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Beta Spectrum of Radium E

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The beta spectrum of RaE has been remeasured with improved techniques. The distribution is fitted by a shape factor appropriate to a once forbidden transition with a spin change, $\Delta J=1$ involving interference between the S and T couplings. The experimental measurement is used to set limits on the ratio of the beta moments which are treated as parameters.

INTRODUCTION

THE RaE beta spectrum has had a long history of measurements and interpretations. It was the first spectrum definitely proved[1] to have a shape which deviates markedly from the "statistical" shape characteristic of the allowed beta transitions. The interpretation went through three main stages as the general knowledge of beta-decay phenomena and nuclear states broadened.

1

In the earliest stage, attempts were concentrated on making a "single" form of coupling law work for all beta radiation phenomena. Either the scalar (S), the vector (V), the tensor (T), the pseudovector (A), or the pseudoscalar (P) form of beta coupling was presumed to represent the complete interaction. Under this restriction, Konopinski and Uhlenbeck[2] were unable to fit the observed shape with any of the alternative predictions for *once* forbidden transitions. They did report that the expectations for *twice* forbidden transitions were sufficiently elastic to permit a fair fitting of the measurements within the accuracy then current.

The situation was changed by the subsequent advent of the shell model for nuclear states, in two main respects. The consequent classification of all known beta transitions showed that the comparative half-life of RaE, $ft=10^8$ sec, was about a factor 10^4 shorter than that characteristic of well authenticated *twice* forbidden transitions. Furthermore, the shell model predicted a change of parity in the RaE transition. This violates the selection rules for allowed and twice forbidden transitions and hence makes the classification of the RaE decay as a *once* forbidden (parity change) transition difficult to escape.

2

A second stage in the interpretations arrived when all the evidence in beta decay began to make it clear that the fundamental coupling law must consist of more than one component of the five: S, V, T, A, or P. The existence of S and T components is essential[3]; the *absence* of V and A components is also essential, but the existence of a P component is neither confirmed nor contradicting to the evidence in general.

Petschek and Marshak[4] tried all the possible linear combinations of the couplings in attempting to reproduce the observed RaE spectrum, on the basis that it is a once forbidden transition. They concluded that only a TP combination could work. The P component contributes only to once forbidden transitions with no spin change. Hence, it was supposed that RaE undergoes a $0-\rightarrow 0+$ transition. For such transitions, the S component of coupling is unable to contribute.

The Petschek-Marshak analysis provided practically the only evidence for the existence of a P component in the beta coupling law.

Konopinski and Langer[5] pointed out the essential feature which allowed Petschek and Marshak to reproduce the RaE spectrum shape. All normal once forbidden transitions with $\Delta J=0$ or 1 are expected and observed to yield very nearly statistical spectrum shapes. The strong deviations observed for RaE are possible only through nearly complete destructive interference between the radiations due to two components of coupling (T and P in the Petschek and Marshak analysis). The destructive interference should also prolong the comparative half-life and, indeed, RaE has an ft value about a factor $10^{2.5}$ greater than observed for other comparable (nearly doubly magic) once forbidden transitions.

3

The final stage of interpretation arrived when Smith[6] measured the spin of RaE and thus showed that it actually undergoes a $1-\rightarrow 0+$ transition.

Meanwhile, Yamada[7] showed that the Petschek-Marshak analysis was incomplete in an essential respect. It is usually found that it is highly accurate to neglect the radial variation of the electronic wave

* Assisted in part by the joint program of the Office of Naval Research and the U. S. Atomic Energy Commission, and a grant from the Frederick Gardner Cottrell Fund of the Research Corporation.

