than the usual visual estimates and hence has been used in the analysis. If one accepts these values of the ionization, the masses of the particles are: M_1: 840–1440 m_e and M_2: 250–520 m_e where the range of values results from the combined uncertainty in the momenta and ionizations. The simplest explanation of this event is that a K^- particle comes to rest in the plate and causes the ejection of a light meson. If the K^- is assumed to have a mass of $\sim$1000 m_e, it should come to rest very near the center of the copper plate, and a light meson (π or μ) would lose about 15 Mev/c before emerging from the plate with a momentum of 116 Mev/c. Thus this event can be interpreted either as a K^--produced star which ejects a π meson of 51-Mev kinetic energy or as a K^- decay at rest with the emission of a light meson. The former possibility seems to be more likely because the stopping material has a high atomic number and similar events have been reported elsewhere.[4] However, it should be emphasized that both of these interpretations hinge on the ionization determinations. If ionization is ignored, the momentum change implies either that a particle of mass $\sim$600 m_e traversed the plate or that a lighter particle (π meson or electron) suffered an anomalously large momentum loss on traversal. The possibility that this event represents a positive π meson which goes upward and gives rise to a proton which in turn emerges from the plate in an upward direction is energetically acceptable, but considered highly unlikely in view of the ionization of track 1 and the geometry of the experiment. Thus the simplest and most consistent explanation of this event seems to be that a K^- particle stops in the plate and ejects a π^- meson from a nuclear interaction.

The author would like to express his thanks to Dr. R. B. Leighton who constructed the magnet and cloud chamber and to Mr. E. K. Bjørnerud for performing the ionization measurements referred to above. He would also like to express his appreciation to Professor C. D. Anderson for many stimulating and enlightening discussions.

[4] Lal, Pal, and Peters, Phys. Rev. **92**, 438 (1953); Hill, Salant, and Widgoff, Phys. Rev. **94**, 1794 (1954); Major, Macpherson, Parkash, and Rochester (private communication).

PHYSICAL REVIEW VOLUME 96, NUMBER 6 DECEMBER 15, 1954

Nuclear Fine Structure in the μ-Mesonic Atom*

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(Received August 23, 1954)

The $2p-1s$ transition of a μ-mesonic atom is split not only by the spin-orbit energy but also by the electric quadrupole interaction between meson and nucleus. If a high-Z nucleus has excited states at several hundred kilovolts or less this interaction can be expected to mix nuclear states, so that for example an even-even nucleus with ground state spin zero can produce a quadrupole fine structure. General formulas are given for the calculation of the line patterns; these are worked out for W, U^{238}, and Ta^{181}. The resulting fine structure is spread over a region of 300–500 kev. The details of the positions and intensities of the lines depend on the quadrupole moments of the excited nuclear states as well as on other parameters which can in principle be measured in other types of experiments; the effect therefore offers a method for investigating these excited moments. A certain fraction of the mesonic gamma rays can be followed by a nuclear radiation; this fraction can be as high as 0.50.

I. INTRODUCTION

THE μ-mesonic atom has proved to be a useful tool in the measurement of nuclear size.[1,2] The reasons for its utility as a probe for nuclear electromagnetic effects are well summarized by Wheeler,[3] who emphasizes the smallness of the Bohr orbits and the transparency of nuclear matter to μ mesons.

Atomic electrons are also commonly used as a tool in nuclear physics, the fine structure of their spectrum yielding values of magnetic dipole and electric quadrupole moments for the ground states of nuclei. There are certain striking differences, however, between an electronic- and a μ-mesonic atom which enable the spectrum of the latter to yield additional nuclear information. The most important of these, for our purposes, concerns the relative fineness of the atomic and nuclear level spacings. (All qualitative arguments in the following shall be made for nuclei at the heavy end of the periodic table.) The difference between the energies of a $2p$ and $1s$ μ meson is several Mev; if now the nucleus in question has low-lying levels with spacings $\sim$100 kev, the meson sees the nucleus not as a rigid structure in its ground state, but as a system in an almost degenerate mixture of several states. A fine structure is induced in the μ levels which depends on the nondiagonal as well

* A preliminary report on this work was given at the May, 1954 Washington meeting of the Americal Physical Society [Phys. Rev. **95**, 654 (1954)]. It has subsequently been called to the author's attention that a paper on the same subject by L. Wilets is to be published.
[1] V. L. Fitch and J. Rainwater, Phys. Rev. **92**, 789 (1953).
[2] L. N. Cooper and E. M. Henley, Phys. Rev. **92**, 801 (1953).
[3] J. A. Wheeler, Phys. Rev. **92**, 812 (1953).

as the diagonal matrix elements of the nonspherical electromagnetic interaction between μ and nucleus. Thus, an even-even nucleus which, since its ground state spin is zero, would look spherically symmetric to an atomic electron might appear to be a mixture of $I=0$ and $I=2$ to a μ meson in a low Bohr orbit. In the former case, no hyperfine structure would appear; in the latter it would.

