# Measurement of Coulomb Interference in 113-Mev $\pi^+$-$p$ Scattering*

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Pion-proton scatterings with a small angle cutoff of 12.5° in the center-of-mass system were detected using the area-scanning, nuclear-emulsion technique. Ilford G5 plates 600 microns thick were exposed to a flux of $4.4\times 10^5$ mesons/cm² in the 122-Mev $\pi^+$ beam of the Chicago cyclotron. Various checks indicated a scanning efficiency greater than 90 percent for scatterings on the hydrogen content of the emulsion between 12.5° and 160°. Out of a total 420 hydrogen events only two scatterings were less than 20°. The data when analyzed according to the two possible sets of signs of the phase shifts give a strong preference for destructive interference, i.e., attractive $p_{\frac{3}{2}}$ interaction. In this case the maximum likelihood solution is $\alpha_3=-(10.9\pm 2)°$, $\alpha_{33}=(27.0\pm 1.5)°$, and $\alpha_{31}=-(3.2\pm 1)°$ for the $s$- and $p$-wave phase shifts with no definite indication of a $d$-wave component. The total nuclear cross section is $(79\pm 5)$ millibarns. The mean energy of pions in the emulsion is $(113.0\pm 1.6)$ Mev as determined from 33 events where the proton stops in the emulsion.

## I. INTRODUCTION

SINCE the only interaction positive pions of 113 Mev can have with protons is an elastic scattering, the interaction may be described in terms of the partial wave phase shifts. These phase shifts can be obtained from the shape of the angular distribution except for their sign. Knowledge of whether the pion-proton interaction is attractive or repulsive would establish the sign. Since we do have knowledge that part of this interaction (the Coulomb part) is repulsive, we can in principle determine the absolute signs of the phase shifts. The quantitative separation of the scattering into nuclear and Coulomb parts has been discussed by several workers.[1]

The scattering of negative pions by hydrogen may also be described by these phase shifts if the pion-nucleon interaction is charge independent. Allowances can be made for the Coulomb part of the interaction which is, of course, not charge independent.[1] Under these assumptions the signs of the phase shifts may also be found by observing the Coulomb interference effects in $\pi^-$ scattering. This determination has been made for 65-Mev pions at Columbia.[2] They were able to fit all their data with both sets of signs for the phase shifts. The set with large positive $\alpha_{33}$ (commonly called the Fermi solution) connects smoothly with results at lower and higher energies.[2,3] However, the set with negative $\alpha_{33}$ (commonly called the Steinberger solution) requires such a small $|\alpha_{33}|$ and such a large $|\alpha_3|$ that reasonable assumptions about the energy dependence of the phase shifts would rule out a negative $\alpha_{33}$.

The present experiment which has the advantage that it requires no assumptions about charge independence also predicts a positive $\alpha_{33}$. For the case of destructive interference the maximum likelihood solution is $\alpha_3=-10.9\pm 2°$, $\alpha_{33}=27.0\pm 1.5°$, and $\alpha_{31}=-3.2\pm 1°$. This is a much better fit than is obtained in the case of construction interference. Positive $\alpha_{33}$ gives a destructive Coulomb interference in the forward direction because the scattered wave (predominantly $p_{\frac{3}{2}}$) is then from an attractive nuclear force which is partially "neutralized" by the repulsive Coulomb force. Quantitatively the expression for pions of relativistic energies as obtained by Solmitz[1] is [neglecting terms of the order $(v_p/c)^2$]:

$$\frac{d\sigma}{d\omega}=\left|\frac{1}{2ik}(P+Q\cos\chi)+f^{(nf)}(\chi)\right|^2+\left|\frac{1}{2ik}R\sin\chi+f^{(f)}(\chi)\right|^2,$$

$$P=\exp(2i\alpha_3)-1,$$

$$Q=\exp(2i\alpha_{31})+2\exp(2i\alpha_{33})-3, \tag{1}$$

$$R=\exp(2i\alpha_{33})-\exp(2i\alpha_{31}),$$

$$f^{(nf)}=-\frac{e^2}{2p(v_\pi+v_p)\sin^2\chi/2}\left[1+\frac{v_\pi v_p}{2c^2}(1+\cos\chi)\right],$$

$$f^{(f)}=\frac{e^2}{2p(v_\pi+v_p)\sin^2\chi/2}\frac{\mu_p v_\pi v_p}{2c^2}$$

(all quantities in the center-of-mass system).

