$E1$ transitions in odd-nucleon nuclei, as for example, in the decay of Am^{241} [16] and Am^{243} [17] where the states involved lie 60 and 75 kev respectively above the ground states.

Rasmussen[18] has discussed some of the implications of the Am^{241} decay scheme and his considerations may be applicable to the heavy element region in general. He pointed out that for protons in the 82–126 shell the only even parity orbital is $i_{13/2}$. The odd parity orbitals should have considerably lower j values and the only manner in which an $E1$ transition could arise is if j is not a good quantum number. His explanation is through the unified nuclear model involving strong coupling between the single particle and nuclear surface deformation which is a collective property. If strong coupling is assumed, the particle angular momentum vector j precesses rapidly around the symmetry axis of the spheroidal nucleus and the projection of j on the symmetry axis defines the spin. We then have a situation in which transitions between states of similar spin can involve large changes in the single particle wave function. Rasmussen suggests that such a situation may be responsible for the abnormally long lifetime for the 60-kev $E1$ transition. He also has found it necessary to employ this picture to explain features of β^- decay processes which would be anomalous on the basis of conventional selection rules.

The pertinence of this discussion to the problem at hand is that, at least in the heavy-element region, it may be necessary to look more deeply into the meaning of spectroscopic states than would appear just from spin assignments. Some of the implications of this idea have also been discussed in reference to the alpha-decay process.[18,19]

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[17] F. Asaro and I. Perlman, Phys. Rev. **93**, 1423 (1954).
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Capture-Positron Branching Ratios*

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Expressions are given for allowed and first-forbidden K and L capture probabilities, and rules are formulated for obtaining such expressions for capture of any order of forbiddenness from any orbit. The effect of screening on branching ratios is discussed, and a table for the rapid calculation of allowed branching ratios, including screening effects, is given. The results are then applied to the decays of Zn^{65}, Na^{22}, Sc^{44}, and V^{48}.

I. INTRODUCTION

In attempting to classify a negatron decay as to order of forbiddenness and to gain information as to the form of lepton-nucleon interaction, one generally uses as evidence the shape of the spectrum, the logft value of the decay, and the spin and parity changes involved, the last either measured or obtained from shell-model considerations.[1]

In the case of radioactive nuclei which emit positrons, however, there is another piece of evidence available, namely the branching ratio between orbital electron capture and positron emission. The branching ratio (b.r.) is defined by the relation

$$\text{b.r.}=\lambda_c/\lambda_+, \quad (1)$$

where λ_c and λ_+ are, respectively, the probabilities per unit time of electron capture and positron emission.

The quantity λ_+ is given by the well-known relation for an nth forbidden decay,

$$\lambda_+=\frac{1}{2\pi^3}\sum_{x,y} g_x g_y \tfrac{1}{2}(1+\delta_{xy})\times\int_1^{W_0} pW(W_0-W)^2F_0(W,Z)C_{nxy}(W,Z)dW=\int_1^{W_0} N(W,Z)dW, \quad (2)$$

where the notation is the same as Greuling's[2] except that here we allow for the possibility of a linear combination of interactions—hence the presence of two indices, x and y. Greuling tabulates the C_{nxy} for pure interactions while Smith[3] and Pursey[4] give the interference terms which arise when more than one form of interaction is assumed to be present.

In calculating λ_+, numerical values for the Fermi

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† Operated for the U. S. Atomic Energy Commission by the General Electric Company.
[1] Mayer, Moskowski, and Nordheim, Revs. Modern Phys. **23**, 315 (1951); L. W. Nordheim, Revs. Modern Phys. **23**, 322 (1951).
[2] E. Greuling, Phys. Rev. **61**, 568 (1942).
[3] A. M. Smith, Phys. Rev. **82**, 955 (1951).
[4] D. L. Pursey, Phil. Mag. **42**, 1193 (1951). We adopt Pursey's notation rather than Smith's.

function $F_0(W,Z)$ were obtained from the tables of Dismuke *et al*,[5] while the radial function combinations appearing in C_{nxy} were taken from the tables prepared by Rose, Perry, and Dismuke.[6] The effect of screening by the orbital electrons is treated in the manner discussed in Sec. III. The integral in Eq. (2) was then performed numerically.

In Sec. II, expressions for λ_c are given. The screening effect also enters into the calculations of λ_c, although the discussions of Secs. III and V indicate that the screening correction is smaller for capture probabilities than for positron emission probabilities. The discussion of Sec. V concerns the decays of Zn^{65}, Na^{22}, Sc^{44}, and the 0.802 branch of the V^{48} decay. We conclude from the available evidence that the decays of Zn^{65} and Na^{22} are l-forbidden, the 0.802-Mev decay of V^{48} may be first-forbidden unique, and that further experiments should be performed on Sc^{44}, the last decay being allowed from all except the branching-ratio evidence.