[1] L. M. Langer and M. D. Whitaker, Phys. Rev. **51**, 713 (1937); E. M. Lyman, Phys. Rev. **51**, 1 (1937).
[2] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).
[3] H. M. Mahoud and E. J. Konopinski, Phys. Rev. **88**, 1266 (1952).
[4] A. G. Petschek and R. E. Marshak, Phys. Rev. **85**, 698 (1952).
[5] E. J. Konopinski and L. M. Langer, Ann. Revs. Nuclear Sci. **2**, 261 (1953).
[6] K. F. Smith (private communication).
[7] M. Yamada, Progr. Theoret. Phys. (Japan) **10**, 252 (1953).

function across the nucleus. Two circumstances conspire to invalidate this assumption, in the special case of RaE. It is one of the most highly charged nuclei studied; the strong Coulomb attraction shortens the de Broglie wavelength of the electron. More important, the destructive interference between the component RaE radiations cancels the usually largest contributions and leaves small ones which are much more sensitive to this so-called[8] "finite de Broglie wavelength effect." Taking it into account, Yamada showed that Petschek and Marshak were wrong in their conclusion that the combination *ST* could not lead to the RaE spectrum shape.

Yamada was successful in reproducing the observed RaE shape on the basis that it undergoes a $1-\longrightarrow 0+$ transition. Now the *S* component of the coupling contributes and a *P* component cannot, even if it exists. Hence, now it is the destructive interference between *S* and *T* contributions which accounts for the singular RaE spectrum shape in Yamada's interpretation. His result removes the evidence for the existence of *P* coupling. (There is also no evidence against its existence because theoretically it would elude giving observable effects in all cases known so far.)

At least two unsatisfactory features remain in Yamada's analysis which occasion the present remeasurement and refitting of the RaE spectrum.

First, there is a small discrepancy between the low-energy part (<175 kev) of the experimental spectrum and the theoretical spectrum arising from Yamada's analysis. The source of this discrepancy may be at least partially theoretical.

Yamada used electronic wave functions appropriate to a point charge nucleus; it has been shown[9] that the spread of the charge on the nucleus may sometimes have appreciable effects. However, Yamada's argument that this should not be important for RaE seems correct. The finite nuclear size affects mainly the large singular terms ($\sim\alpha Z/R$) of the electronic wave functions and these are just the ones cancelled out by the destructive interference in the RaE radiation. The remaining "finite de Broglie wavelength" effects are determined by parts of the wave function farther from the center and less sensitive to redistribution of the point charge. A fully precise conclusion on this point must await numerical analysis by machine computation.

On the other hand, it is not impossible that the low-energy discrepancy is due to errors of measurement associated with the inherent difficulties encountered in attempting to obtain such low energy information in such a high-*Z* element. Also, the accuracy of the experimental data used by Yamada did not impose the more stringent limits on the theoretical fit which are required by the results of the present investigation. The experimental work reported here was meant to test these points, taking advantage of improved techniques developed since the earlier measurements.[10]

Perhaps the second reason for the present reinvestigation of RaE, is the more important. Early in his analysis, Yamada quite arbitrarily chose a special ratio for two of the three nuclear moments (matrix elements) responsible for the beta radiation. The successful conclusion of his work implies that the special assumption he thus employed is an essential one. As we shall see here, this is not at all the case. Yamada's special assumption was justified insofar as the interest was concentrated only on showing that by using it the RaE spectrum is at all explainable. However, the spectrum measurements should ultimately help in the evaluation of the moments possessed by the RaE transition for the radiation of beta particles. To this end, it is necessary to show what the necessary limits are that the experiments put on the values of the nuclear beta moments. As will be seen here, the correct values of the beta moments are probably somewhat different from those used by Yamada.

EXPERIMENTAL PROCEDURE AND RESULTS

The beta spectrum was measured by using the high-resolution, 40-cm radius of curvature, 180-degree shaped magnetic field spectrometer.[11]

An intense source of RaE was obtained by a carrier free separation from an equilibrium mixture of RaD, E, and F with the use of an anion exchange resin column.[12] The source thus produced was superior to those obtained by electroplating separation because of the increased yield. The RaE activity was deposited from a dilute nitric acid solution on a backing of 3 μg/cm^2 of zapon. The source was spread quite uniformly with the aid of insulin[13] covering an area of 0.5 cm by 2.5 cm and was then covered by a 1.5 μg/cm^2 layer of zapon. Grounding of the source was obtained by means of electron emission from an oxide-coated-filament assembly[14] mounted directly below the source in the spectrometer.

