Comparison of H^3 and He^3 Production in High-Energy Deuteron-Deuteron Collisions*

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The relative production of H^3 and He^3 from deuteron-deuteron collisions at 190 Mev has been measured at 30° (center of mass). The method of detection required that the particles traverse a magnetic channel and a pulse-height counter telescope. The pulses were displayed on a fast cathode-ray oscilloscope and recorded photographically. A liquid deuterium target was employed. The comparison shows a ratio of H^3 to He^3 of 0.86 ± 0.14. These data support the hypothesis that charge symmetry of nuclear forces exists at high energy.

INTRODUCTION

THE production of H^3 and He^3 in deuteron-deuteron collisions has been studied extensively by investigators using deuterons of energy up to 10 Mev.[1–6] The fact that the cross sections for these particles are essentially equal (except where Coulomb forces predominate) has been considered as evidence that charge symmetry exists up to these energies. The term "charge symmetry" is used herein to denote the equality of *n-n* nuclear forces to *p-p* forces of the same angular momentum and spin state, neglecting effects due to Coulomb forces and mass differences between the particles. Many other data have been published which confirm this conclusion. Ajzenberg and Lauritsen[7] conclude from their studies of nuclear spectroscopy that charge symmetry of nuclear forces, insofar as it is manifested in low-lying energy states, can be regarded as an established fact. A study of the three-body stable nuclei[8] leads to the conclusion that the binding energies of H^3 and He^3 can be understood most simply under the assumption that charge symmetry exists for the ^1S state. Further evidence of charge symmetry can be inferred from low-energy scattering[9,10] data which indicate that the dineutron is probably just unbound and that the assumption of equal scattering lengths for *n-n* and *p-p* scattering in the ^1S state is not inconsistent with these data.

Evidence of charge symmetry at high energies, where many higher states, strong tensor forces and meson fields are expected to play important roles, is very meager. Yet all theoretical approaches to high-energy scattering assume its existence. The most conclusive evidence would of course be a comparison of high-energy *p-p* scattering with *n-n* scattering. Since targets of neutrons do not exist, however, one must turn to less direct methods. Evidence is available in a comparison of *n-d* and *p-d* scattering at the same energy. The width of available neutron spectrums, however, and the differences of experimental technique required to perform the experiments make a direct comparison of results difficult. A comparison of the results of Chamberlain and Stern[11] for *d-p* scattering and those of Powell[12] and Youtz[13] for *n-d* scattering shows discrepancies in shape and absolute value that make any conclusion suspect. Barkas and Wilson[14] have examined the ratio for the production of π^+ and π^- mesons at 90° when carbon is bombarded with 390-Mev alpha particles. The mesons, however, are relatively low in energy and are appreciably affected by the Coulomb barrier. They conclude that the data are not inconsistent with the assumption of charge symmetry. A direct comparison of the production of H^3 and He^3 in 190-Mev deuteron-deuteron collisions can be made. The assumption of charge symmetry would require that these two cross sections be equal.

EXPERIMENTAL PROCEDURE

To provide the necessary separation and identification of the H^3 and He^3 particles, they were made to pass through a magnetic channel and pulse-height detectors. In order to provide a direct comparison, both types of particles should have the same angular resolution, the same energy resolution, and the same multiple-scattering. For a given scattering angle, both H^3 and He^3 will have essentially the same energy (about one Mev difference because of their mass difference). Originating in the same target, however, and passing through identical amounts of intervening windows in going from target to detector, they will have different scattering and energy distributions. The general scheme of attack had three basic features:

* This work was performed under the auspices of the U. S. Atomic Energy Commission.
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(1) The same magnetic channel (physically) was used for both particles. To detect He^3, the magnetic field intensity was reduced to half that used for H^3. This procedure had the advantage of insuring identical geometry for both particles.

(2) The energy spread of the H^3 was made to equal that of the He^3. Any loss of counting efficiency due to energy spread was thereby balanced out.

(3) The mean scattering angle of H^3 from the material in and near the target was made to approximate that of He^3. Multiple scattering was made to occur as near the target as possible. The path of the particles from the vicinity of the target to the vicinity of the detectors was evacuated to minimize multiple scattering errors along this path.

