be equal at some specific angle in spite of an asymmetry in the nuclear forces. Moreover, the matter of degree must be considered. Since the cross sections cannot be calculated explicitly in terms of the n-n and p-p forces, it is not possible to say how much effect an asymmetry in the forces would have upon the cross sections. The following conclusion, however, seems justified. At low energy, charge symmetry is an established fact and the cross sections for the production of H^3 and He^3 from deuteron-deuteron collisions are equal (neglecting of course Coulomb and mass-difference effects). The fact that the cross sections are nearly equal at high energy supports the hypothesis of charge symmetry at high energy.

ACKNOWLEDGMENTS

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Electron Tridents between 0.1 and 10 Bev

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Data on the direct production of pairs by fast electrons of the cosmic radiation have been collected in nuclear emulsions by a track-following method. Multiple-scattering measurements were made on the secondaries of 61 tridents with energies between 0.1 and 10 Bev; of these, 14 originated on arms of pairs. A mean free path of 134±42 cm was deduced for the latter, corresponding to a cross section of $0.92\pm0.29\times10^{-25}$ cm^2. An appropriate statistical treatment for small numbers has been adopted, and a correction has been made for the inclusion of pseudo-tridents. The theoretical cross sections of Bhabha and Racah, which apparently disagree by a factor of about 2 in this energy region, have been shown to be consistent. The theoretical cross section averaged over the experimental energy distribution is 0.82×10^{-25} cm^2. The distributions in fractional energy transfer to the created pair and internal energy partition in the pair have been derived and are compared with our observations. It is concluded that trident theory is in agreement with this experiment.

I. INTRODUCTION

OBSERVATIONS of the direct creation of electron pairs arising from the interaction of fast charged particles with the Coulomb fields of nuclei have been extensively studied.[1–12] These phenomena, called tridents, consist of an incoming, or primary track of a relativistic charged particle, and three emergent tracks as shown by the photomicrograph in Fig. 1. Experiments made chiefly in photographic emulsions indicate that the great majority of tridents have electron primaries. In this text we refer to both electrons and positrons as electrons. In those instances where identification has been possible, it has been demonstrated that the secondaries are of electronic mass. Energy measurements have indicated that the primary energy is conserved among the three secondary tracks.

The theory of the direct pair creation process has been discussed by a number of authors, notably Bhabha,[13] Nishina *et al.*,[14] and Racah.[15] The cross sec-

FIG. 1. A photomosaic of a trident in a G5 emulsion. In this example, less than one percent of the primary energy is transferred to the directly created pair.

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[14] Nishina, Tomonaga, and Kobayasi, Sci. Papers Inst. Phys. Chem. Research Tokyo **27**, 137 (1935).
[15] G. Racah, Nuovo cimento **14**, 93 (1937).

tion is predicted to increase slowly with primary energy, in qualitative agreement with experiment.[8] Theory shows that the cross section is $\sim 10^{-25}$ cm^2 for primary electron energies of several hundred Mev, the corresponding cross section for primary mesons being smaller by a factor of ~ 50. Bhabha, Nishina *et al.* and Ravenhall[16] have considered the effect of screening and have found it to be small up to ~ 1 Bev. However, the magnitude of the trident cross section given by Bhabha is about twice that given by Racah for the energies found in the cosmic radiation.

Experimental identification of a trident rests upon the observer's ability to distinguish between pair origins truly coincident with the primary track and those due to chance superposition. The latter arise chiefly from the materialization of bremsstrahlung photons indistinguishably close to the parent track. A recent study[17] has been made of 200 bremsstrahlung pairs found in this experiment. This work has established the magnitude of bremsstrahlung pair confusion, or pseudo-trident occurrence, and shows that for primary energies between 0.1 and 10 Bev this effect is not large. Calculations further indicate that for the high electron energies found in the cores of "jets," the proportion of pseudo-tridents among the tridents becomes markedly greater. An additional experimental factor encountered in "jet" observations is the difficulty in distinguishing the individual tracks due to the locally high track densities. We have therefore confined our study to a general cosmic ray exposure with events of energy below 10 Bev and of low track density, in order to minimize the possibility of misinterpretation.

In the present experiment the data have been collected by a track following method which allows a quantitative measure of the cross section. The number of tridents of primary energy between 0.1 and 10 Bev, initiated by particles originating in pairs, was compared with the total length of pair tracks within the same energy interval. The division of the primary energy among the three secondaries has also been examined. In the course of the present study a comparison has been made between the work of Bhabha and Racah in order to determine the basis of their numerical disagreement. It has been found that a more detailed application of Bhabha's method leads to a result in substantial agreement with that of Racah. A comparison between theory and experiment has been made.