Before we consider the conditions necessary for this effect to be appreciable, we should mention several other differences between mesonic and electronic atoms. First we recall the fact that, in low orbits, the μ spends an appreciable amount of time inside the nucleus. This fact, which has proved so useful in the measurement of nuclear size, shall prove for us to be somewhat of a nuisance. It makes the fine structure depend not only on gross properties of the nucleus, such as the quadrupole moment, but also on other things such as the distribution of the quadrupole-producing charge. At some future time, if experiments such as those outlined here should prove feasible, such dependence might prove advantageous; in the present calculation, it forces us to introduce a nuclear model midway in the formulas (Sec. II).

Another important difference is that, although magnetic dipole and electric quadrupole hyperfine structure effects are of approximately equal importance for electrons, the former will be completely negligible for a μ.[3] The ratio of quadrupole to dipole interaction energy is, in order of magnitude,

$$\frac{\langle e^2Q_N/r^3\rangle}{\langle \mu_N\mu_a/r^3\rangle}=\frac{e^2Q_N}{\mu_N\mu_a},$$

where μ_N and μ_a are the magnetic moments of the nucleus and the atomic particle, respectively. This ratio, which is ~ 1 in the electronic case, is instead ~ 100 for the meson if its magnetic moment is one mesonic magneton. We shall ignore all magnetic effects except for the meson spin-orbit interaction, which is comparable to the quadrupole energy. Moreover, although many arguments can be given in favor of omitting from consideration all electric interactions other than quadrupole, we shall bypass them by pointing out that in this calculation we shall be concerned with orbital μ's with $l=0$ and 1; these can produce at the most a quadrupole field at the nucleus. Since we shall restrict ourselves to nuclei which have low states all of the same parity, electric dipole effects shall also be ignored.[4]

We shall now estimate the amount of mixing of nuclear levels to be expected. This will be appreciable whenever the matrix elements of the quadrupole interaction are comparable to the separation of two nuclear levels a and b; i.e., if

$$1 \lesssim \left\langle\frac{e^2}{r^3}\right\rangle_{2p} \frac{|(b|Q|a)|}{|E_a-E_b|} \sim \frac{e^2 Z_{\mathrm{eff}}^3}{24 a_\mu^3}\frac{|Q_a|}{\Delta E} \sim (4\times 10^{20}\ \mathrm{kev/cm^2}) Z_{\mathrm{eff}}^3 |Q_a|/\Delta E. \quad (1)$$

[Equation (1) makes use of the fact pointed out by Bohr and Mottelson[5] that many nuclei have large off-diagonal quadrupole matrix elements (collective transitions) comparable in size to the large ground state quadrupole moments.] Thus nuclei with fairly high Z and with excited states at about 100 kev can be expected to show this effect, provided that the quadrupole moments in the region of the periodic table under consideration are $>1\times 10^{-24}$ cm^2. As BM has shown, the last two requirements are likely to be satisfied simultaneously in regions of the periodic table far from the magic numbers.

The formulas developed in the following sections include as a special case the static quadrupole effect considered by Wheeler;[3] his results are valid when ΔE is large enough to reverse the inequality (1).

II. GENERAL THEORY

The Hamiltonian to be used for the system of meson and nucleus is

$$H=H_N+H_\mu+H_S+H_{\mu N}. \quad (2)$$

H_N operates only on the nuclear coordinates and has eigenvalues E_1, E_2, $\cdots$, the experimentally known nuclear levels. H_μ depends on the meson coordinates and contains the spherically symmetric potential energy $V(r)$ arising from the nucleus in its ground state.[2] In the equation

$$H_S=g\left(\frac{\hbar}{2Mc}\right)^2\frac{1}{r}\frac{dV}{dr}(\boldsymbol{\sigma}\cdot\mathbf{L}/\hbar), \quad (3)$$

$M=$ meson mass and $g=-[2Mc/\hbar|e|]\times$[meson magnetic moment]. Henceforth, we shall take $g=1$.

The last term in Eq. (2) contains all electric effects not already included in $V(r)$; we expand it and, as previously explained, keep only the electric quadrupole part. Thus

$$H_{\mu N}\rightarrow H_q=-e^2\textstyle\sum_P F(r,R_P)P_2(\cos\Theta_P)$$
$$=-e^2\textstyle\sum_P F(r,R_P)\mathbf{C}^{(2)}(\theta_P,\varphi_P)\cdot\mathbf{C}^{(2)}(\theta,\varphi), \quad (4)$$

$$F(r_1,r_2)=(r_<)^2/(r_>)^3. \quad (5)$$

All the notation used here for angular factors and matrix elements is taken from Racah.[6] The coordinates with subscripts P refer to the protons, those without subscripts to the meson.