A pulse-height telescope behind the magnet provided pulses corresponding to specific ionization (dE/dx) and total energy (E). It may seem that the three parameters —$H\rho$, dE/dx, and E—overdetermine the separation problem. The separation of deuterons from tritons, however, is marginal under optimum conditions when only two parameters are used. It was felt that wider limits on the resolutions of both the magnetic channel and the pulse-height equipment could be allowed by the use of all three parameters.

The experimental arrangement for comparison of H^3 and He^3 is shown schematically in Fig. 1. The incident deuterons were collimated to a beam one inch in diameter and monitored with an ionization chamber. The

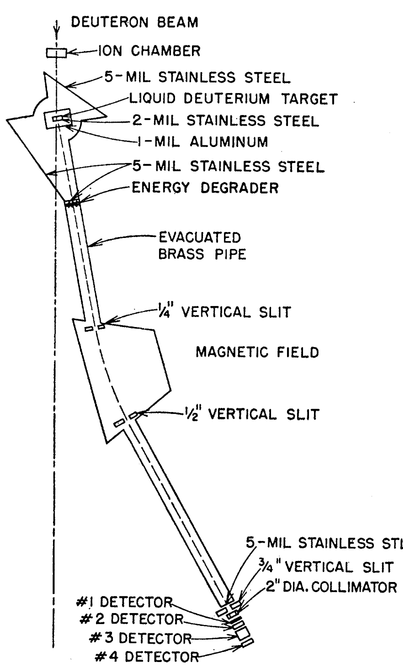

FIG. 1. Schematic diagram showing experimental arrangement for comparing production of H^3 and He^3 in deuteron-deuteron collisions.

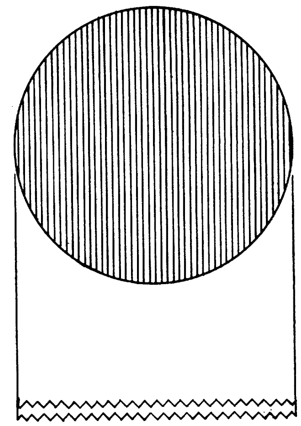

FIG. 2. Energy degraded for H^3.

energy of the deuteron beam has been measured as 192 ± 2 Mev.[15]

In order to keep background counts at a minimum, a liquid-deuterium target was used. This target had a volume of deuterium $1\frac{1}{2}$ inches in diameter and approximately $\frac{1}{2}$ inch thick. The windows were 2-mil stainless steel. Blank runs were obtained by closing the valve leading to the top of the deuterium target. The evaporation from the surface of the liquid built up enough pressure to force the liquid deuterium out of the target into a reservoir. The blank runs, therefore, were not truly blank, since deuterium gas at approximately 21°K remained in the target. Since this experiment was a comparison of two cross sections, no error was introduced by this residual gas.

The magnet used was a pair-spectrometer magnet readily capable, with a three and one-half inch gap, of providing 14 kilogauss over a path length of 30 inches. The path traversed by the H^3 particles is shown in Fig. 1. The path traversed by the He^3 is identical except that the energy degrader is removed. The location of this path was determined by using the well-known current-carrying wire to trace the orbit of the desired particle through the magnetic field.

The procedure used was as follows. A wire was strung down the channel from the center of the target to the center of the detector. Tension was provided by a weight strung over a pulley. The calculated current for a H^3 of proper energy was applied through the wire. The magnetic field was adjusted so that the wire passed down the centers of the brass pipes. The collimating slits were all put in position so that the wire bisected them. The magnetic-field potentiometer reading was recorded. Next the calculated current for a He^3 of proper energy was applied to the wire. The magnetic field was then adjusted so that the wire again bisected the channel and collimating slits. The magnetic-field potentiometer reading was recorded. Thus a single physical channel was defined which could be adjusted

[15] A. L. Bloom and O. Chamberlain, Phys. Rev. 94, 659 (1954).

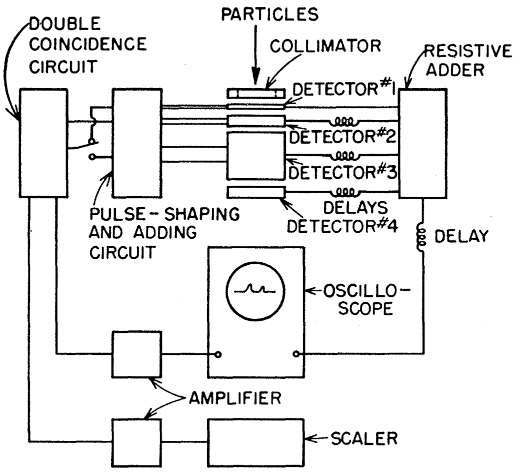

FIG. 3. Block diagram of electronics for comparing H^3 and He^3.

to accept either H^3 or He^3 of the proper energy by changing the magnetic field.