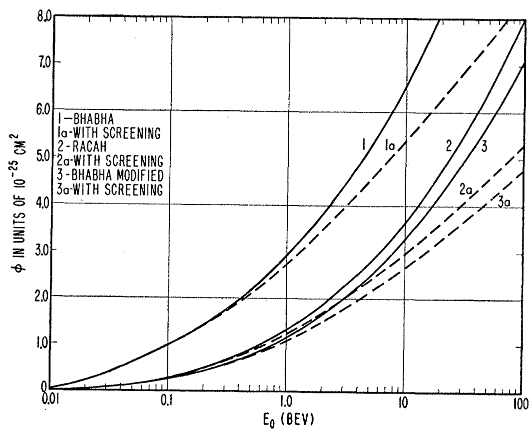

FIG. 2. The theoretical total cross section for the direct creation of a pair by a fast electron, as a function of primary energy.

II. THEORY

A. Total Cross Section

According to Bhabha[13] the expression for the electron trident cross section ϕ is, neglecting screening,

$$\phi(E_0)=\frac{28}{27\pi}\left(\frac{r_0 Z}{137}\right)^2 \ln^3\left(\frac{kE_0}{mc^2}\right), \tag{1}$$

where E_0 is the energy of the primary electron which initiates the process, mc^2 the electron rest energy, r_0 the classical electron radius, and Z the atomic number of the target nucleus. k is a number of the order of unity. This expression is accurate only when the energy of the primary electron is extremely large in relation to its rest energy, so that terms of lower powers in $\ln(E_0/mc^2)$ can be neglected in comparison to the cubic term in (1). In order to study the behavior of the cross section at lower energies where this condition is no longer valid, we have attempted a more detailed integration of the differential cross section $d^2\phi$ which is given by Eq. (32) of Bhabha's paper,

$$d^2\phi(E_0,E_+,E_-)=\frac{8}{\pi}\left(\frac{r_0 Z}{137}\right)^2 \frac{E_+^2+E_-^2+\frac{2}{3}E_+E_-}{(E_+ + E_-)^4} \times\ln\frac{kE_+E_-}{(E_+ + E_-)mc^2}\ln\frac{k'E_0}{E_+ + E_-}dE_+dE_-; \tag{2}$$

k and k' are again numbers of the order of unity, and E_+ and E_- are the energies of the created electrons. Following Bhabha, we introduce a new variable E_T, the total energy of the created pair, and perform the integration with respect to E_-. However, we depart from Bhabha's treatment in that we retain all terms, making approximations only after completing all integrations. We then obtain the modified differential cross section $d\phi_m$, which, after putting $k=k'=1$, becomes

$$d\phi_m(E_0,E_T)=\frac{56}{9\pi}\left(\frac{r_0 Z}{137}\right)^2 \frac{1}{E_T}\ln\frac{E_0}{E_T} \times\left\{\ln\frac{E_T}{mc^2}-\frac{44}{21}+\frac{18}{7}\frac{mc^2}{E_T}+\frac{2}{7}\left(\frac{mc^2}{E_T}\right)^2-\frac{1}{3}\left(\frac{mc^2}{E_T}\right)^3 +\frac{9}{14}\left(\frac{mc^2}{E_T}\right)^4+\cdots\right\}dE_T. \tag{3}$$

[16] D. G. Ravenhall, Proc. Phys. Soc. (London) **A63**, 1177 (1950).

[17] D. T. King and M. M. Block, Phys. Rev. **95**, 648 (1954); M. M. Block and D. T. King, Phys. Rev. **95**, 171 (1954).

In the derivation of (3), the terms in mc^2/E_T which, when integrated over E_T would contribute to lower powers of $\ln(E_0/mc^2)$ in the total cross section, have been preserved. This result is to be contrasted with the corresponding expression derived by Bhabha [(34) of his paper, when $k=k'=1$],

$$d\phi(E_0,E_T)=\frac{56}{9\pi}\left(\frac{r_0Z}{137}\right)^2\frac{1}{E_T}\ln\left(\frac{E_0}{E_T}\right)\ln\left(\frac{E_T}{mc^2}\right)dE_T. \quad (4)$$

Integrating (3) between the limits $2mc^2$ and $E_0-2mc^2\approx E_0$, we obtain the modified expression for the total cross section

$$\begin{aligned}\phi_m(E_0)=\frac{28}{27\pi}\left(\frac{r_0Z}{137}\right)^2\Bigg\{&\ln^3\left(\frac{E_0}{mc^2}\right)-\frac{44}{7}\ln^2\left(\frac{E_0}{mc^2}\right)\\&+\left(\frac{54}{7}+\frac{88}{7}\ln2-3\ln^22\right)\ln\left(\frac{E_0}{mc^2}\right)\\&-\frac{54}{7}-\frac{54}{7}\ln2-\frac{44}{7}\ln^22+2\ln^32+\cdots\Bigg\},\end{aligned} \quad (5)$$

which is to be compared with (1). For energies of several hundred Mev, numerical evaluation of (5) shows that the terms in $\ln^2(E_0/mc^2)$, $\ln(E_0/mc^2)$ and the constant are not negligible in comparison to $\ln^3(E_0/mc^2)$ which is the only corresponding term in (1). In Fig. 2 we have plotted the Bhabha cross section (1) and the modified cross section (5), together with the corresponding cross section obtained by Racah.[15] The cross sections have been evaluated for photographic emulsions, with effective $Z=22.1$. Bhabha's original cross section is about twice as large as that found by Racah; the close agreement of our modified result (5) and that of Racah is satisfactory in view of the approximations involved in both treatments. There is thus a simpler theoretical basis against which experiment is to be compared.

