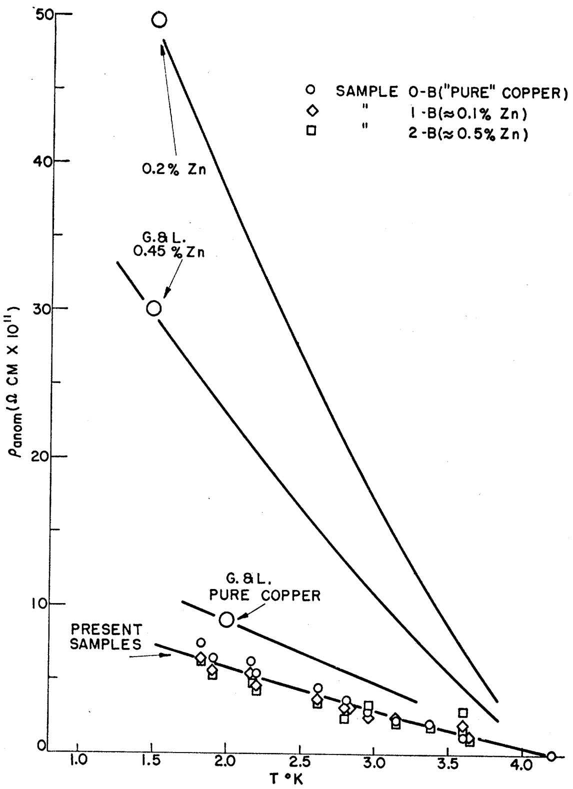

FIG. 1. $\rho_{\text{anom}} \equiv \rho_T - \rho_{4.2°\text{K}}$ *vs* temperature for copper-zinc alloys.

from their smoothed curves of resistance ratio *vs* temperature for their alloys. Our samples, which were prepared by melting the components in sealed-off, evacuated quartz tubes, show a smaller ρ_{anom} than do their samples. Moreover, the magnitude of the anomalous resistance does not vary as a function of either the zinc content or the state of anneal. Thus, no distinction is made in the figure between the data taken on a particular sample in its various stages of anneal (cold worked; 200°C for about two hours; 350° for about two hours). Distinction is made, however between the different samples consisting of pure copper (American Smelting and Refining Company high-purity—nominal 99.999 percent—copper stock[2] treated and prepared in the same manner as the alloys), of ~0.1 atomic percent zinc, and of ~0.5 atomic percent zinc in American Smelting and Refining copper. Figure 1 clearly shows that ρ_{anom} *vs* temperature is the same for all samples. There is thus no evidence allowing one to ascribe the observed anamolous temperature-dependent resistivity to zinc in either of the alloys.

The anomalous temperature-dependent resistivity observed in the particular sample of "pure" copper used in these experiments is larger than any we have observed in other samples of pure American Smelting and Refining copper[3] even though it is somewhat less than that reported by Gerritsen and Linde for their copper. The indication is, then, that our method of preparing the alloys, to which the pure copper is also subjected, may result in picking up very small amounts of minimum-producing impurities.

A more detailed report of these measurements, along with measurements in the hydrogen temperature range and on higher percentage zinc alloys, will be given soon.

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Correspondence between Semiclassical and Quantum Treatments of Coulomb Excitation*

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FROM the experimental evidence[1] regarding correctness of the dependence of the cross section on incident energy, it is not possible to conclude directly that the absolute value of the cross section is correctly given by the usual semiclassical treatment, referred to as SCT. In this and the following notes only the simple case of one nuclear proton, initially in an *s* state, being responsible for the interaction, is considered, the generalization to several nuclear protons being irrelevant to the question of accuracy of the SCT. The first order Born approximation with the nuclear quadrupole interaction as the small quantity is taken to be adequate.

The collision cross section for the reaction can be represented as

$$\sigma = (4/25)(mk_f/v_i\hbar^3)Z_1^2e^2B(2)\mathcal{A}, \tag{1}$$

$$\mathcal{A} = \Sigma\mathcal{A}_L, \tag{2}$$

$$\mathcal{A}_L = 16\pi^2 k_i^{-2}k_f^{-2}\left\{\frac{3L(L+1)}{2(2L+1)}\times\left[\left|\int F_{L-1}(k_ir)F_{L+1}(k_fr)r^{-3}dr\right|^2 + \left|\int F_{L+1}(k_ir)F_{L-1}(k_fr)r^{-3}dr\right|^2\right] + \frac{L(L+1)(2L+1)}{(2L-1)(2L+3)}\left|\int F_L(k_ir)F_L(k_fr)r^{-3}dr\right|^2\right\} = 16\pi^2\mathcal{C}_L. \tag{2'}$$