The energy degrader is shown in Fig. 2. It was placed in position as shown in Fig. 1 for H^3 and removed for He^3. It had a dual function: to provide the same spread in energy of the H^3 as for the He^3, and to provide the same multiple scattering for both H^3 and He^3. The principle of the degrader is obvious from Fig. 2. Particles go through different thicknesses of absorber and suffer various energy losses. Ideally, if the grooves were very close together, the energy losses would be quite random horizontally. Since the horizontal distance across the degrader to go from maximum attenuation to minimum attenuation is only $\frac{1}{16}$ inch, it was felt that the random ideal was approached. The depth of the grooves was calculated to equalize the H^3 energy spread to that of the He^3. The residual thickness of the degrader was calculated to equalize the multiple scattering of the mean H^3 to that of the He^3. Fortuitously, this also served to drop the mean energy of the H^3 to a value equal to that of He^3. In as much as the energy of the He^3's from the target was 163 ± 6 Mev, it is apparent that the effects of such an energy spectrum or the multiple scattering for such an energetic particle would not be large, even without the use of the energy degrader.

The exact value of the magnetic field was later checked during the run by varying the magnetic field of the magnet over a range of values close to the calibrated value, and plotting the counting rate. A magnetic field was used corresponding to the peak of the counting rate. The value of the field thus determined was found to lie very close to the expected value.

The detectors used were liquid scintillators of the general type described by Garwin,[16] with one difference: in addition to a 5819 photomultiplier tube to give pulse height, each liquid cell was viewed by two 1P21 photomultiplier tubes. These latter tubes were used to drive a coincidence circuit.

Three pulse-height detectors were employed. The first detector served to measure dE/dx of the He^3. The second detector served to measure both E of the He^3 and dE/dx of the H^3. The third detector served to measure E of the H^3. The fourth detector was merely a liquid cell viewed by one 1P21 tube. Since the desired particles did not have sufficient energy to get into this detector, any particle making a pulse in this detector could be discarded.

By placing the detector telescope directly in the deuteron beam and by varying the amount of absorber between the beam and the detector, the detector telescope was calibrated for various energy deuterons. The pulses from these essentially monoenergetic deuterons were used to obtain resolution curves of the individual detectors. These data were obtained immediately after the H^3 data were obtained, without changing voltages or relative positions of the detectors. This procedure made it possible to use these resolution curves in differentiating between H^3's and deuterons.

A block diagram of the electronics is shown in Fig. 3. Double coincidences were made either between the first and second detector (for He^3) or between the second

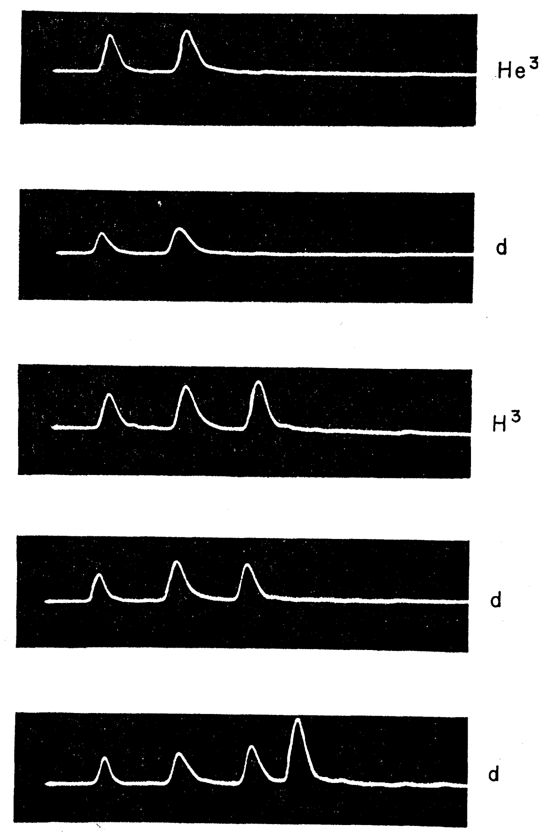

FIG. 4. Film traces of typical He^3, and H^3 and background pulses. First two during He^3 run; last three during H^3 run.