Screening has been neglected in the above calculations. Ravenhall[16] has estimated the effects of screening in photographic plates, and concludes that the cross section is reduced by 6 percent for primary electron energies of 1 Bev, and by 18 percent for 10 Bev. Since the energies of the tridents found in this study are mainly in the vicinity of 500 Mev, the effect of screening is trivial, reducing the cross section by $\sim$3 percent. The dashed curves in Fig. 2 take screening[13,14,16] into account.

In our experiment the energies of the primary electrons are distributed over a wide range. To achieve a meaningful comparison with theory it is necessary to average the theoretical cross section over the energy spectrum of the primary electrons. A power law of the form

$$dN(E_0)=\text{constant }E_0^{-1.6}dE_0 \quad (6)$$

has been assumed for the differential distribution in electron energy, derived from the experimentally determined π^0 meson spectrum.[18] The average cross section is then given by

$$\bar{\phi}=\int_{E_0(\min)}^{E_0(\max)}\phi(E_0)dN(E_0)\Big/\int_{E_0(\min)}^{E_0(\max)}dN(E_0), \quad (7)$$

where $E_0(\min)$ and $E_0(\max)$, the minimum and maximum primary electron energies of our experiment, are 0.1 Bev and 10 Bev, respectively. Using the modified Bhabha cross section $\phi_m(E_0)$ we obtain 0.82×10^{-25} cm^2 for $\bar{\phi}$. From Fig. 2, this corresponds to an average primary electron energy of $\sim$400 Mev. Assuming that screening reduces the cross section by 3 percent at this energy, we conclude that the average theoretical cross section for the tridents found in our study is 0.80 $\times10^{-25}$ cm^2.

B. Energy Partition

In order to facilitate further comparison with experiment it is instructive to introduce new variables $R=E_T/E_0$ and $r=E_+/E_T$ into (2). R is the fractional transfer of primary energy to the trident pair, whereas r expresses the fraction of the trident pair energy carried off by the positron. The theory is completely symmetric in E_+ and E_- as seen in (2). In terms of R and r, (2) becomes

$$\begin{aligned}d^2\phi(E_0,R,r)=\frac{8}{\pi}\left(\frac{r_0Z}{137}\right)^2\left(1-\frac{4}{3}r+\frac{4}{3}r^2\right)\frac{1}{R}\ln\left(\frac{1}{R}\right)\\\times\ln\left(\frac{E_0Rr(1-r)}{mc^2}\right)dRdr.\end{aligned} \quad (8)$$

When the dependence on r is integrated out of (8), we obtain

$$\begin{aligned}d\phi(E_0,R)=\frac{56}{9\pi}\left(\frac{r_0Z}{137}\right)^2\frac{1}{R}\ln\left(\frac{1}{R}\right)\Bigg\{&\ln\left(\frac{E_0R}{mc^2}\right)\\&-\frac{44}{21}+\frac{18}{7}\frac{mc^2}{E_0R}+\frac{2}{7}\left(\frac{mc^2}{E_0R}\right)^2\\&-\frac{1}{3}\left(\frac{mc^2}{E_0R}\right)^3+\frac{9}{14}\left(\frac{mc^2}{E_0R}\right)^4+\cdots\Bigg\}dR.\end{aligned} \quad (9)$$

This is the equivalent of (3) in terms of the new variables. Similarly, by integrating R out of (8), we find

$$\begin{aligned}d\phi(E_0,r)=\frac{4}{3\pi}\left(\frac{r_0Z}{137}\right)^2&\left(1-\frac{4}{3}r+\frac{4}{3}r^2\right)\\&\times\Bigg[\ln^3\left(\frac{E_0}{mc^2}\right)+3\ln^2\frac{E_0}{mc^2}\ln[r(1-r)]\\&-6\ln\frac{E_0}{mc^2}\{\tfrac{1}{2}\ln^2r-\ln r\ln[r(1-r)]\}\\&-2\ln^3r+3\ln^2r\ln[r(1-r)]\Bigg]dr.\end{aligned} \quad (10)$$

[18] Carlson, Hooper, and King, Phil. Mag. **41**, 701 (1950).