Furthermore, it is concluded that unless the effect of screening on the positron spectrum is considered, fallacious results for the branching ratio may easily be obtained. In particular, the unscreened branching ratio of Zn^{65} is outside of experimental limits, and screening brings the theoretical branching ratio of Na^{22} into better agreement with experiment so that some conclusions previously reached by other authors as to the amount of Fierz interference present in this decay appear to be invalidated.

II. CAPTURE PROBABILITIES

The probability per unit time that a radioactive nucleus consisting of Z protons and $A-Z$ neutrons will decay via orbital electron capture into a nucleus consisting of $Z-1$ protons and $A-Z+1$ neutrons, with the accompanying emission of a neutrino, may be written in the form,

$$\lambda_c=\sum_{x,y} \lambda_c^{xy}\tfrac{1}{2}(1+\delta_{xy}). \qquad (3)$$

The summation indices, x and y, take on five values corresponding to the five covariant forms of the nucleon-lepton interaction (i.e., S, V, T, A, or P). When x and y are equal, the term is said to be "pure," while terms for which $x\neq y$ are referred to as "interference" terms.

Expressions[7] for λ_c^x have been given by many authors including Nataf and Bouchez,[8] Marshak,[9] and DeGroot and Tolhoek.[10] None of these gives a complete treatment, although the Nataf and Bouchez paper, which quotes results for allowed and first-forbidden K capture, is the most nearly complete. These results are

TABLE I. Radial function combinations for capture.

Orbit of captured electron:	K ($n=1, \kappa=-1$)	L_{I} ($n=2, \kappa=-1$)	L_{II} ($n=2, \kappa=+1$)	L_{III} ($n=2, \kappa=-2$)
L_0	g^2_{-1K}	g^2_{-1L}	f^2_{1L}	0
L_1	0	0	0	$\rho^{-2}g^2_{-2L}$
M_0	$\rho^{-2}f^2_{-1K}$	$\rho^{-2}f^2_{-1L}$	$\rho^{-2}g^2_{1L}$	0
N_0	$\rho^{-1}f_{-1K}g_{-1K}$	$\rho^{-1}f_{-1L}g_{-1L}$	$-\rho^{-1}f_{1L}g_{1L}$	0
P_0	g^2_{-1K}	g^2_{-1L}	$-f^2_{1L}$	0
P_1	0	0	0	$\rho^{-2}g^2_{-2L}$
Q_0	$-\rho^{-2}f^2_{-1K}$	$-\rho^{-2}f^2_{-1L}$	$\rho^{-2}g^2_{1L}$	0
R_0	$\rho^{-1}f_{-1K}g_{-1K}$	$\rho^{-1}f_{-1L}g_{-1L}$	$\rho^{-1}f_{1L}g_{1L}$	0

extended here to L_{I}, L_{II}, and L_{III} capture and to include "interference" terms. Also, rules are given for finding capture probabilities from all orbits for all orders of forbiddenness, although the usefulness of such formulas is questionable.

The calculation of the λ_c^{xy} are carried out in the same manner in which Konopinski and Uhlenbeck[11] and Greuling[2] calculated the probability of radioactive electron emission, except that a sum over discrete energy levels of the captured electron replaces the integration over a continuous energy spectrum. We write

$$\lambda_c^{xy}=\sum_{\kappa,n} g_x g_y\left(\frac{W_0+\epsilon_{n\kappa}}{2\pi}\right)^2 C_{n\kappa}^{xy} \qquad (4)$$

where κ and n are the (Dirac) quantum numbers of the orbital electron, g_x and g_y measure the strength of the interaction as well as the amount of admixture, while $C_{n\kappa}^{xy}$ is the "capture correction factor." The quantity $\epsilon_{n\kappa}$ is the (screened) energy of the bound electron before capture.

The $C_{n\kappa}^{x}$ and $C_{n\kappa}^{xy}$ are identical with the C_{nx} formulas (given by Konopinski and Uhlenbeck or Greuling) and the $C_n(x,y)$ formulas (given by Smith or Pursey) for positron emission, if the radial function combinations (L_0, M_0, P_0, Q_0, etc.), now functions of the indices n and κ, are redefined as in Table I.

Here the f's and g's are the radial functions given by Bethe.[12] The subscript numbers refer to the κ value, while the subscript letters give the principal quantum number n. The symbol ρ represents the nuclear radius.

Correction factors for higher orbits may be obtained simply by changing the definitions of the radial function combinations given above in such a manner that the subscripts give the quantum numbers of the orbit under consideration. Thus, for M_1 capture, L_0 becomes $g_{-1M}{}^2$, etc.