[16] R. L. Garwin, Rev. Sci. Instr. 23, 755 (1952).

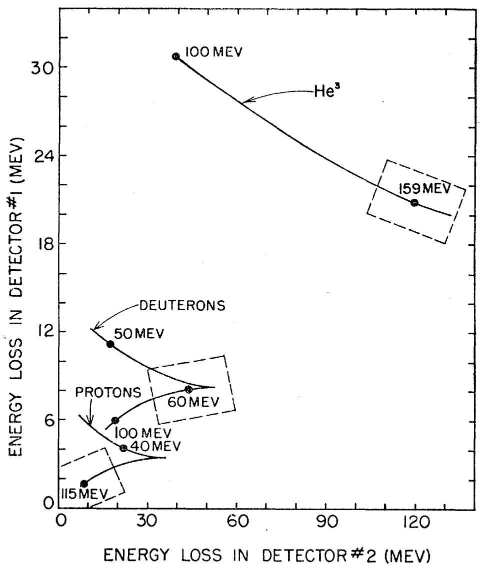

FIG. 5. Plot of calculated energy loss in first detector *vs* calculated energy loss in second detector for He^3 and other particles. Areas outlined indicate regions where pulses expected to lie.

and third detector (for H^3). The pulse-shaping and coincidence units were similar to those described by Frank.[17] Amplified outputs from the coincidence unit were used to trigger the sweep of the oscilloscope and to drive a scalar. The use of separate photomultipliers to provide coincidence pulses and pulse-height pulses was considered advantageous, so that optimum voltages could be used on each and so that a fast coincidence could be made. The resolving time of the coincidence unit was about three millimicroseconds. The success of this circuit in eliminating accidentals can be seen from the fact that plugging the magnetic channel when running at full beam intensity reduced the coincidence counts essentially to zero.

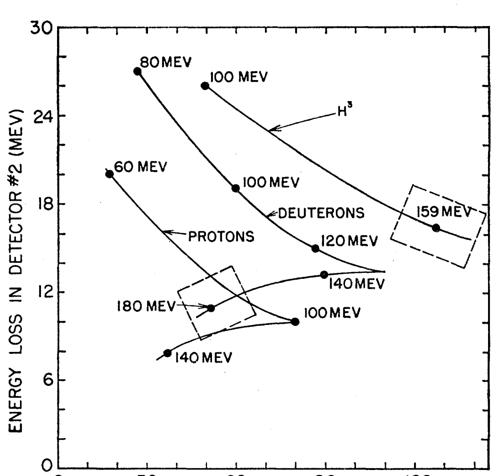

FIG. 6. Plot of calculated energy loss in second detector *vs* calculated energy loss in third detector for H^3 and other particles. Areas outlined indicate regions where pulses expected to lie.

The pulse-height signals themselves were delayed with respect to each other in order to separate them on the oscilloscope screen, were added, were fed into a Tektronix model 517 oscilloscope, and were photographed on a slow moving film. Several traces of typical pulses are shown in Fig. 4. These traces were reduced to usable data by measuring the height of the dE/dx and E pulses and plotting each trace as a single point in the dE/dx *vs* E plane.

ANALYSIS OF DATA

In Figs. 5 and 6 are plotted the calculated energy losses of various particles traversing the detector system.[18] Figure 5, for example, shows that He^3's of various energies fall along a line. If the mechanism by which the energy loss of the particle is converted to pulse height were perfectly linear, these curves would also represent the pulse-height locus for He^3's of various energy (assuming the scales to be suitably chosen). Owing primarily to saturation effects in the detector for high values of specific ionization and to nonlinearity of the amplification system, one would not expect the actual pulse-height locus to have the same slope or position as the energy-loss curve. The over-all resolution of the detecting system, moreover, as manifested by the Landau-effect,[19] the statistical fluctuation in the number of photoelectrons released from the photocathode, and the optical resolution of the detector spreads the expected line out into a band. The