[4] Even if low-lying states of opposite parity should be present, electric dipole effects would be small since (a) the meson would be in a state which mixes $2s$ and $2p$, introducing a large energy denominator [see J. A. Wheeler, Revs. Modern Phys. **21**, 133 (1949)], and (b) the nuclear dipole matrix elements are presumably not enhanced by collective effects.

[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd **27**, No. 16 (1953). We shall continually refer to this paper as BM.

[6] G. Racah, Phys. Rev. **62**, 438 (1942).

To solve the Hamiltonian (2) for large Z and for small nuclear level spacing, we ignore all elements $(n'l'\cdots|H|nl\cdots)$ unless $n=n'$, and $l=l'$. This is fully justified if we wish to treat only the low orbits; for example, the nearest orbit connected to the $2p$ by H_q is the $3p$, which lies one or two Mev higher in energy. An immediate consequence of this approximation is that H_q has no effect for the $l=0$ orbits; the $1s$ group consists of pure nuclear states with energies

$$E=E_\mu(1s_{\frac{1}{2}})+E_N(I). \quad (6)$$

For the $2p$ orbit, we write down the matrix elements of expression (4) in a scheme in which the following are diagonal: H_N (with energy E_N and angular momentum I), j (total angular momentum of the meson), and J ($\mathbf{J}=\mathbf{I}+\mathbf{j}$). Equations (38) and (53) of reference[6] yield immediately

$$(I_1j_1J|H_q|I_2j_2J)=e^2(-1)^{I_1+\frac{3}{2}-J}(4/5)^{\frac{1}{2}}W(I_1j_1I_2j_2;J2)\times\langle(I_1\|\sum F(r,R_P)C^{(2)}(\theta_P,\varphi_P)\|I_2)\rangle_{2p}. \quad (7)$$

W is a Racah coefficient; extensive numerical tables of these have recently appeared.[7]

If the meson orbit were always outside of the nucleus, we could write $F(r,R_P)=R_P^2/r^3$. The double-barred symbol of Eq. (7) could then be factored into $\langle r^{-3}\rangle_{2p}\times\{$a factor proportional to the nuclear quadrupole moment if $E_N(1)=E_N(2)$, or the quadrupole matrix element for $E_N(1)\neq E_N(2)\}$. We utilize this fact to guide us in making some new definitions which separate out the factor which depends on the penetration of the meson into the nucleus.

First, for those cases in which $E_N(1)=E_N(2)$, $I_1=I_2=I$, we define the quadrupole moment of the nuclear state in the customary way:

$$Q(I)\equiv(I,+I|\sum R_P^2(3\cos^2\theta_P-1)|I,+I). \quad (8)$$

If we now define the "penetration function" $P(I,I;r)$ for $I\geqslant 1$ (the double-barred symbol is zero for $I<1$) by

$$Q(I)\frac{P(I,I;r)}{r^3}\equiv\left\{\frac{4I(2I-1)}{(I+1)(2I+1)(2I+3)}\right\}^{\frac{1}{2}}\times(I\|\sum F(r,R_P)C^{(2)}\|I), \quad (9)$$

it follows from the formulas of Racah[6] that $P(I,I;r)=1$ for all r larger than the nuclear radius R_0. For $r<R_0$, $P(I,I;r)$ is a form factor depending on the radial distribution of the quadrupole-producing charge; the model of a slightly ellipsoidal uniform charge distribution yields

$$P(I,I;r)=(r/R_0)^5 \quad \text{for} \quad r<R_0, \quad (10)$$

which has been used here in all numerical calculations.

Similarly one can separate the form factor for those matrix elements in Eq. (7) which are nondiagonal in the nuclear states. From the customary definition[5] for the reduced quadrupole transition probability,

$$B(I_1\rightarrow I_2;2)=e^2\textstyle\sum_{\mu,M_2}|(I_1M_1|\sum R_P^2Y_{2\mu}|I_2M_2)|^2, \quad (11)$$

the function $P(I_1,I_2;r)$ is defined by

$$\left\{\frac{4\pi}{5e^2}(2I_1+1)B(I_1\rightarrow I_2;2)\right\}^{\frac{1}{2}}\frac{P(I_1I_2;r)}{r^3}\equiv(I_1\|\sum F(r,R_P)C^{(2)}\|I_2), \quad (12)$$

for $E_2>E_1$. (The cases with $E_2<E_1$ need not be written down; the required matrix elements are most easily found from the Hermitian property of H_q.) With the phases of the nuclear wave functions chosen appropriately, it can easily be shown that $P(I_1I_2;r)=1$ for $r>R_0$. In the numerical calculations, the Bohr-Mottelson result for a collective transition has been used:

$$P(I_1I_2;r)=(r/R_0)^5 \quad \text{for} \quad r<R_0. \quad (10')$$

All the formulas necessary for calculating the $2p$ energy levels and wave functions are contained in Eqs. (3), (7), (9), and (12). These are completely independent of any model of the nucleus, and correspondingly contain many unknown quantities. In principle, a precise measurement of the fine structure combined with other experiments which measure the $B(2)$'s and the ground state Q could be made to yield information about some of the form factors. However, since for all but the largest Z's a $2p$ meson does not spend a great fraction of its time inside the nucleus, the spectrum is not extremely sensitive to the penetration factors. This enables us to decrease the number of unknowns by inserting Eqs. (10) and (10′) for the penetration factors (R_0 is here the electromagnetic nuclear radius). The results then depend only weakly on the nuclear model; for example, for $Z\sim 70$ the very extreme assumption that all the quadrupole-producing charge is concentrated at the center of the nucleus increases our estimate of $\langle P(r)/r^3\rangle_{2p}$ by a factor of only 1.6.

With this approximation, the positions and intensities (as we shall see in Sec. III) of the fine structure of the $2p-1s$ lines are completely determined in terms of the nuclear electromagnetic radius, the nuclear energy levels, and the nuclear quadrupole moments (static and dynamic). Some of these parameters are susceptible to measurement by other methods; a study of the effect here considered could then yield information about the remaining parameters, such as the quadrupole moments of certain excited nuclear states. For this reason, we prefer not to insert as yet definite predictions derived from a nuclear model for these parameters. They shall be left free so that the sensitivity of the spectrum to variations in the quadrupole moment of an excited state can be examined.

For those nuclei which follow the Bohr-Mottelson strong-coupling sequence of states ($I=0$, 2, 4, $\cdots$ for A even; $I=I_0$, I_0+1, I_0+2, $\cdots$ for A odd), it is convenient to introduce one more bit of notation.

[7] Simon, Van der Sluis, and Biedenharn, Oak Ridge National Laboratory Report, 1679, 1954 (unpublished).

TABLE I. The factors $f(I,I';I_0)$ of Eq. (14) where I_0 is the ground state spin, and equals zero for an even-even nucleus.

I'	$f(I,I';I_0)$
I	$\left\{\frac{1}{5}\frac{2I+1}{I(2I-1)(I+1)(2I+3)}\right\}^{\frac{1}{2}}[3I_0^2-I(I+1)]$
$I+1$	$\left\{\frac{3}{5}\frac{I_0^2[(I+1)^2-I_0^2]}{I(I+1)(I+2)}\right\}^{\frac{1}{2}}$
$I+2$	$\left\{\frac{3}{10}\frac{[(I+1)^2-I_0^2][(I+2)^2-I_0^2]}{(I+1)(I+2)(2I+3)}\right\}^{\frac{1}{2}}$

Instead of the quadrupole moments $Q(I)$, we define a set of numbers $Q_0(I,I)$:

$$Q(I)\equiv\frac{3I_0^2-I(I+1)}{(I+1)(2I+3)}Q_0(I,I), \tag{13}$$

where I_0 is the ground-state spin. $Q_0(I,I)$ has been chosen so that it would be independent of I and just equal to the BM intrinsic quadrupole moment Q_0 if the nucleus were correctly described by the strong coupling collective model and if all the quadrupole moment arose from the core. Similarly we define $Q_0(I_1,I_2)=Q_0(I_2,I_1)$ (we choose these to be positive) for $I_1\neq I_2$ by Eqs. (VII. 17, 18, and 19) of BM, inserting into these definitions $K=I_0$. Again these would all equal $|Q_0|$ if the transitions were the BM pure rotational ones.

The final expressions for the quadrupole matrix elements can be written as

$$(I_1j_1J|H_q|I_2j_2J)=e^2(-1)^{I_1+\frac{3}{2}-J}W(I_1j_1I_2j_2;J2)\times f(I_1I_2;I_0)Q_0(I_1,I_2)\langle P(r)/r^3\rangle_{2p}, \tag{14}$$

for $I_2\geqslant I_1$, where the factors $f(I_1I_2;I_0)$ are listed in Table I.

If we shift the zero of energy so that $H_S=0$ for a $p_{\frac{1}{2}}$ state, then the only nonvanishing matrix elements of the spin-orbit interaction are

$$(I\tfrac{3}{2}J|H_S|I\tfrac{3}{2}J)=\frac{3}{4}\left(\frac{\hbar}{Mc}\right)^2\left\langle\frac{1}{r}\frac{dV}{dr}\right\rangle_{2p}\equiv E_S. \tag{15}$$

In order to carry out the appropriate averages over the $2p$ meson wave function in (14) and (15) we have assumed a uniformly charged nucleus in H_μ. The averages were then carried out using nonrelativistic hydrogenic wave functions in which there was left a variation parameter (Z') chosen to minimize the $2p$ energy of H_μ. The value $R_0=r_0A^{\frac{1}{3}}$ with $r_0=1.1\times10^{-13}$ cm was used.[8]

III. INTENSITIES OF SPECTRAL LINES

The intensity of a given line arising from a meson transition $a\rightarrow b$ is proportional to the probability of excitation of a times the probability that a, if excited, decays into b. We shall first consider the latter question.