In order to maintain a counting efficiency which is independent of the incident electron energy over the entire RaE spectrum, the usual side window G-M tube whose sensitivity had a slight dependence on energy was replaced with a counter of unusual design. This is essentially an end window counter, the effective volume of which is confined between an aluminum covered mylar window of 0.9 mg/cm^2 total thickness and a loop of three mil stainless steel wire. The counter slit

[8] M. E. Rose and C. L. Perry, Phys. Rev. **90**, 479 (1953).
[9] M. E. Rose and D. K. Holmes, Phys. Rev. **83**, 190 (1951).
[10] A Flammersfeld, Z. Physik **112**, 727 (1939); G. J. Neary, Proc. Roy. Soc. (London) **A175**, 71 (1940); L. M. Langer, Phys. Rev. **75**, 328 (1949); R. Morrissey and C. S. Wu, Phys. Rev. **75**, 1288 (1949).
[11] L. M. Langer and C. S. Cook, Rev. Sci. Instr. **19**, 249 (1948).
[12] B. A. Raby and E. K. Hyde, U. S. Atomic Energy Commission Declassified Document AECD-3525, 1952 (unpublished); and University of California Radiation Laboratory Report UCRL-2069 (unpublished).
[13] L. M. Langer, Rev. Sci. Instr. **20**, 216 (1949).
[14] L. M. Langer and R. J. D. Moffat, Phys. Rev. **88**, 689 (1952).

was adjusted to 0.4 cm in width and 2.5 cm in height. A gas mixture of nine parts of argon to one part of ethylene was maintained at a constant pressure of 10 cm of Hg with the use of a Cartesian manostat.[15] These conditions yield a counting plateau of well over 100 volts at a threshold of 950 volts and with a slope of 1.5 percent rise per 100 volts.

Calibration for the spectrometer is in terms of the K-line from the 0.661-Mev[16] gamma ray of Cs^{137} and the magnetic field measurements were made by use of a continuously rotating coil arrangement.[17]

Figure 1 shows the conventional Fermi-Kurie plot of the data obtained with the source described above. A minimum of 10 000 counts were recorded for each experimental point below the energy corresponding to $W=2.8$ so that the statistical deviation of these points on the F-K plot is ± 0.5 percent.

These data were combined with similar results obtained with a much weaker source, which had been prepared from electrochemically separated RaE, and all the experimental points together with their statistical errors were plotted on a greatly expanded scale so that a best fitting curve could be drawn through them. Table I gives a tabulation of values $[N(p)/p^2F]^{\frac{1}{2}}$ read from this curve at convenient intervals of the electron momentum, p. In both Fig. 1 and Table I the Coulomb factor, F, has been corrected for screening.[18]

TABLE I. Beta spectrum of RaE.[a]

p	W	$N(p)$	$(N/p^2F)^{\frac{1}{2}}$
0.6	1.1662	0.5594	0.1676
0.7	1.2207	0.5939	0.1579
0.8	1.2806	0.6219	0.1496
0.9	1.3454	0.6395	0.1409
1.0	1.4142	0.6452	0.1327
1.1	1.4866	0.6413	0.1244
1.2	1.5621	0.6225	0.1162
1.3	1.6401	0.5990	0.1078
1.4	1.7205	0.5639	0.09950
1.5	1.8028	0.5243	0.09158
1.6	1.8868	0.4846	0.08410
1.7	1.9723	0.4390	0.07676
1.8	2.0591	0.3940	0.06961
1.9	2.1471	0.3435	0.06250
2.0	2.2361	0.2959	0.05590
2.1	2.3259	0.2464	0.04931
2.2	2.4166	0.2021	0.04318
2.3	2.5080	0.1616	0.03732
2.4	2.6000	0.1249	0.03178
2.5	2.6926	0.09205	0.02638
2.6	2.7857	0.06384	0.02133
2.7	2.8792	0.04034	0.01648
2.8	2.9732	0.02262	0.01200
2.9	3.0676	0.009950	0.00776
3.0	3.1623	0.002601	0.00386
3.05	3.2097	0.000862	0.00220
3.10	3.2573	0.000113	0.00079
3.134	3.2900	0	0

[a] p and W are the momentum and energy in relativistic units; N is the number of particles per unit momentum interval; F is the Coulomb factor and is corrected for screening. The estimates of error are: p less than 0.1%; N less than 1.0% for $p<2.7$.