In Figs. 3 and 4 we have plotted $d\phi(E_0,R)/dR$ and $d\phi(E_0,r)/dr$, respectively, for several primary energies. It is instructive to compare the R-distribution with the corresponding distribution in the Bethe-Heitler theory[19] for bremsstrahlung, and the r-distribution with the corresponding distribution for energy partition in pair production. With increasing trident primary energy the similarity becomes marked. It is interesting to conjecture that this similarity is due, in the direct creation process, to the virtual photon exhibiting a behavior more and more like that of a real photon, with increasing primary energy.

Let us now consider $P(R,E_0)dR$, the probability distribution in fractional energy transfer, namely, the probability that R lie between R and $R+dR$ for a given E_0. This is given by the ratio of (9) to (5), i.e., $d\phi(E_0,R)/\phi(E_0)$. In order to enable a comparison with experiment where the primary energies range between 0.1 and 10 Bev, we average $P(R,E_0)$ over the energy distribution (6). Thus

$$\bar{P}(R)dR=\frac{\int_{E_0(\min)}^{E_0(\max)} P(R,E_0)dN(E_0)dR}{\int_{E_0(\min)}^{E_0(\max)} dN(E_0)}. \qquad (11)$$

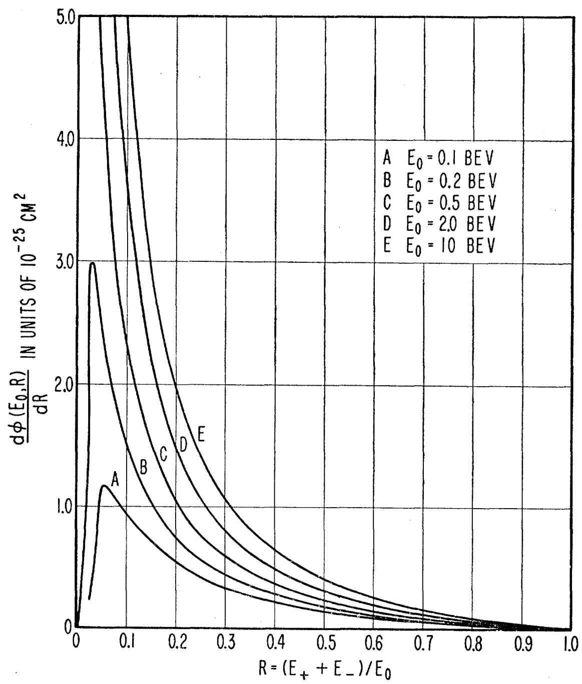

FIG. 3. The theoretical differential cross section $d\phi(E_0,R)/dR$ as a function of R, illustrating the fractional transfer of primary energy to the trident pair. E_0 is the primary energy and $R=E_{\text{pair}}/E_0$.

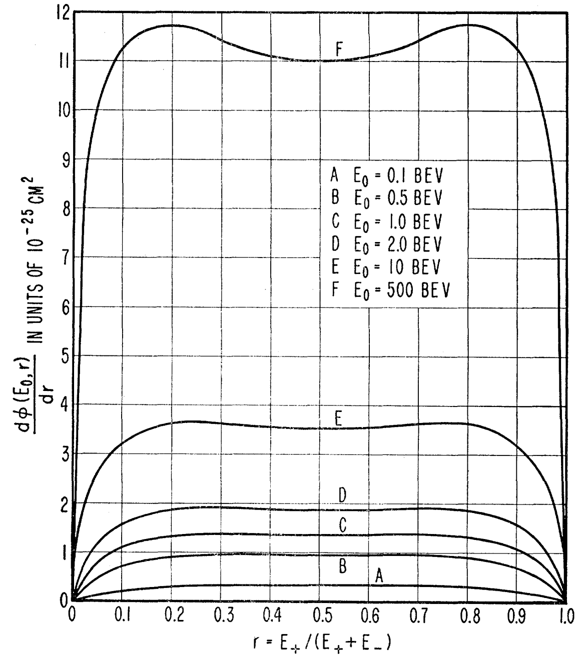

FIG. 4. The theoretical differential cross section $d\phi(E_0,r)/dr$ as a function of r, illustrating the internal energy partition within trident pairs. E_0 is the primary energy and $r=E_+/E_{\text{pair}}$.

In the same manner we obtain

$$\bar{P}(r)dr=\frac{\int_{E_0(\min)}^{E_0(\max)} P(r,E_0)dN(E_0)dr}{\int_{E_0(\min)}^{E_0(\max)} dN(E_0)}, \qquad (12)$$

where $P(r,E_0)=d\phi(E_0,r)/\phi(E_0)$. We have plotted (11) and (12) as the smooth curves in Figs. 6 and 7, respectively.

III. EXPERIMENT

A. Exposure, Search Method, and Measurement

Twenty-six Ilford G5 nuclear emulsions 400 microns thick were exposed to the cosmic radiation at altitudes exceeding 95 000 feet and at geomagnetic latitude 55°N for a period of 6 hours. The plates employed in this study were arranged under differing amounts of Pb and C absorber, as several simultaneous experiments with energetic particles were being made. In such exposures a large number of tracks attributable to the soft component can be found.