Consider a single state a (labeled by J, M, and an additional index ν) when the meson is in a p orbit. When the Hamiltonian discussed in the last section is diagonalized, its wave function is found to be

$$\psi(2p;\nu JM)=\sum_{jI}x(jI\nu J)\varphi(2p;jIJM),$$

and its energy is $E(\nu J)$. With the μ in a $1s$ orbit, the label ν can be replaced by I, and

$$\psi=\varphi(1s;IJM),$$

with energy $E_N(I)$. (We omit all contributions to the energies which are independent of I, J, and ν.) The probability of a mesonic transition with frequency $[E(\nu,J)-E_N(I)]$ from a single state $(2p;\nu JM)$ to the entire level $(1s;I)$ can straightforwardly be shown to be independent of M and proportional to

$$\sum_j|x(jI\nu J)|^2\equiv p(I\nu J). \tag{16}$$

Next must be considered the question of the excitation of the various $2p$ states. The meson arrives in one of these by falling from some higher state, utilizing some non-nuclear process such as radiative emission or the Auger effect. Since in the initial state the meson is far from the nucleus, the mixing of nuclear levels should be small. If we make the simplifying assumption that all states which populate the $2p$ level have the nucleus in the ground state I_0, then the relative population of a single state $(2p;\nu JM)$ can be shown to be $p(I_0\nu J)$, independent of the process by which the state was formed as long as the meson was initially unpolarized.

The frequencies and intensities of the spectral lines are thus given by

$$\hbar\omega=E(\nu J)-E_N(I)+\text{constant}; \tag{17}$$

$$\text{intensity}=(2J+1)p(I\nu J)p(I_0\nu J), \tag{18}$$

where $\sum_I p(I\nu J)=1$.

A simple consequence of Eq. (18) is that only nuclear states connected by H_q to the ground state (directly or indirectly) need be considered.

IV. NUCLEAR TRANSITIONS

Since the lifetimes for nuclear transitions are very long (10^{-8} or 10^{-9} sec for the nuclei considered here) compared to the meson lifetimes (10^{-18} sec for $2p-1s$), we can ignore spontaneous nuclear radiation until after the meson settles into the $1s$ orbit. If the nucleus is left in an excited state, it will then radiate with the same frequency it would have in the absence of the meson (we recall that a $1s$ meson produces only a spherically symmetric field at the nucleus), *except* that the process of meson K-capture may occur before the nuclear transition. The K-capture mean life has been measured to be ~5 to 10×10^{-8} second for heavy

[8] Hofstadter, Hahn, Knudsen, and McIntyre, Phys. Rev. 95, 512 (1954).

nuclei,[9] and should successfully compete with the nuclear decay for a nucleus such as U with an excited state at 44 kev. On the other hand, the rapid dependence of nuclear lifetime on energy ($\tau^{-1}\sim\omega^5 Q_0^2$) enables one to predict that, for Ta (first excited state at 137 kev), K-capture in excited nuclear states will be small. In Secs. V and VI, the fraction of $2p-1s$ gamma rays which should be followed by nuclear gammas is calculated, ignoring the possible effect of K-capture in depleting the excited states.

V. EVEN-EVEN NUCLEI

In an even-even nucleus with low-lying states of even parity and $I=0$, 2, 4, only the ground and first excited states are involved in the fine structure. A $2p$ meson combines with the nuclear spins $I=0$ and 2 to give two levels with $J=\frac{1}{2}$ and three with $J=\frac{3}{2}$. Levels with higher J are not connected to the nuclear ground state and give no contribution to the spectrum [see Eq. (18)]. We introduce a parameter α with the dimensions

TABLE II. $(I'j'|H|Ij)$ for meson in $2p$ orbit; even-even nucleus with $I=0,2$.