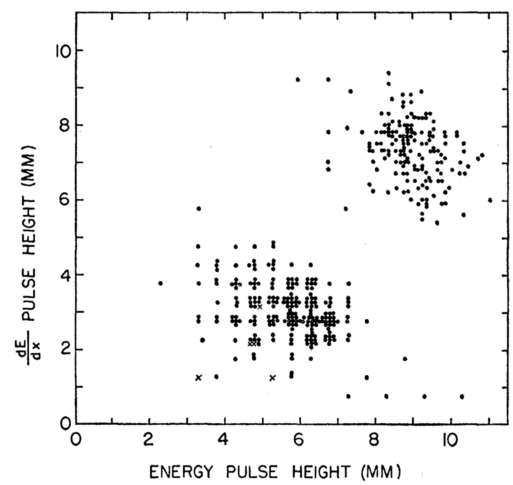

FIG. 7. Data for counting rate of He^3 using liquid deuterium target. Data obtained in 10 integrated beam units. Crosses represent particles making a pulse in the third detector.

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[19] L. Landau, J. Phys. (U.S.S.R.) **8**, 201 (1944).

width of this band obtained is a measure of the over-all resolution.

In Figs. 5 and 6, therefore, the general regions outlined by dashed lines are where pulse-height pairs were expected to fall. The energies of the particles in these regions correspond to the energies of the particles that can come down the channel.

Figure 7 shows the total data collected for He^3 with liquid deuterium target. Figure 8 shows the corresponding data for H^3. (Blank target data is not shown but counting rates are listed in Table II.) The He^3 data are unambiguous. The He^3 fall in an island well separated from the expected slow-deuteron and proton background. It should be noted that several He^3 particles have the proper dE/dx, but have an energy pulse height lying below that to be expected from the resolution of the equipment. These are assumed to be He^3 particles that suffered an inelastic event in stopping in the energy detector. This argument is developed in the discussion of the H^3 data, where a calculation of this effect is required.

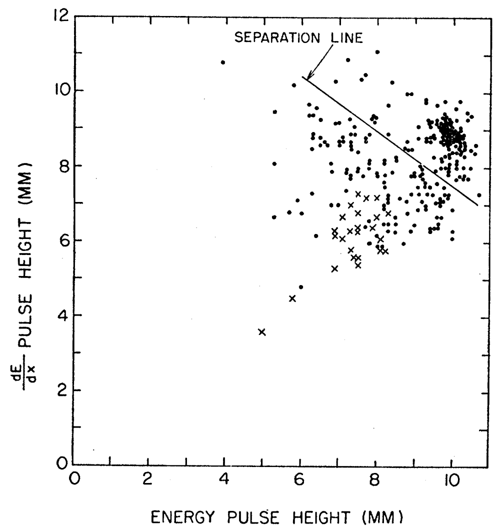

FIG. 8. Data for counting rate of H^3 using liquid deuterium target. Data obtained in 10 integrated beam units. Crosses represent particles making a pulse in the fourth detector.

The H^3 data in Fig. 8 do not resemble the anticipated results. Instead of having two well-defined areas of H^3 and fast deuterons, they show that deuterons of lower energy than those anticipated are being detected. By a process of elimination, it was determined that these deuterons did come from the target and did come down the channel. With the collimation used, however, they could not come down the channel in a direct path. It was finally determined that they represented deuterons that had scattered off one edge of the collimating slit at the entrance to the magnet. A further check of the geometry showed that deuterons might reach the detectors by such a process with energies down to approximately 120 Mev.

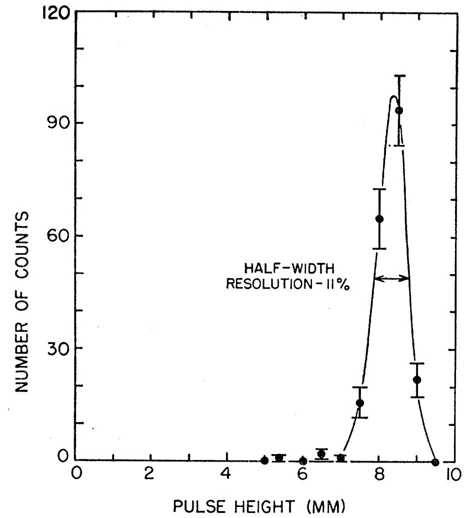

FIG. 9. Distribution in pulse height for 200 random deuterons of 130-Mev initial energy as measured by third detector.