An appropriate search procedure, track following, was employed in order to detect more efficiently those configurations consisting only of lightly ionizing tracks. In earlier experiments[8,10] on tridents the material was found by a systematic examination of the emulsion volume; this led to criticism[20] that tridents of highly unequal secondary energies tended to escape observa-

[19] W. Heitler, *Quantum Theory of Radiation* (Oxford Press, London, 1944).

[20] G. Gottstein and K. Ott (private communication).

tion. With the present search method it is found that tridents of all types are relatively conspicuous. The examination is limited to those tracks which are inclined to the emulsion plane at angles less than $\sim 5°$, and which have grain densities less than three times the minimum. These criteria describe the long paths of charged particles which penetrate the emulsion with velocities exceeding about one-half that of light.[21] Our scan proceeds by following along such tracks using oil immersion objectives. The observer picks at random the first satisfactory track and follows it until it leaves the volume under examination. He then makes a small arbitrary displacement of the microscope stage, selects a second track and repeats the procedure. An area of 4×7 cm^2 of each plate is scanned in this manner. If a plate is examined by this method for a total time of $\sim$40 hours, it can be shown that about 30 percent of the tracks are found more than once and approximately 20 percent of the material escapes detection altogether. Among the observations recorded were electron pairs, tridents, and electromagnetic cascade phenomena including bremsstrahlung pairs which have been discussed previously.[17] The quantum energies of the pairs were estimated from the opening angles. The energies of the secondary particles from tridents were measured by multiple scattering and in favorable circumstances the energy of the primary was similarly determined.

The measurement of electron energies by multiple scattering is complicated by the appreciable radiative losses which may occur along the necessary minimum length of track. An additional difficulty arises from the fact that great path lengths are required for even a rough energy measurement of the very energetic particles, so that the determinations for high energies are relatively uncertain. For very low energies there is a tendency for the particles to scatter out of the emulsion so that insufficient track length is available for measurement. Thus there is a considerable measurement bias at both high and low energies. Further it is well known that the determination of quantum energies from opening angles of pairs is at best a crude estimate. We conclude that the majority of energy measurements made in this work are subject to considerable error.

B. Cross-Section Determination

The total cross section is derived from the experimentally determined mean free path; for this purpose the number of tridents found is compared with the length of electron track examined. Since the emulsion is traversed by an admixture of fast protons, mesons and electrons, it is not generally possible to estimate the total track length of the electrons alone. However, it is possible to determine that length of electron track which originates from pairs formed in the emulsion. In order, therefore, to evaluate the electron trident cross section, only those tridents occurring on arms of pairs can be used. Out of 67 tridents found through our scanning procedure, only 14 occurred on pairs and could be employed for the cross section determination.

The quantum energies of the pairs found in our experiment ranged from several Mev to many Bev. Since $\sim$90 percent of the tridents were found to have primary energy between 0.1 and 10 Bev, we have restricted our analysis to those pairs and the 61 tridents in this interval of energy. Our choice of 10 Bev as the upper limit is arbitrary; the evaluation of total track length from pairs is not greatly influenced by this selection since from the spectrum given by (6) the major intensity contribution is from lower energy electrons.

(*i*) *Total Track Length from Pairs*

A direct measurement of length and energy for all pair tracks would be unduly costly in time. We have therefore measured the track lengths in a sample of 3 out of the 26 plates. In order to exclude tracks due to electrons of energy less than 100 Mev, we have made two corrections. Firstly, the pairs with quantum energies estimated from opening angle to be less than 100 Mev were eliminated. Secondly, the contribution due to electrons of energies less than 100 Mev arising from pairs with quantum energies exceeding 100 Mev was subtracted. For the latter correction the distribution (6) and a random energy partition within the pairs was assumed. Details of the path length determination together with the estimated uncertainties are shown in Table I. The total length of pair tracks within the energy interval was found to be 1440 ± 490 cm. The indicated probable error includes both the uncertainty in correcting for the amount of electron track with energies less than 100 Mev and the error in extrapolation of the path length from the sample. The major source of error arises from the attempt to apply a sharp cutoff at 100 Mev where the spectral intensity is greatest.

(*ii*) *Correction for Pseudo-Tridents*

In the same energy interval and volume of emulsion in which the 61 tridents were found, a study of 200

TABLE I. Evaluation of mean free path for tridents formed on arms of pairs in the energy interval 0.1–10 Bev. The determination was confined to the track length from pairs created in the emulsion and to tridents formed on arms of such pairs.