J	(I,j)			
		$(0,\frac{1}{2})$		$(2,\frac{3}{2})$
$\frac{1}{2}$	$(0,\frac{1}{2})$	0		$-\sqrt{2}$
	$(2,\frac{3}{2})$	$-\sqrt{2}$		$\epsilon_N+\epsilon_S-\rho$
		$(0,\frac{3}{2})$	$(2,\frac{1}{2})$	$(2,\frac{3}{2})$
	$(0,\frac{3}{2})$	ϵ_S	1	1
$\frac{3}{2}$	$(2,\frac{1}{2})$	1	ϵ_N	$-\rho$
	$(2,\frac{3}{2})$	1	$-\rho$	$\epsilon_N+\epsilon_S$

of energy

$$\alpha=\frac{e^2}{10}Q_0(0,2)\left(\frac{P(r)}{r^3}\right)_{2p}, \tag{19}$$

and write

$$E_N(2)=\alpha\epsilon_N,\quad E_S=\alpha\epsilon_S,\quad \rho=Q_0(2,2)/Q_0(0,2). \tag{20}$$

If E_S were known from experiments on a "stiff" nucleus such as Pb, and E_N and $Q_0(0,2)$ from Coulomb excitation experiments or life-time studies, the fine structure pattern would depend on a single parameter not susceptible to measurement by other experiments: the quadrupole moment of the excited state (essentially ρ). The extreme BM strong-coupling model predicts $\rho=+1$ for a prolate, -1 for an oblate nuclear shape.

The H matrices to be diagonalized are presented in Table II. (H_μ has been omitted, since it gives no contribution to the fine structure.) The eigenvalues of these matrices are the energies $E(\nu J)$ [Eq. (17)] in units of α; the eigenfunctions yield the intensities of the spectral lines [Eq. (18)].

For $_{74}W^{\rm even}$, the fine structure has been calculated

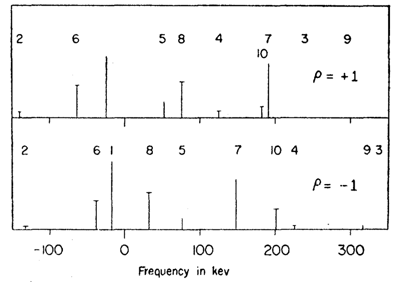

FIG. 1. $2p-1s$ fine structure for $_{74}W^{\rm even}$ for two values of ρ [see Eq. (20)]. The quadrupole moment of the first excited state is $-2.0\rho\times10^{-24}$ cm^2. The numbers serve to indentify lines which go continuously into each other as ρ is varied. Even numbered lines (drawn with horizontal bars at their tops) leave the nucleus in the first excited state. If all quadrupole effects were absent, all lines would disappear except No. 1, which would have $\frac{1}{3}$ the total intensity at 0 kev, and No. 7, with $\frac{2}{3}$ of the intensity at 155 kev.

by using as parameters[10] $E_N=115$ kev, $E_S=155$ kev, and $Q_0(0,2)=7\times10^{-24}$ cm^2; the latter gives $\alpha=53$ kev. The Bohr-Mottelson value of ρ is $+1$, since neighboring odd-A nuclei have positive quadrupole moments. The positions and relative intensities of the spectral lines are presented in Fig. 1 for $\rho=+1$ and $\rho=-1$. In this as well as all other spectra, the zero of the abscissa has

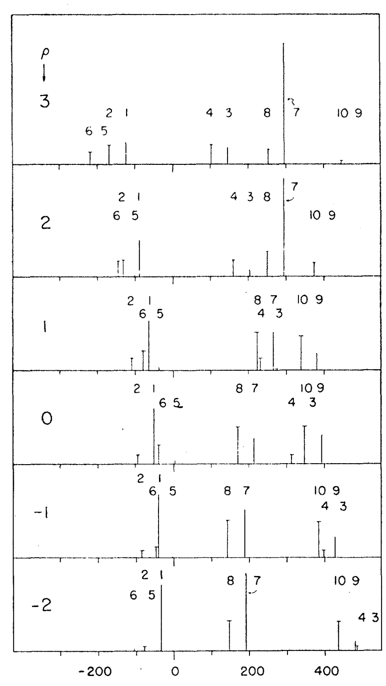

FIG. 2. Fine structure of the $2p-1s$ line for $_{92}U^{238}$ for several values of ρ. The quadrupole moment of the first excited state is $-2.9\rho\times10^{-24}$ cm^2. The lines are numbered as in Fig. 1. The arbitrary intensity units are the same throughout the diagram. If all quadrupole effects were absent, there would be one line at 0 kev, and a second with twice the intensity at 260 kev.

[9] Keuffel, Harrison, Godfrey, and Reynolds, Phys. Rev. **87**, 942 (1952).

[10] T. Huus and C. Zupancic, Kgl. Danske Videnskab. Selskab, Mat. fys. Medd. **28**, No. 1 (1953).

TABLE III. The fraction (e) of the intensity of the $2p-1s$ transition in $_{92}U^{238}$ which leaves the nucleus in the 44-kev excited state for various values of ρ [see Eq. (20)].

ρ	e
3	0.293
2	0.375
1	0.504
0	0.482
−1	0.413
−2	0.312

been chosen at the energy which a $2p_{\frac{1}{2}}-1s$ gamma ray would have if all quadrupole effects were absent. The height of each line is proportional to its intensity, and those with horizontal lines at their tops are followed by a nuclear gamma ray. The fraction of the intensity of the lines which leave the nucleus excited is 0.408 for $\rho=+1$ and 0.412 for $\rho=-1$.