DISCUSSION OF RESULTS

The experimental RaE spectrum represented by Fig. 1 and Table I differs slightly from the older results. Part of this difference may arise from the fact that some of the earlier measurements were made with separate low-energy and high-energy counters and the exact normalization of the data was therefore made somewhat uncertain by the possibilities of partial transmission through the grid supporting the low-energy counter window and the energy sensitive transmission through the mica window of the high-energy counter. The present results were obtained with a single thin window counter whose detection sensitivity was found to be independent of energy over the entire region of the RaE distribution.

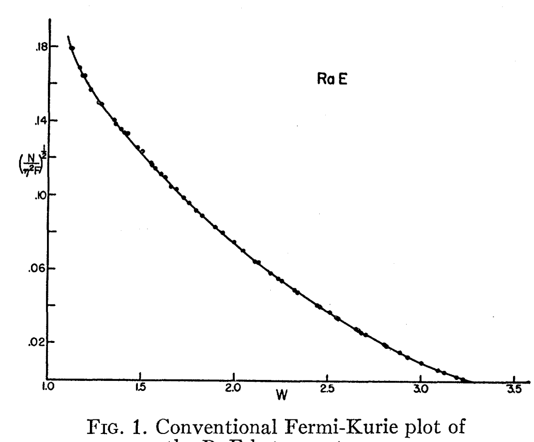

FIG. 1. Conventional Fermi-Kurie plot of the RaE beta spectrum.

The new measurements, if anything, *increase* the discrepancy between the experiments and Yamada's theoretical curve. The statistical accuracy of the present data demands a more exacting fit than the data used by Yamada. Therefore it was felt worthwhile to attempt a complete refitting in place of just taking over Yamada's final curve.

Moreover, as discussed further in the section on "Theoretical Interpretation", instead of presuming a value for the ratio of the matrix elements, as Yamada did, here *measured* limits are put on the ratio of the moments through the fitting procedure.

THEORETICAL INTERPRETATION

The beta spectrum for a once forbidden transition is described by an equation which gives the number of electrons $N(W)$ emitted in the energy range between W and $W+dW$, as

$$N(W)dW=(G^2/2\pi^3)F(Z,W)pW(W_0-W)^2C(W)dW, \quad (1)$$

where the energy W, and the momentum p of the electron are in relativistic units. W_0 is the end point

[15] L. M. Langer and R. D. Moffat, Phys. Rev. **80**, 651 (1950).
[16] L. M. Langer and R. D. Moffat, Phys. Rev. **78**, 74 (1950).
[17] L. M. Langer and R. F. Scott, Rev. Sci. Instr. **21**, 522 (1950).
[18] The screening correction was carried out using *Tables for the Analysis of Beta Spectra*, Applied Mathematics Series No. 13, National Bureau of Standards (U. S. Government Printing Office, Washington, D. C., 1952).

energy of the electron spectrum. G is the Fermi coupling constant. In (1), the "statistical shape", $\sim pW(W_0-W)^2$ is modified not only by the Coulomb factor, $F(Z,W)$, but also by the "shape factor" (or correction factor), $C(W)$. It is this shape factor which is of special interest. Its precise form depends on the particular nuclear moments (matrix elements) which are responsible for the beta radiation.