In order to separate these deuterons from the H^3, the resolution of the second and third detectors for deuterons of various energy was first determined from the deuteron calibration data. Figure 9 shows a resolution function for detector No. 3 as determined by 200 random deuterons of 130 Mev. These resolution functions not only determine the width of the deuteron band, but locate accurately the center of the distribution on the dE/dx *vs* energy plot for each calibration energy. In Fig. 10 the locus of the center of these distributions is shown. (For the sake of illustration, there are also plotted the pulse heights of 160 random deuterons of various energies ranging from 110 Mev to 150 Mev.) A line was drawn parallel to this line and lying midway between the center of the deuteron distribution and the center of the triton distribution of Fig. 8.

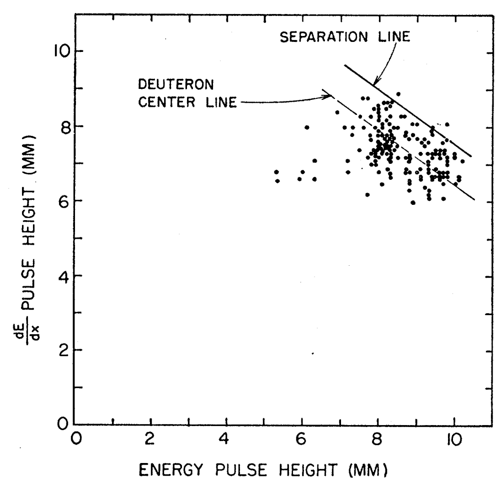

FIG. 10. 160 random traces made by deuterons of various energy. One curve shows locus of center of deuteron distribution. Other curve shows separation line between H^3 and deuterons.

TABLE I. Correction factors resulting from inelastic collisions of H^3 and He^3 particles in detectors.

Particle	Attenuation before or in dE/dx detector	Degrading in energy detector	Total
H^3	1.07	1.13	1.21
He^3	1.01	1.00	1.01

By superimposing this new separation line on the resolution plots, it was determined that from 2 to 3 percent of the deuterons would fall above this line. Assuming the detectors had the same resolution for H^3 as for deuterons, this would mean approximately 2 to 3 percent of the H^3 might fall below the line. This much was deemed satisfactory to effect a separation between the two.

CORRECTIONS

It was necessary at this point to introduce the only correction factors applied to the data. The H^3 and He^3 can suffer inelastic events in the detectors that affect their pulse heights. The most common event is a stripping off of one or two particles in a manner similar to the familiar deuteron stripping. In the event that such a process occurs before or in the dE/dx detector, the particle is very probably lost, since neither the dE/dx or the energy pulse is proper for a H^3 or He^3. Crandall[20] has measured the total inelastic cross section of high-energy He^3 particles for various elements. Assuming the probability of a nuclear event at high energy is the same for H^3 as for He^3, one can calculate the attenuation of H^3 and He^3 prior to entering the energy detector.

The effect of an inelastic event in the energy detector is impossible to predict rigorously. Since the resultant deuterons, neutrons, or protons from the stripping process will be less ionizing than the original particle, it is assumed that the energy pulse is degraded in all cases. The degrading of the H^3 pulses is greater than that of the He^3, since there is a greater probability of neutrons' carrying off energy with no ionization in the former case. Moreover, the probability of an inelastic event is approximately four times as great for the H^3 as for the He^3, since it will go through four times as much matter in stopping (assuming that the inelastic cross sections of the two are equal over the major portion of their range).

The probability of an event in which all the energy of H^3 or He^3 would become unavailable for ionization seems quite small. It might occur if the proton of a H^3 suffered a knock-on collision with a neutron in which charge was exchanged. In this event the three neutrons would carry off the energy of the H^3 with little probability of further ionization. It might also occur if the proton were stripped from the H^3 by a nucleus in such a manner that the resulting excited nucleus produced little or no ionization. The possibility of an event in which a He^3 would fail to produce a substantial amount of ionization is hard to conceive.

Since the degraded He^3 particles are still readily separable from the background, it is apparent that no correction need be applied to the He^3 from this source. It is reasonable to assume, however, that a considerable number of the particles in Fig. 8 that fall in the category of deuterons, are actually H^3, whose energy pulse has been degraded sufficiently to make them indistinguishable from deuterons. That this is a reasonable assumption can be further argued from the fact that the scattering of deuterons off the collimating slit can account only for detected deuterons of energies ranging down to about 120 Mev. It is felt that a majority of the particles in Fig. 8 whose dE/dx pulse was larger than about 8.5 on the ordinate scale were actually degraded H^3.