A second even-even calculation has been carried through for $_{92}U^{238}$ to see the effect of changes in the parameters; in this case the sensitivity of the spectrum to variations in ρ has been examined more thoroughly. We have used[11] $E_S=260$ kev, $E_N=44$ kev, $Q_0(0,2)=10\times10^{-24}$ cm^2, and $\alpha=95$ kev. Figure 2 shows the spectrum for various values of ρ. The fractional intensities of the transitions to the excited nuclear state are listed in Table III, which shows that these fractions form a rather insensitive measure of the quadrupole moment.

Figures 1 and 2 show that an experimental resolution somewhat better than the 100 kev reported by Fitch and Rainwater[1] will be needed to see the complete structure.

Before we leave the even-A case, a special peculiarity of the matrices displayed in Table II should be mentioned. If by chance a certain nucleus has E_N equal to E_S, the first two states for $J=\frac{3}{2}$ which are listed are degenerate to zeroth order in the quadrupole effect. Then even for a very small quadrupole effect the off-diagonal elements are important and the nuclear states are strongly mixed. In this special case, one third of the $2p-1s$ intensity leaves the nucleus in the excited state. In Fig. 3 the spectrum is shown for (a), $\alpha\ll|E_N-E_S|$ and (b), $|E_N-E_S|\ll\alpha\ll(E_N+E_S)$; in the latter case, there is an inversion of the intensity ratio for the two groups of lines.

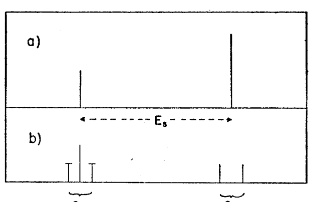

FIG. 3. (a) The $2p-1s$ fine structure for $\alpha\ll|E_N-E_S|$ (i.e., spin-orbit splitting only) and (b) the special case $|E_N-E_S|\ll\alpha\ll E_N+E_S$.

[11] N. P. Heydenburg and G. M. Temmer, Phys. Rev. **93**, 906 (1954); also G. M. Temmer (private communication).

The parameters for W roughly approximate the special case while those for U do not at all; this should be remembered when comparing the qualitative aspects of Figs. 1 and 2.

VI. ODD-A NUCLEI

The odd-A nuclei give patterns considerably more complicated than those considered above, since there is more than one excited state connected by quadrupole matrix elements to the ground state. We have worked out the formulas explicitly for the case of a nucleus with $I_0=7/2$, $I_1=9/2$, and $I_2=11/2$ (all parities equal), omitting any possible effects of higher nuclear states. An enumeration of the possible mixings in this case predicts a $2p-1s$ fine structure with 37 lines; although many of these can be expected to be extremely weak, the resultant pattern is considerably harder to resolve and to interpret than those shown in Figs. 1 and 2.

In this case, the fine structure depends on the six quantities $Q_0(I,I')$ (i.e., the quadrupole moments of the three states and the matrix elements connecting

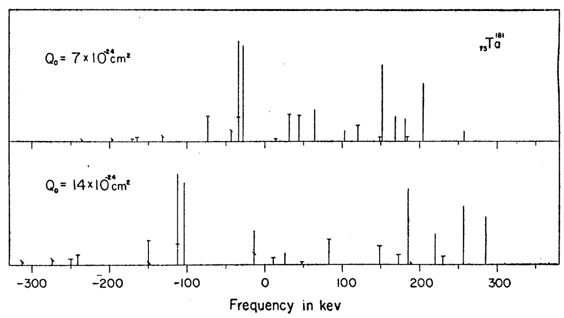

FIG. 4. The $2p-1s$ fine structure for $_{73}Ta^{181}$ for two values of Q_0 using the Bohr-Mottelson model ($\rho=+1$). The lines with horizontal tops leave the nucleus in the first excited state; those with slanting tops leave it in the second excited state.

them) in addition to the energies $E_N(9/2)$, $E_N(11/2)$, and E_S. We introduce the energy α just as in Eq. (19), substituting $Q_0(7/2,9/2)$ for the $Q_0(0,2)$ appearing there, and define $\rho(I,I')=Q_0(I,I')/Q_0(7/2,9/2)$. The energies and eigenfunctions can then be found by diagonalizing the matrices appearing in Table IV, where, however, to achieve the general case each matrix element of H_q must be multiplied by the ρ appropriate to the indices of that matrix element. For example,

$$(4,7/2,3/2|H|4,7/2,1/2)=-(28/45)^{\frac{1}{2}}\rho(7/2,7/2).$$

The solution has been carried out for $_{73}Ta^{181}$ by using the Bohr-Mottelson model of pure rotational states: $\rho(I,I')=+1$ for all I, I'. The parameters used were $E_S=151$ kev, $E(9/2)=137$ kev and $E(11/2)=300$ kev; the spectrum plotted in the upper part of Fig. 4 is for $Q_0=7\times10^{-24}$ cm^2, a value taken from Coulomb excitation cross sections;[10] that in the lower part of Fig. 4 is for $Q_0=14\times10^{-24}$ cm^2 (which agrees with the measured quadrupole moment of the ground state).