$$B(2) = 5e^2\left|\int_0^\infty r_p^4R_i(r_p)R_f(r_p)dr_p\right|^2, \tag{3}$$

with the normalization $\int_0^\infty R^2 r^2 dr = 1$. Here i,f refer to initial and final states, $k/(2\pi)$ is the wave number of the incident particle, r is the distance of the projectile from the target, Z_1 is the charge of the projectile, r_p is the distance of the nuclear proton from the nuclear center, the R's are radial functions, F_L is the Coulomb function in the notation of Yost *et al.*[2] In Eq. (2′), the limits of integration are understood to be 0 and ∞. The effect of the proton and nuclear spin results in a factor common to both treatments and is therefore left out of account. The corresponding expression on the SCT is

$$\sigma_{\mathrm{cl}} = (2\pi^2/25)[k^2/(Z_2^2e^2)]B(2)g_2(\xi), \tag{4}$$

according to Ter-Martirosyan[3] in the notation of Bohr and Mottelson[4] which is followed below except where symbols are otherwise defined. Agreement of the two imples

$$(\pi^2/2)g_2(\xi) \cong \eta_{\mathrm{cl}}^2(k_f/k_i)\mathcal{A}, \tag{5}$$

where

$$\eta_{\mathrm{cl}} = Z_1Z_2e^2/(\hbar v_{\mathrm{cl}}) \tag{5'}$$

is the value of the Coulomb parameter η corresponding to the value of the classical velocity, v_{cl} used in the SCT. Since the dependence of σ on $\xi = \Delta Et/\hbar$ is established experimentally, the approximate correctness of σ_{cl} as an approximation to σ for $\xi = 0$ may be considered as a partial argument for the validity of σ_{cl} also if $\xi \neq 0$. For $\xi = 0$ the comparison may be carried out by attributing an SCT contribution of an interval $\delta\epsilon$ with the contribution to $\mathcal{A}$ caused by a given L to $\mathcal{A}$ in Eq. (2). The connection of ϵ with L is given by

$$\epsilon^2 = 1 + L^2/\eta^2. \tag{6}$$

The correspondence between the quantum and SCT results postulated to apply for corresponding ranges of angular momenta gives

$$(\pi^2/2)\int_1 \sum_\mu |S_\mu^{(2)}|^2 \epsilon d\epsilon \cong \eta_{cl}^2(k_f/k_i)\mathcal{A}_L, \tag{7}$$

where $\int_1$ indicates integration over a range of L of width 1 with a mean L approximately that of $\mathcal{A}_L$. In Eq. (2′), integrals with the same average of the L values of the F's are grouped together. Correspondence to the SCT is not sensitive to the type of grouping.

A qualitative argument for the validity of Eq. (7) can be made by means of the JWKB approximation. For small L/η the quantity $\epsilon^2 - 1$ is small, and hence for $\xi = 0$

$$|S_2^{(2)}| = |S_{-2}^{(2)}| \cong (3/2)^{\frac{1}{2}}|S_0^{(2)}|.$$

The three integrals entering $\mathcal{A}_L$ are approximately equal in this case and hence Eq. (7) yields

$$64\left|\int F_L^2(r/a')^{-3}d(r/a')\right|^2 \cong \sum_\mu |S_\mu^{(2)}|^2 \cong 4|S_0^{(2)}|^2, \tag{8}$$

where a' is half the distance of closest approach. The same condition holds for $L/\eta \gg 1$. In this case,

$$|S_2^{(2)}| = |S_{-2}^{(2)}| \cong (1/6)^{\frac{1}{2}}|S_0^{(2)}|,$$

and the integrals with unequal L entering $\mathcal{A}_L$ are each $\frac{1}{3}$ of that with equal L. These changes give Eq. (8) again. The latter may be rewritten as

$$16\left|\int_0^\infty F_L\,(r/a')^{-3}d(r/a')\right|^2 \cong 4\left|\int_{r_{\mathrm{min}}}^\infty (v/\dot{r})(r/a')^{-3}d(r/a')\right|^2, \tag{9}$$

where $\dot{r}$ is the classical velocity of the SCT and v is the velocity for infinite r. For short wavelengths of the r motion, the JWKB approximation applies and F_L^2 may then also be replaced by $\frac{1}{2}$ of A_L^2, the square of the amplitude of F_L. But

$$[A_L^2]_{\mathrm{JWKB}} = v/\dot{r}, \tag{10}$$

where $\dot{r}$ is again the "classical" value. In this approximation the left and right sides of Eq. (9) are equal, as expected from the general connection of classical mechanics and geometrical optics.

For large η and moderate L, the JWKB method gives errors of the order of 5 percent in the value of the amplitude A_L at the first maximum and is qualitatively wrong at the classical turning point. The replacement of F_L^2 by $\frac{1}{2}A_L^2$ together with the omission of the region from $\rho = 0$ to the turning point have no obvious justification except for large L. On the other hand the contributions to the integrals from the turning point to the first node are of the order of 60 percent of the whole. For these reasons Eq. (7) has been investigated numerically for $\xi = 0$ with results described in the following note. In this case, Eq. (7) becomes

$$32\eta^2\mathcal{C}_L \cong \int_1 \sum |S_\mu^{(2)}|^2 L dL/\eta^2. \tag{11}$$

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