Track length per plate from pairs with quantum energy $E_\gamma > 100$ Mev.	84.3 ± 13.3 cm
Calculated length correction factor (allows for the inclusion of electrons of energy less than 100 Mev originating in pairs with quantum energies exceeding 100 Mev; includes error in estimate of E_γ).	0.66 ± 0.20
Number of plates	26
Corrected track length	1440 ± 490 cm
Number of tridents on arms of pairs	14
Number corrected for pseudo-tridents	11.8
Maximum likelihood value of mean free path	134 ± 42 cm

[21] These characteristics were adopted because studies of meson emission were pursued concurrently.

bremsstrahlung pairs has been made in order to estimate the incidence of pseudo-tridents.[17] In this work the calculations on the proximity distribution of bremsstrahlung pairs indicated that ~ 6 percent of these phenomena are experimentally indistinguishable from tridents. In order to estimate the true number of tridents among the observations we apply a correction factor of $(1-0.06\times 200/61)=0.81$ to the number, 14, of tridents initiated by pair particles. Thus the appropriate number of tridents upon which to base the mean free path determination is $0.81\times 14=11.8$.

(*iii*) *Evaluation of Mean Free Path*

In experiments of this type the mean free path is conventionally taken to be $\bar{\lambda}$, the total path length divided by the number of events. However, if the total path length is finite, the true mean free path is in general somewhat larger than $\bar{\lambda}$ and approaches $\bar{\lambda}$ as the number of events increases. This effect is due to events occurring near the end of the total path where the available remaining path length may be smaller than the mean free path. The magnitude of this effect is evaluated by employing the statistical method of "maximum likelihood," as shown below.

We assume that all events are characterized by the same mean free path λ. A convenient model is to picture all of the scanned track laid end to end to form one continuous path of length Y, in which M events are observed. Let us measure the path lengths of the individual events from the beginning of the composite track and denote them by $y_1, y_2, y_3\cdots y_M$, where $y_1\leq y_2\leq y_3\leq\cdots\leq y_M\leq Y$. Define $x_j=y_j-y_{j-1}$, and $X_j=Y-y_{j-1}$, where $y_0=0$; x_j is the distance traveled to event j from the preceding event, and X_j is the amount of track length available in which event j can take place. The probability that event 1 occurs between the origin and Y (or in distance X_1) is unity. Similarly, the probability that event 2 occurs between y_1 and Y (or in distance X_2) is unity. In general, the probability that event j occurs in distance X_j is unity. Thus, the probability that event j occurs between x_j and x_j+dx_j is given by

$$P(\lambda,x_j)dx_j=\exp\{-x_j/\lambda\}(1-\exp\{-X_j/\lambda\})^{-1}dx_j/\lambda.$$

The compound simultaneous probability that all x_j lie in their corresponding dx_j is given by

$$G(\lambda,x_1,x_2,\cdots x_M)dx_1dx_2\cdots dx_M =\prod_{j=1}^{M}\exp\{-x_j/\lambda\}(1-\exp\{-X_j/\lambda\})^{-1}dx_j/\lambda.$$

Following Annis *et al.*,[22] we obtain λ_l, the "maximum likelihood" estimate of λ, by maximizing G, i.e., by solving the equation

$$\left.\frac{\partial G}{\partial\lambda}(\lambda,x_1,x_2,\cdots x_M)\right]_{\lambda=\lambda_l}=0 \quad\text{for}\quad \lambda_l.$$

In this fashion we obtain

$$\lambda_l\approx\bar{\lambda}+M^{-1}\sum_{j=1}^{M}X_j(\exp\{X_j/\bar{\lambda}\}-1)^{-1},\quad X_j\gg\bar{\lambda}. \quad (13)$$

Equation (13) is formally identical with Eq. (54) of Annis *et al.*, but arises from consideration of a different physical situation. In the same manner, a measure of the statistical uncertainty is given by

$$\epsilon(\lambda)\approx\lambda_l\left\{M-\sum_{j=1}^{M}\left(\frac{X_j}{\bar{\lambda}}\right)^2\left[1-\exp\left(-\frac{X_j}{\bar{\lambda}}\right)\right]^{-2}\times\exp\left(-\frac{X_j}{\bar{\lambda}}\right)\right\}^{-\frac{1}{2}}. \quad (14)$$

Since in our experiment we have no information regarding the individual values of the X_j's, but only know $\bar{\lambda}$ and M it is reasonable to assume that on the average, when $j=1$, $X_1=M\bar{\lambda}$; $j=2$, $X_2=(M-1)\bar{\lambda}$; $\cdots j=j$, $X_j=(M-j+1)\bar{\lambda}$. If for simplicity we put $i=M-j+1$, and subsitute in (13), we obtain

$$\lambda_l=\bar{\lambda}\{1+M^{-1}\sum_{i=1}^{M}i(e^i-1)^{-1}\},$$

which, for $M\gtrsim 8$, becomes

$$\lambda_l\approx\bar{\lambda}(1+1.18M^{-1}). \quad (15)$$

[22] Annis, Cheston, and Primakoff, Revs. Modern Phys. **25**, 818 (1953).