TABLE IV. $(I'j'|H|Ij)$ for meson in $2p$ orbit; odd-A nucleus with $I=7/2$, $9/2$, $11/2$. Every off-diagonal matrix element is the positive square root of the appropriate entry unless the entry is followed by an asterisk, in which case it is the negative square root.

J	(I,j)					
		(7/2,3/2)				
2	(7/2,3/2)	ϵ_S+1				
		(7/2,1/2)	(7/2,3/2)	(9/2,3/2)		
3	(7/2,1/2)	0	4/3	2/3		
	(7/2,3/2)		$\epsilon_S-1/3$	8/9*		
	(9/2,3/2)			$\epsilon(9/2)+\epsilon_S+1/3$		
		(7/2,1/2)	(7/2,3/2)	(9/2,1/2)	(9/2,3/2)	(11/2,3/2)
4	(7/2,1/2)	0	28/45*	0	98/99*	64/165*
	(7/2,3/2)		$\epsilon_S-13/15$	56/45*	56/495*	112/825
	(9/2,1/2)			$\epsilon(9/2)$	16/99	98/165
	(9/2,3/2)				$\epsilon(9/2)+\epsilon_S-5/33$	1568/1815*
	(11/2,3/2)					$\epsilon(11/2)+\epsilon_S+1/55$
		(7/2,3/2)	(9/2,1/2)	(9/2,3/2)	(11/2,1/2)	(11/2,3/2)
5	(7/2,3/2)	$\epsilon_S+7/15$	392/1815	3136/5445	192/605	624/3025
	(9/2,1/2)		$\epsilon(9/2)$	32/363*	0	7938/7865*
	(9/2,3/2)			$\epsilon(9/2)+\epsilon_S-10/33$	147/121*	588/7865*
	(11/2,1/2)				$\epsilon(11/2)$	4/7865
	(11/2,3/2)					$\epsilon(11/2)+\epsilon_S-7/715$

The corresponding values of α are 53 and 105 kev, respectively.

The lines with single horizontal bars over them leave the nucleus in its first excited (9/2) state; those with slanting bars leave it in the second excited (11/2) state. The fraction of the total intensity going to the 9/2 state is 0.241 for $Q_0=7$ barns and 0.235 for $Q_0=14$ barns; the corresponding fractions for the 11/2 state are 0.035 and 0.049.

Perturbation theory would predict a group of lines of intensity 1 near 0 kev and another group of intensity 2 near 150 kev, accompanied by a sprinkling of very weak lines shifted by 137 and 300 kev toward the low-frequency end of the spectrum. Figure 4 indicates that the groups spread apart as Q_0 increases and also spread considerably within themselves; moreover, the one to two intensity ratio is considerably changed even for $Q_0=7\times10^{-24}$ cm^2.

Free variation of all six parameters $Q_0(I,I')$ could be made to produce a large variety of spectral features, especially since it is possible to get approximate degeneracies of some of the diagonal elements in Table IV. There is little point in thus refining the calculations at this stage.

VII. SUMMARY

The electric quadrupole field of a $2p$ μ meson in a heavy atom can be expected to mix several of the lowest nuclear levels provided that (a) these are separated by $\sim$100 kev and (b) the quadrupole matrix elements connecting them are large. A large number of nuclei are known (from Coulomb excitation experiments) to satisfy both these conditions. $_{92}U^{238}$, the even isotopes of $_{74}W$, and $_{73}Ta^{181}$, have been singled out for a detailed calculation of the fine structure of the $2p-1s$ line resulting from this interaction. The spectrum in these cases is found to be spread out over a region of 300–500 kev in a way which is sensitive to the quadrupole moments and transition probabilities of the nuclear states involved. The interpretation of an experimental spectrum would be clearest in the case of an even-A nucleus. Here the single parameter which strongly influences the spectrum and which is not easily measureable in other types of experiments is the quadrupole moment of the first excited state. Figure 2 shows most clearly how this parameter would influence the pattern in one case. Finally, the mixing of the nuclear levels should result in a nuclear gamma ray following the $2p-1s$ transition (unless K-capture competes); this will happen 25–50 percent of the time in the cases considered here.

The author wishes to express his appreciation to Professor Robert Oppenheimer and the Faculty of the Institute for Advanced Study for the extension of a grant and of warm hospitality at the Institute, where this work was begun.