With the RaE transition having the character $1-\to 0+$, any P coupling which may exist will not contribute, and only S and T couplings need be considered. The relative strength of these are denoted by G_S and G_T; other beta decay evidence has shown these to be roughly equal. The S coupling acts on the nuclear moment $\int\beta\mathbf{r}$, in standard[2] notation, and the T coupling depends on $\int\beta\boldsymbol{\sigma}\times\mathbf{r}$ and $\int\beta\boldsymbol{\alpha}$. Only ratios of the moments affect the spectrum shape, and it is convenient to define:

$$\xi_1 = i\frac{G_S}{G_T}\frac{\int\beta\mathbf{r}}{\int\beta\boldsymbol{\sigma}\times\mathbf{r}},\qquad \eta_1=\left(\frac{\alpha Z}{2R}\right)^{-1}\frac{\int\beta\boldsymbol{\alpha}}{\int\beta\boldsymbol{\sigma}\times\mathbf{r}}. \tag{2}$$

The factor $\alpha Z/R$ is the Coulomb energy in units of mc^2, at the nuclear radius R. The ξ_1 and η_1 are real numbers which depend on the details of the nuclear states. There is no satisfactory theory for calculating them and they should be regarded as nuclear properties to be determined by measurement. Here, they will be at least partially measured by being treated as parameters in the fitting of the theoretical shape factor to the observed spectrum. Yamada quite arbitrarily put $\xi_1=\frac{1}{2}$, before attempting the fitting. We find this unnecessary.

A shape factor appropriate to RaE will have terms proportional to ξ_1^2, due to the S coupling, others proportional to ξ_1 arising from interference between the S and T couplings, and further terms due to the T coupling alone. Precise expressions are most conveniently obtained from Konopinski and Uhlenbeck[2] and from Smith.[19] They are usually easily evaluated by recourse to the tables of Rose, Perry, and Dismuke.[20] However, these tables turn out to have an insufficient number of significant figures for cases like RaE.

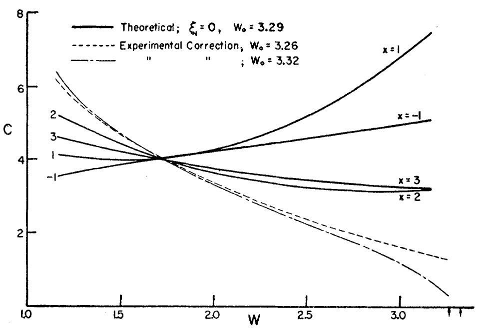

FIG. 2. Shape factors for the RaE spectrum for $\xi_1=0$. This corresponds to zero S coupling.

[19] A. M. Smith, Phys. Rev. **82**, 955 (1951).

RaE is peculiar because its spectrum deviates substantially from the statistical shape, i.e., $C(W)$ is observed to be heavily energy dependent, yet its Coulomb energy, $\alpha Z/R\approx 30$, far exceeds the kinetic energy $W_0-1\approx 2.3$. Normally, once forbidden spectra with $\Delta J\leq 1$ show a closely statistical shape (constant C) when $\alpha Z/R\gg W_0-1$. The large energy independent terms $\sim(\alpha Z/R)^2$ must be nearly cancelled from C in the case of RaE. One must presume the existence of ξ_1 and η_1 values which cause such cancellations, and these are to be understood as a destructive interference between contributions from the three moments involved.

For an accurate treatment of the cancellations we follow Yamada's procedure closely and define the presumably small difference ($x\ll\alpha Z/R$):

$$x=\frac{\alpha Z}{2R}\left(\eta_1-2\frac{1+\xi_1}{1+S}\right), \tag{3}$$

where $S=(1-\alpha^2Z^2)^{\frac{1}{2}}$. Our definition (3) coincides with Yamada's x only if we put $\xi_1=\frac{1}{2}$. Yet we lose none of the essential advantages asserted for his definition by Yamada. As did Yamada, we replace η_1 with x in the shape factor formulas. With our definition of x, we obtain:

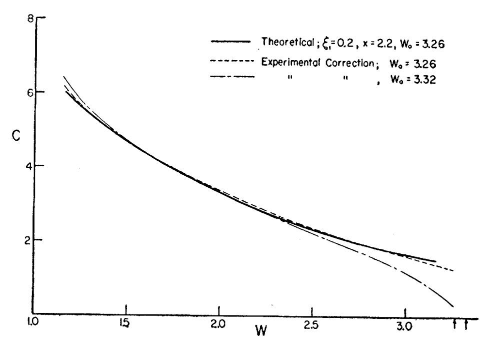

FIG. 3. Shape factors for the RaE spectrum for $\xi_1=0.2$, $x=2.2$. This is the best fit of theory to experiment.

$$C=x^2-\tfrac{2}{3}(1-\xi_1)qx-4x(1+\xi_1)\frac{p^2+\alpha^2Z^2}{(S+1)(2S+1)W}+\frac{q^2}{6}(1+2\xi_1^2)+\frac{4}{3}q(1-\xi_1^2)\frac{p^2+\alpha^2Z^2}{(S+1)(2S+1)W}+4(1+\xi_1)^2\frac{p^2+\alpha^2Z^2}{(S+1)^2(2S+1)^2}+\frac{(1-2\xi_1)^2}{1+S}L_1, \tag{4}$$

within an irrelevant constant multiplying factor. This reduces to Yamada's (13) if we put $\xi_1=\frac{1}{2}$. The

[20] Rose, Perry, and Dismuke, Oak Ridge National Laboratory Report ORNL-1459, 1953 (unpublished).

small quantity L_1 is adequately given by the tables of Rose *et al.*[20] The electron momentum is $p=(W^2-1)^{\frac{1}{2}}$ and q is the momentum of the neutrino.

Our procedure was to choose various values of ξ_1 and then find the scale factor and the value of x which make (4) coincide most closely with the experimental shape factor.

The experimental curves for C are obtained from the observed spectrum, $N(W)$, through the relation (1). This involves division by $(W_0-W)^2$ and hence the experimental C is very sensitive to the precise value of the maximum electron energy W_0 when W approaches W_0. Close inspection of the experimental points near $W=W_0$ only can set the limits $3.26<W_0<3.32$, because of the inevitably low intensity near the end point. We have then worked mainly with two experimental C curves corresponding to these limits. The choice of W_0 is not so critical for the theoretical C curves. It was found, however, that the value of $W_0=3.26$ leads to the best agreement between theory and experiment. For a case like RaE, where the ordinary F-K plot cannot be easily extrapolated, this is probably the best method of determining the maximum energy release. Since no theory provides for a shape factor concave downward near the end-point, the best value for the maximum energy can be set at $W_0=3.26\pm0.01$. This corresponds to a kinetic energy of $E_0=1.155$ Mev and is somewhat lower than the value of 1.17 Mev usually assigned to the RaE decay.

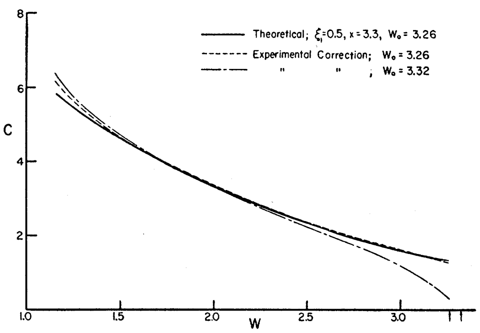

FIG. 4. Shape factors for the RaE spectrum for $\xi_1=0.5$. This is the value used by Yamada.

We tried values of ξ_1 ranging from $\xi_1=0$ to $\xi_1=2$. Only for $\xi_1\geq0.17$ could reasonable fits be obtained, for any value of x. The degree of disparity obtained for $\xi_1=0$ is shown in Fig. 2. Comparisons are also shown for $\xi_1=0.2$, $\xi_1=0.5$ (Yamada's value), and $\xi_1=1$ in Figs. 3, 4, and 5 respectively. All attempts with $\xi_1<0.17$ led to detailed discrepancies which we consider to exceed the experimental limits of error. In all cases, the curves are normalized at $W=1.72$ (or $p=1.4$).