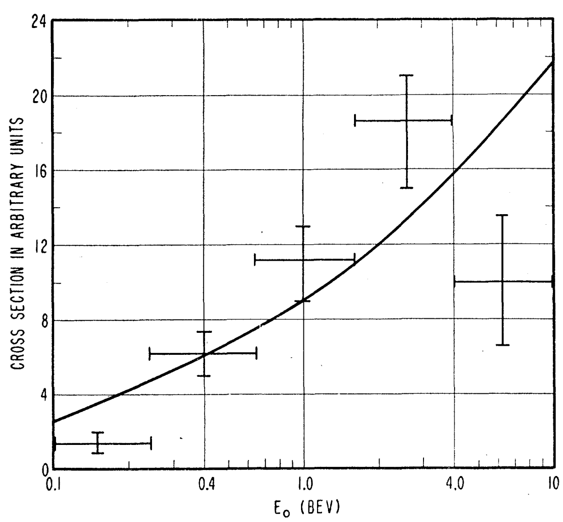

FIG. 5. Comparison of the experimental number of tridents per unit energy interval weighted by the primary electron spectrum, with the theoretical total cross section as a function of primary energy. The theoretical cross section, which includes screening, has been arbitrarily normalized to the experimental point at a primary energy of 0.4 Bev.

Thus, as $M\to\infty$, λ_t approaches $\bar{\lambda}$, as is expected. Similarly, from (14), the statistical uncertainty is given by

$$\epsilon(\lambda)=\lambda_t(M-2.39)^{-\frac{1}{2}}. \quad (16)$$

For our experiment, where M is 11.8, we obtain from (15) and (16), $\lambda_t=1.1\bar{\lambda}$ and $\epsilon(\lambda)=0.33\lambda_t$. Thus we see that the mean free path is 10 percent greater than the usual determination and the associated statistical uncertainty is 14 percent greater than the uncertainty conventionally taken as $M^{-\frac{1}{2}}$ (standard deviation).

Consideration of the total path length from pairs (1440 ± 490 cm), the corrected number of tridents (11.8) and the appropriate statistical error leads to a value of trident mean free path $\lambda_t=134\pm42$ cm for our experiment. This value of λ_t corresponds to a cross section of $0.93\pm0.29\times10^{-25}$ cm^2, where the number of atoms per cm^3 is taken to be 8.03×10^{22} for G5 emulsion and the uncertainties indicated are probable errors.

(*iv*) *Comparison with Theory*

The experimental result, $0.93\pm0.29\times10^{-25}$ cm^2, is to be compared with the theoretical prediction from Sec. II, 0.80×10^{-25} cm^2. The agreement is satisfactory. In order to investigate whether this agreement is fortuitous, we have attempted a test of the assumptions upon which the theoretical prediction is based. The theoretical cross section averaged from 0.1 to 10 Bev is compounded of the energy variations of both the trident cross section and the incident electron spectrum. A simultaneous check of the assumed forms of these variations, (5) and (6), is afforded by a comparison of the experimental excitation function with theory. All 61 tridents were employed; a plot of the relative yield with energy is given in Fig. 5. The experimental number of tridents per unit energy interval was divided by the spectrum (6) and compared to (5), modified by screening. The curve was arbitrarily normalized to the experimental value at 400 Mev. The indicated errors are standard deviations due to statistics. In view of the limited number of observations and the errors in energy determination, the prediction is consistent with experiment. It therefore appears justifiable to employ (5) and (6) in an evaluation of the averaged theoretical cross section.

C. Energy Division Among the Secondaries

It is not possible to identify the created pair among the three secondaries. Theory, however, suggests that a large majority of the pairs produced have energies less than one-half that of the primary electron. In Fig. 3, the plot of the differential cross section as a function of R, where $R=E_{\text{pair}}/E_{\text{primary}}$, illustrates this point. Following earlier experiments,[10] we employ the convention in which the two secondaries of lowest energy are taken to be the created particles. Under this assumption the largest attainable value of the quantity R is $\frac{2}{3}$. The energies of the outgoing particles have been measured by multiple scattering; for each trident R is determined as the ratio of the sum of the two lowest energies to the sum of all three. The experimental probability distribution in R is given as a histogram in Fig. 6. All 61 tridents found in the energy interval 0.1 to 10 Bev were used in this plot, and indicated uncertainties are standard deviations.