For second and higher-forbidden capture, the same fules for forming λ_c apply as do for allowed and first-forbidden capture. However, now other radial function combinations besides those given above will appear. In this instance the modified expressions applicable to

[5] Dismuke, Rose, Perry, and Bell, unclassified Oak Ridge National Laboratory Report ORNL-1222 (unpublished).
[6] Rose, Perry, and Dismuke, unclassified Oak Ridge National Laboratory Report ORNL-1459 (unpublished).
[7] For $x=y$, only one superscript will be retained.
[8] R. Nataf and R. Bouchez, J. phys. et radium **13**, 190 (1952).
[9] R. E. Marshak, Phys. Rev. **61**, 431 (1941).
[10] S. R. DeGroot and H. A. Tolhoek, Physica **16**, 456 (1950).
[11] E. J. Konopinski and G. E. Uhlenbeck, Phys. Rev. **60**, 308 (1941).
[12] H. A. Bethe, *Handbuch der Physik* (Springer, Berlin, 1933), second edition, Vol. 24, Part II, pp. 311 ff.

TABLE II. Allowed K to positron branching ratios.

W_0/mc^2 \ Z	16	29	49	84	92
1.28	93.11	711.3	10 070	5.066×10^5	1.620×10^6
1.44	17.29	112.7	1 317	5.000×10^4	1.528×10^5
1.60	5.654	33.78	354.2	1.148×10^4	3.335×10^4
1.76	2.468	13.94	136.2	3961	9.100×10^3
1.92	1.280	6.954	64.70	1741	4850
2.08	0.7449	3.933	35.27	895.8	2462
2.40	0.3070	1.605	13.67	320.6	870.6
2.88	0.1224	0.6005	4.883	107.1	286.4
3.84	0.03382	0.1605	1.256	26.23	70.76
4.80	0.01397	0.06517	0.5021	10.53	28.52
5.76	7.114×10^{-3}	0.03298	0.2517	5.356	14.63
6.72	4.111×10^{-3}	0.01896	0.1443	3.129	8.627
7.68	2.593×10^{-3}	0.01192	0.09062	2.000	5.564
8.64	1.739×10^{-3}	7.978×10^{-3}	0.06077	1.364	3.824
9.60	1.256×10^{-3}	5.633×10^{-3}	0.04278	0.9759	2.753
10.56	8.946×10^{-4}	4.119×10^{-3}	0.03138	0.7245	2.055
11.52	6.736×10^{-4}	3.086×10^{-3}	0.02340	0.5535	1.579
12.48	5.200×10^{-4}	2.382×10^{-3}	0.01825	0.4365	1.244

electron capture may be found from Rose's definitions of the radial function combinations[6] if

a. The normalization factor $(2p^2F_0)^{-1}$ is set equal to unity.

b. For capture from a given subshell (i.e., specified values of n and κ) only those radial functions with the appropriate κ value are retained. Then the n subscript is added. For example, Rose defines

$$L_2=\frac{1}{2p^2F_0\rho^4}(g_{-3}^2+f_3^2).$$

For electron capture, only those orbits for which $\kappa=\pm3$ will contribute to L_2, just as only those orbits for which $\kappa=\pm2$ can contribute to L_1.

It should be noted however that this does not imply that only such orbits contribute to second-forbidden K and L capture, inasmuch as radial function combinations with all subscripts up to and including n appear in the nth forbidden spectral correction factors.

III. SCREENING CORRECTIONS

The wave functions of both the bound electrons and the positrons are affected by the perturbation on their electrostatic potential energy of the (other) orbital electrons.

Reitz[13] has calculated the screening effect on allowed positron spectra by the use of a Thomas-Fermi-Dirac model of the atom. In order to extend these calculations to the forbidden spectra, the Thomas-Fermi-Dirac wave functions obtained by Reitz[14] were used to form the radial function combinations appearing in expressions for allowed and first-forbidden positron decay probabilities. In this way, it was found numerically that the first-forbidden positron spectra are screened by approximately the same factor as are the allowed spectra. That is, if $N(W,Z)dW$ and $N_{\mathrm{Coul}}(W,Z)dW$ are the screened and Coulomb positron spectra respectively, then the equation,

$$N(W,Z)dW=N_{\mathrm{Coul}}(W,Z)S(W,Z)dW, \qquad (5)$$

holds for both the allowed and first-forbidden positron spectra with the screening function $S(W,Z)$ being approximately the same for both cases. Good[15] and Huster[16] have discussed this point in some detail.

Thus, positron screening was taken into account by the use of Reitz's allowed screening factors, with intermediate values being obtained by graphical interpolation.