The uncertainties are best discussed in relation to the conventional "linearized" plots of

$$[N(W)/pWFC]^{\frac{1}{2}}\sim W_0-W, \quad (5)$$

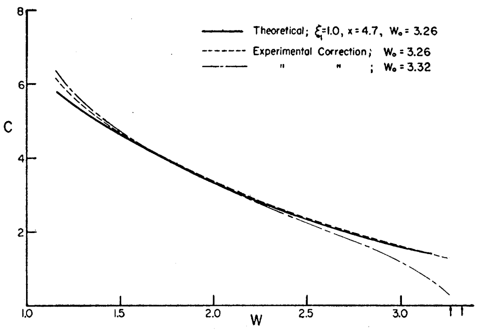

FIG. 5. Shape factors for the RaE spectrum for $\xi_1=1.0$.

where N is taken from the observed spectrum and C from (4) with the stated values of ξ_1 and x. Such plots are much less sensitive than those giving C directly, but represent the current accuracy of measurement (especially that of W_0) more appropriately.

The linearized plot in Fig. 6 shows that the fitting we achieved is as good as any obtained, so far, for any beta spectrum. The low energy discrepancy for $\xi_1=0.2$ and $x=2.2$ is much smaller than that obtained using Yamada's value of $\xi_1=0.5$, and $x=3.3$ as shown in Fig. 7. It may well be that this residual low energy discrepancy lies within the accuracy of even the present measurements. While attempting to find the best values for the parameters in the formula for C, it was noticed that ξ_1 could be chosen over a fairly wide range to obtain shape factors which were not too inconsistent with the experimental correction curve. It is perhaps worth noting that if ξ_1 is plotted against the corresponding value of x which yields the best fit, as is done in Fig. 8, it is found that an almost linear plot results.

Finally, we gave some consideration to what values of ξ_1 and $\eta_1(x)$ are consistent with the little which can be surmised theoretically. We restricted our consideration to predictions from the formulas given by Rose and Osborn.[21] This necessitates decisions about the probable orbital characters of the nuclear states.

The current ideas based on the shell model consider RaE and its daughter each to have two nucleons

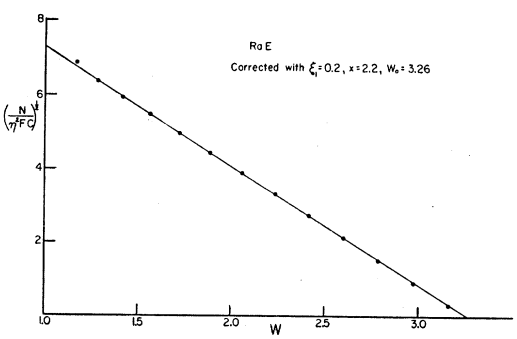

FIG. 6. Linearized plot of the RaE spectrum using the best fit values for the shape factor, C.

[21] M. E. Rose and R. K. Osborn, Phys. Rev. 93, 1326 (1954).

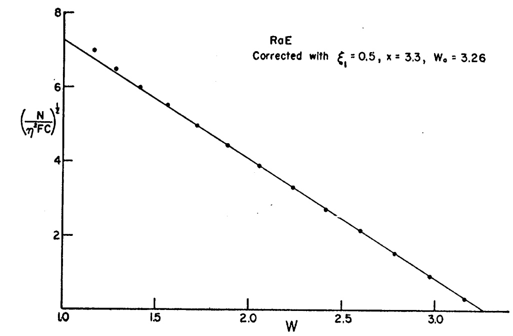

FIG. 7. Linearized plot of the RaE spectrum using Yamada's choice for $\xi_1=0.5$.

outside a doubly-closed shell of 82 protons and 126 neutrons. The daughter presumably has two $h_{9/2}$ protons outside closed shells. The parent has an $h_{9/2}$ proton with a neutron which may either be $g_{9/2}$ or $i_{11/2}$ to give a resultant spin $J'=1$. We applied Rose and Osborn formulas to both possibilities.