It is not necessary to make this assumption to get a rough correction factor for this effect. It was assumed that in three-quarters of the inelastic events, the energy of the H^3 was degraded sufficiently to either lose the count completely or to throw it into the deuteron band.

TABLE II. Corrected counting rates in counts per volt of integrated beam. First column for target filled with liquid deuterium. Second column for blank target.

Particle	Corrected deuterium rate (counts/volt)	Corrected blank rate (counts/volt)	Difference rate
H^3	15.6 ± 1.3	2.4 ± 1.1	13.2 ± 1.7
He^3	16.5 ± 1.3	1.2 ± 0.7	15.3 ± 1.5

Crandall's data were used to determine this correction factor. The correction factors are summarized in Table I.

RESULTS

The final counting rates are summarized in Table II.

The final ratio for the production of H^3 to He^3 at the center-of-mass scattering angle of 30 degrees is 0.86 ±0.14. The errors shown are the statistical deviations related to the number of counts constituting the original data. Within statistics, therefore, the production probabilities of He^3 and H^3 are equal. Although the correction factors are relatively small, they may introduce an uncertainty the order of 5 percent. An additional 5 percent uncertainty is estimated for possible differences in angular resolution and other systematic errors.

CONCLUSIONS

The existence of charge symmetry would require that the cross sections for the production of H^3 and He^3 be the same. Unfortunately, the inverse is not necessarily true. The cross sections might conceivably

[20] W. Crandall (private communication).

be equal at some specific angle in spite of an asymmetry in the nuclear forces. Moreover, the matter of degree must be considered. Since the cross sections cannot be calculated explicitly in terms of the *n-n* and *p-p* forces, it is not possible to say how much effect an asymmetry in the forces would have upon the cross sections. The following conclusion, however, seems justified. At low energy, charge symmetry is an established fact and the cross sections for the production of H^3 and He^3 from deuteron-deuteron collisions are equal (neglecting of course Coulomb and mass-difference effects). The fact that the cross sections are nearly equal at high energy supports the hypothesis of charge symmetry at high energy.

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Electron Tridents between 0.1 and 10 Bev

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Data on the direct production of pairs by fast electrons of the cosmic radiation have been collected in nuclear emulsions by a track-following method. Multiple-scattering measurements were made on the secondaries of 61 tridents with energies between 0.1 and 10 Bev; of these, 14 originated on arms of pairs. A mean free path of 134±42 cm was deduced for the latter, corresponding to a cross section of $0.92\pm0.29\times10^{-25}$ cm². An appropriate statistical treatment for small numbers has been adopted, and a correction has been made for the inclusion of pseudo-tridents. The theoretical cross sections of Bhabha and Racah, which apparently disagree by a factor of about 2 in this energy region, have been shown to be consistent. The theoretical cross section averaged over the experimental energy distribution is 0.82×10^{-25} cm². The distributions in fractional energy transfer to the created pair and internal energy partition in the pair have been derived and are compared with our observations. It is concluded that trident theory is in agreement with this experiment.

I. INTRODUCTION

OBSERVATIONS of the direct creation of electron pairs arising from the interaction of fast charged particles with the Coulomb fields of nuclei have been extensively studied.[1–12] These phenomena, called tridents, consist of an incoming, or primary track of a relativistic charged particle, and three emergent tracks as shown by the photomicrograph in Fig. 1. Experiments made chiefly in photographic emulsions indicate that the great majority of tridents have electron primaries. In this text we refer to both electrons and positrons as electrons. In those instances where identification has been possible, it has been demonstrated that the secondaries are of electronic mass. Energy measurements have indicated that the primary energy is conserved among the three secondary tracks.

The theory of the direct pair creation process has been discussed by a number of authors, notably Bhabha,[13] Nishina *et al.*,[14] and Racah.[15] The cross sec-

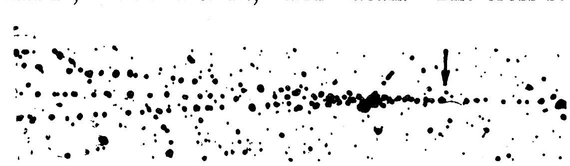

FIG. 1. A photomosaic of a trident in a G5 emulsion. In this example, less than one percent of the primary energy is transferred to the directly created pair.

* On military leave of absence from Duke University, Durham, North Carolina.
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