90° cross section could be caused by singlet scattering. Further information on the singlet scattering at 90° may be obtained from the value of R at 90° since this is determined completely by singlet-triplet interference: a nonzero value of R sets a lower limit on the singlet scattering. If we take the value of D at 90° as 0.7 the maximum possible absolute value of R is close to 0.5, and if this were the actual value of R it would fix the singlet scattering at about 30 percent. Of course there is no simple converse conclusion from a zero value of R. It may also be noted that $D(\theta)$ may contain odd terms as a function of $\cos\theta$ due to the contribution of singlet-triplet interference so that a measurement of the triple scattering asymmetry e_{3n} on both sides of 90° could yield information on the singlet scattering at other angles.

Once again we may ask the extent to which I, P, D, and R are independent as functions of the scattering angle θ. If we assume the phase shifts are real[12] and

[9] Equation (7a) of A. In some of the equations in the present paper the obvious generalization to a target of spin s is made. In place of the notation $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$, in the present paper $\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}_t$ are used.

[10] This result Eq. (3.3) is the same as Eq. (15) of reference 5, except that there the trace is omitted; it is essential to take the trace when the scattering depends on the target spin.

[11] Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis (private communication). More recent results suggest a value of D around 0.5. It must be emphasized that these are preliminary results and may be subject to large corrections.

[12] The imaginary part of the phase shifts can be estimated from the analysis of data on meson production.

ignore the mixing of different partial waves due to tensor forces there are then $2(L+1)$ real numbers to be determined where L is the maximum orbital angular momentum (assumed for convenience to be odd). Since $I(\theta)$ contains $(L+1)$ coefficients when expanded in powers of $\cos^2\theta$ and $P(\theta)$ contains L coefficients there remains a one-parameter family of phase shifts fitting this data.[13] Thus $D(\theta)$ or $R(\theta)$ could reduce these possibilities. Actually for the energies for which such a phase-shift analysis is most feasible, a study of Coulomb interference effects[14] may be a simpler method for reducing the possibilities. However, at higher energies where Coulomb interference is hard to measure and the number of partial waves increases the measurement of $D(\theta)$ and $R(\theta)$ should be of real value in determining the scattering matrix M.

4. FURTHER EXPERIMENTAL POSSIBILITIES

For the case of a spin-zero target the triple scattering experiment coupled with the previously-measured P and I_0 completely determines the scattering matrix [Eq. (2.2b)] except for the phase of the scattered amplitude, which we cannot expect to determine directly by any reasonable experiment. For proton-proton scattering, on the other hand Eq. (3.4) contains nine real parameters at each angle θ, whereas the experiments discussed here determine only four: I_0, P, D, and R. It may be of interest to inquire whether it would be possible to acquire any further information by successive scatterings without using a polarized target. The triple scattering experiments considered here are not the most general experiments of this type because they are restricted by the fact that the initial polarization and the part of the final polarization that is detected must be at right angles to the direction of motion. In general we might hope to determine all parameters relating the final polarization vector to the initial. As shown in the appendix there are five independent parameters of this type, which are conveniently chosen as P, D, R, A, and R' of Eq. (1.4). While A and R' might be determined from quadruple scattering experiments, a more practical mothod would be to use a magnetic field directed at right angles both to the path of the proton and the direction of its spin. Because of the anomalous moment of the proton, the proton spin precesses about this field with an angular frequency larger than that of the proton's motion. Such a magnet placed between the first and second scatterers would cause the proton to end up with a spin component along the direction of motion and so would allow a determination of the parameter A. If the magnet were placed between the second and third scatterers it would partially convert the undetectable $\mathbf{k}_2'$-component of spin to a component at right angles to the direction of motion and so allow the determination of R'.

Even these do not exhaust the possibilities without polarized targets. Additional information at any angle θ may be obtained by a suggested experiment on the polarization correlation of the scattered protons in $p-p$ scattering. Such an experiment involves analyzing in coincidence the polarization states of the scattered and recoil protons and would determine the expectation values of operators such as $(\boldsymbol{\sigma}\cdot\mathbf{n})(\boldsymbol{\sigma}_t\cdot\mathbf{n})$. This would be of interest even for the case in which the initial beam is unpolarized.

APPENDIX. TIME-REVERSAL ARGUMENTS

When we take the trace of the factors involving g and $\mathbf{h}$ in Eq. (3.3) these factors reduce to expressions which depend only on the vectors $\mathbf{p}$ and $\mathbf{p}'$. From rotation and reflection invariance it follows that the first term must reduce to form $f_1(\theta)\langle\boldsymbol{\sigma}\rangle_1$, the second term to $f_2(\theta)\mathbf{n}\langle\boldsymbol{\sigma}\rangle_1$, and the third term to

$$f_3(\theta)\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{nn}+f_4(\theta)\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{KK}+f_5(\theta)\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{PP}\\+f_6(\theta)\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{PK}+f_7(\theta)\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{KP}. \quad (A.1)$$

The invariance of M under time-reversal requires that in Eq. (2.2a) the operator g be even and $\mathbf{h}$ be odd under time-reversal. Consequently the third term in Eq. (3.3) is even under time-reversal so that the last two terms in Eq. (A.1) must vanish by the arguments used in A. By combining $f_1(\theta)\langle\boldsymbol{\sigma}\rangle_1$ with the remaining terms in Eq. (A.1) we reduce Eq. (3.3) to the form

$$I_2\langle\boldsymbol{\sigma}\rangle_2=I_0(P\mathbf{n}+J\mathbf{n}\times\langle\boldsymbol{\sigma}\rangle_1+D\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{nn}\\+X\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{KK}+Y\langle\boldsymbol{\sigma}\rangle_1\cdot\mathbf{PP}). \quad (A.2)$$

Since the quantities A, R, A', and R' can be derived directly from Eq. (A.2) and (1.4) in terms of J, X, and Y it follows that only three of these four are independent.

A direct calculation also gives in Eq. (3.3) a term independent of $\langle\boldsymbol{\sigma}\rangle_1$ proportional to $\mathrm{Tr}(\mathbf{h}\times\mathbf{h}^\dagger)$; this same term with opposite sign occurs in the direct calculation of Eq. (3.2) from Eq. (7a) of A. Now rotation-reflection invariance tells us this term must be proportional to $\mathbf{n}$ since it is a pseudovector, but the term must be even under time inversion (since it involves $\mathbf{h}$ times itself). However $\mathbf{n}$ is odd. Therefore this term vanishes. Indeed it is just the vanishing of this term that guarantees the validity of Eq. (8) of A. This paragraph is essentially equivalent to Sec. 3 of A.

[13] A. Garren, Phys. Rev. 92, 213, 1587 (1953), gives an example for s and p waves alone.

[14] R. M. Thaler and J. Bengston, Phys. Rev. 94, 679 (1954); A. Garren, Phys. Rev. (to be published).