A two-nucleon state formed by *j-j* coupling has a degeneracy which prevents a definite conclusion unless it is removed on the basis of further assumptions. It may be removed by presuming the isotopic spin to be a good quantum number, an assumption which is hardly justifiable for as highly charged a nucleus as RaE. Nevertheless, it is of some interest to see just how much isotopic spin selection rules are violated. We obtain sets of values for the moments which are essentially the same irrespective of the choice $g_{9/2}$ or $i_{11/2}$ for the neutron.

The two proton daughter state must have isotopic spin, $I=1$, but the parent state may have $I'=0$ or 1. This has peculiar consequences. We find that the S coupling moment, $\int\beta\mathbf{r}$, nearly vanishes (exactly so for a $g_{9/2}$ neutron) if $I'=1$ whereas both the T coupling moments vanish (only nearly so for the $i_{11/2}$ neutron) if $I'=0$. Thus, we are led to the conclusion that, if the isotopic spin is a good quantum number for RaE, the S and T couplings cannot simultaneously contribute to its radiation. This is clearly wrong, since the nearly complete destructive interference between these two couplings is essential for explaining the spectrum. The unsuccessful attempt at a fit for $\xi_1=0$ (no S coupling) was shown in Fig. 2. With T coupling absent ($\xi_1\rightarrow\infty$), the spectrum would not deviate from the statistical shape. On the other hand, it may have some significance that as low a ξ_1 value as $\xi_1\approx\frac{1}{5}$ works so well (Fig. 3 and Fig. 6). One might say that the RaE state does not deviate far from having a $I'=1$ character.

A more consistent way of removing the degeneracy may be to assume that only the single transforming nucleon state is significant for the transition. We then find that $\xi_1{}^2=1/100$ for a $g_{9/2}\rightarrow h_{9/2}$ transition whereas $\xi_1{}^2=1$ if the transition is $i_{11/2}\rightarrow h_{9/2}$.

Discarding the absurd case in which the T coupling moments vanish, we find for η_1 the same result by either of the two methods for removing the degeneracy. This is

$$\eta_1=\Lambda, \tag{6}$$

which is a number expected to be between 1 and 3 by various investigators.[21–23] This result may have some significance, since it arises irrespective of rather extremely different assumptions.

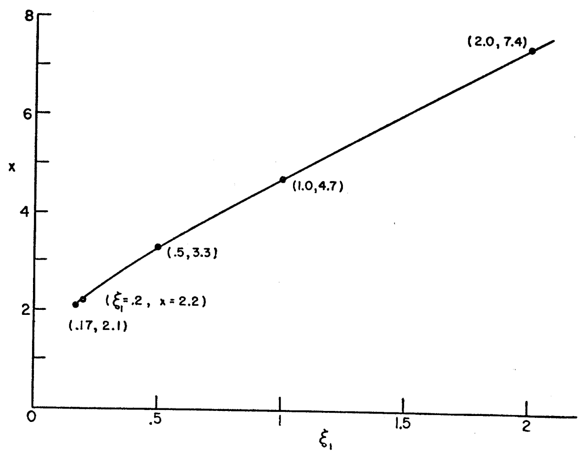

FIG. 8. Plot of values of ξ_1 and x that lead to reasonable fits to the experimental data.

The range of expected values of Λ easily covers the range of uncertainty in our experimental evaluations of ξ_1 and x. Values of these parameters which can be made to yield a reasonably fit of the RaE spectrum are shown in Fig. 8. As (ξ_1,x) range from (0.2, 2.2) to (1.0, 4.7),

$$\Lambda=\eta_1=1.5 \text{ to } 2.5,$$

according to (3).

The authors are greatly indebted to Professor E. J. Konopinski for detailed discussions on the theoretical aspects of the problem. They also wish to thank Mr. R. Johnson for helping with some preliminary measurements.

[22] T. Ahrens and E. Feenberg, Phys. Rev. **86**, 64 (1951).
[23] D. L. Pursey, Phil. Mag. **42**, 1193 (1951).