It is found that in the region of $Z\sim25$, the effect of screening on the K wave functions can be neglected, Bethe's formulas for hydrogen-like atoms giving values about one percent higher than Reitz's wave functions. Incidentally, Slater wave functions[17] are somewhat closer to the Reitz wave functions than are the Bethe wave functions.

For L electrons, on the other hand, the screening effect is much larger, and if L to positron branching ratios are of interest, even Slater wave functions are not adequate. However, in most cases when K capture is energetically possible, the L capture is a small fraction of total capture, so that the choice of wave functions is not of great importance.

IV. TABLE OF ALLOWED BRANCHING RATIOS

Table II gives allowed K to positron branching ratios as a function of W_0 and Z, where W_0 is the end-point energy of the positron spectrum and Z is the charge of the parent nucleus. Branching ratios for intermediate values of W_0 and Z may be obtained by graphical inter-

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[14] J. R. Reitz, *Relativistic Electron Wave Functions for a Thomas-Fermi-Dirac Statistical Atom* (University of Chicago Press, Chicago, 1949).

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[16] E. Huster, Z. Physik **136**, 303 (1953).

[17] J. C. Slater, Phys. Rev. **36**, 57 (1930).

polation, although the results should not be extrapolated beyond the Z-values given.

The effect of screening, both on the positron and capture decay probabilities, was taken into account by using Reitz's wave functions.

Allowed branching ratios for L-capture to positron emission may also be obtained readily if Table II is used in conjunction with the graph given by Rose and Jackson[18] plotting the ratio of allowed L capture to K capture as a function of Z.

V. COMPARISON WITH EXPERIMENT

a. Zn-65

The 0.325-Mev decay of Zn^{65} has been measured by Haynes and Perkins,[19] who find a Kurie plot linear down to 1.12 mc^2, if the experimental points are corrected for screening. Since the shell model[1] predicts this transition to be $f_{\frac{5}{2}}\rightarrow p_{\frac{3}{2}}$, ($\Delta J=1$, "no") the allowed shape is no surprise. The high logft value of 7.34[20] is assumed to be caused by the fact that $\Delta l=2$ (i.e., the transition is "l-forbidden").

Under the assumptions that it is valid to evaluate the lepton wave functions at the nuclear radius[21] and that second-order corrections[22] are unimportant, the K-positron branching ratio is the same for an l-forbidden decay as for an allowed decay.

The theoretical K-positron branching ratio for this decay is 28.9, which is in good agreement with the measured value of 28.0 ± 3.0.[19] If the effect of screening on the positron spectrum is ignored, the theoretical branching ratio for an allowed decay is 33, which is outside the experimental limits.

It is interesting to note that if the experimental points are corrected for screening before the Kurie plot is drawn, the end-point energy is found to be 0.330 Mev rather than the 0.325-Mev figure usually quoted in the literature.

b. Na-22

The 0.54-Mev decay of Na^{22} should be another $\Delta J=1$, "no" allowed transition.[23] The theoretical allowed branching ratio ($L+K$) for such a decay is 0.111, which is in good agreement with recent measurements by Kreger and Cook[24] and Sherr and Miller[25] both of whom measure 11 percent. Our theoretical value is slightly lower than that given by the latter authors because of the effect of screening on the positron spectrum. Since Sherr and Miller failed to include this effect, their conclusions concerning the amount of Fierz interference present are largely invalidated, the discrepancy between the experimental and theoretical branching ratios which they used in their argument becoming much smaller when screening is included.

The high logft value of this decay (logft=7.38) indicates that this too is an l-forbidden transition.

c. Sc-44

The 1.463-Mev decay of Sc^{44} is presumably allowed and shows a linear Kurie plot with a logft value of 5.3.[23,26] Bruner and Langer, however, quote electron capture as being the same order of magnitude as positron emission for this transition. The theoretical allowed K to positron branching ratio for this isotope assuming an allowed transition may readily be calculated to be 0.062, in very bad agreement with experiment, suggesting the need of more work on this decay.

d. V-48

The 0.802-Mev decay of V^{48}, the route by which about 5 percent of the isotope decays, is presumably first-forbidden unique, at least according to the spin and parity assignments of Roggenkamp *et al.*, and the quoted logft value of 7.4.[27]

The theoretical K to positron branching ratio is 2.02, while if the decay were allowed the branching ratio would be only 0.534. Roggenkamp quotes the branching ratio only as >0.7, so that the branching-ratio evidence is not inconsistent with the conclusion that the decay is first-forbidden unique. An experiment designed to measure accurately this branching ratio would be of great value in determining firm spin and parity assignments for the nuclear levels involved in this decay.

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