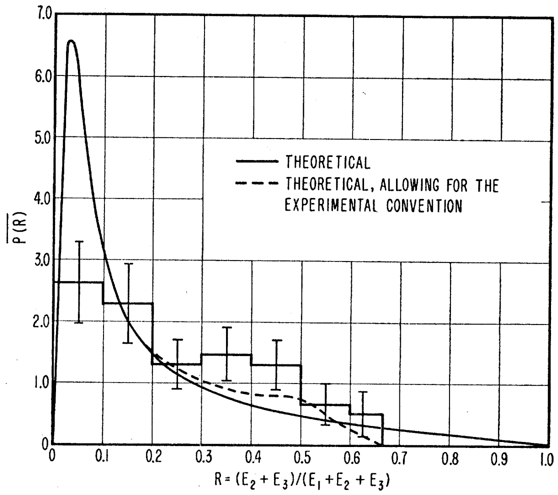

FIG. 6. Comparison of experimental R-distribution with the averaged theoretical probability for fractional transfer of primary energy to trident pairs. E_1, E_2, and E_3 are the energies of the secondaries with $E_1\geqslant E_2\geqslant E_3$.

For comparison with experiment the quantity $\bar{P}(R)$ of (11) is shown in Fig. 6 as the solid curve. It has been modified to take account of the effects of our convention, on the assumption of random energy partition within the created pair. The modification results in a small redistribution of the area under $\bar{P}(R)$ for $R>\frac{1}{2}$ and a cutoff at $R=\frac{2}{3}$. The altered $\bar{P}(R)$ is the dashed curve in Fig. 6. The experimental results are not in marked disagreement with the theory. It appears, however, that the experimental values are higher than the theoretical for large values of R, but the significance of this conclusion is doubtful because of poor statistics and because distortion of the R-distribution may occur through systematic errors in energy measurement.

A previous determination[10] of the R-distribution was in poor agreement with theory; this may have been due to the search procedure by which the material was found, in which possibly there was a discrimination against low values of R. In the present experiment this disadvantage was probably minimized by scanning along tracks.

A similar study has been made of the experimental probability distribution in r. For this purpose, r was taken to be the ratio of the lowest secondary energy to the sum of the two lowest secondary energies. Figure 7 shows the experimental histogram compared with the theoretical averaged probability distribution $\bar{P}(r)$ of (12). We have examined the effects of our experimental convention upon the theoretical r-distribution and have

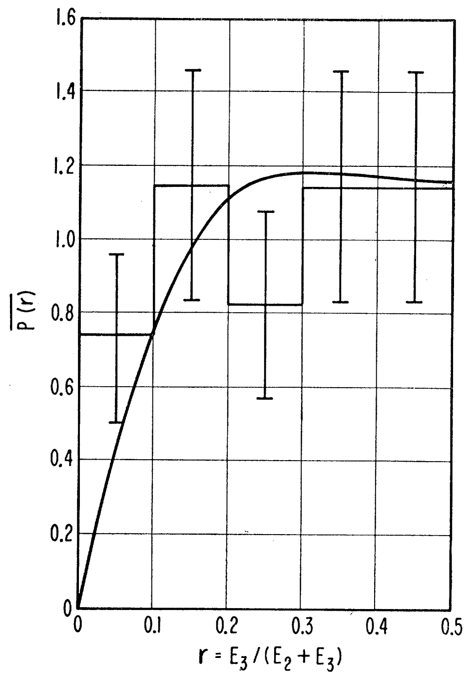

FIG. 7. Comparison of experimental r-distribution with the averaged theoretical probability for internal energy partition within trident pairs. E_2 and E_3 are the two lowest secondary energies with $E_2 \geqslant E_3$.

concluded that in this case the redistribution results in a sufficiently small change in the shape of $\bar{P}(r)$ that we have neglected it. The agreement between theory and experiment is again satisfactory.

IV. CONCLUSIONS

In the course of this text we have made the observations summarized as follows:

(a) a reconsideration of Bhabha's theory shows that it is consistent with Racah's prediction of total cross section;

(b) the experimentally determined total cross section in the energy interval between 0.1 and 10 Bev is consistent with the above predictions;

(c) the experimental variation of total cross section with energy for the same energy region is consistent with theory;

(d) the experimental distributions in both the fractional energy transfer to the trident pair and the energy partition within the pair are consistent with Bhabha's theory.

From the above evidence we conclude that the quantum electrodynamical description of direct pair production by fast electrons, as given by Bhabha, is a satisfactory representation of the trident process in the energy interval 0.1 to 10 Bev.

Freier and Naugle[23] have recently reported an experimental determination of electron trident mean free path which is in serious disagreement with theory. Their measurements, made in the core of a "jet," were of primary energy of the order of 100 Bev; the experimental m.f.p. was several times less than the theoretical value. As a result of these measurements it has been suggested that the theory of direct pair creation might break down at these energies. More recently, Koshiba and Kaplon[24] have presented somewhat similar results at energies $\sim$10 Bev. In a recent communication[17] we have estimated that rather large corrections might be appropriate to these experiments; consideration of the pseudo-trident contribution would lead to a significant increase in the value of the experimental mean free path. We are therefore of the opinion that there is no firm evidence at this time that theory is in serious disagreement with experiment.

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