This function is plotted in Fig. 1 for the values of $\alpha_3$, $\alpha_{33}$, and $\alpha_{31}$ obtained in this experiment assuming both possibilities of sign. The two curves begin to depart significantly below 25°. This interference effect is less noticeable for $\pi^+$-$p$ scattering in the energy region 30 to 70 Mev where the repulsive $s$-wave is the same order of magnitude as the attractive $p$-wave.[4]

Except for the detection of small-angle scatterings down to 12.5°, the scanning technique is the same as that described by the author.[5] The scanning technique

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[1] L. van Hove, Phys. Rev. 88, 1358 (1952); J. Ashkin and L. Smith, Technical Report No. 1, Carnegie Institute of Technology, February 2, 1953 (unpublished); and F. T. Solmitz, Phys. Rev. 94, 1799 (1954).

[2] Bodansky, Sachs, and Steinberger, Phys. Rev. 93, 1367 (1954).

[3] J. Orear, Phys. Rev. 96, 176 (1954).

[4] Orear, Lord, and Weaver, Phys. Rev. 93, 575 (1954).

[5] J. Orear, Phys. Rev. 92, 156 (1953).

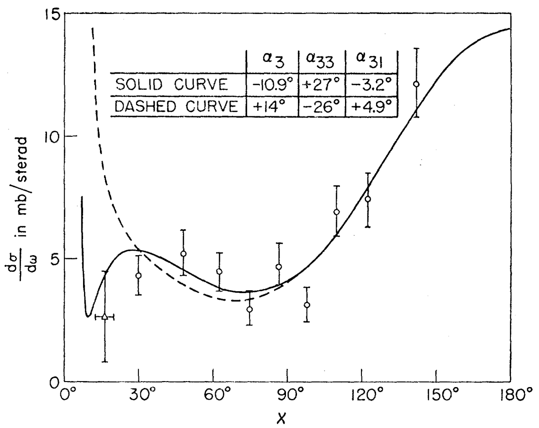


FIG. 1. The average $d\sigma/d\omega$ in mb/sterad corresponding to dividing the data into 9 equal units of solid angle. The two events found in the region $12.5° < \chi < 20°$ are represented by the triangle. The solid curve is the best fit assuming positive $\alpha_{33}$. The dashed curve is the best fit assuming negative $\alpha_{33}$.

used here along with estimates of scanning efficiency is discussed in Sec. III. The experimental results are displayed and analyzed in Sec. IV. The following section gives details of the geometry and beam used.

## II. EXPOSURE

Ilford G5 plates 600 microns thick were exposed in the geometry shown in Fig. 2. One inch of polyethylene was used to filter out the many protons of about the same momentum. This filter reduced the proton-pion ratio at least a factor $10^4$. Since the $\pi^+$ are taken off the target in the backward direction where the production cross section is low, about 20 hours of exposure time were needed to reach the flux of $4.4 \times 10^5$ tracks/cm$^2$. Range curve analysis in this beam indicates a 7 percent contamination caused by muons of the same momentum.[6] Total pion track length corresponding to the region scanned was determined as described in reference 5 by counting a total of 1235 tracks. The product of the track density so determined by the total area scanned gives $L_\pi = (1.56 \pm 0.05) \times 10^5$ cm.

In 46 of the events the proton stops in the emulsion. The incoming pion energy is then determined by the proton energy and scattering angle. The values of incoming pion energy are used to determine the average beam energy in the plate as $(113.0 \pm 1.6)$ Mev. This agrees with the energy loss in the polyethylene and range curve determination of the beam energy. The rms spread of beam energy in the plates is about 7 Mev. The small angle (pion-proton) scatterings are not used in this beam energy determination, because the proton angle cannot be determined accurately. In the case of the small angle scatterings a better value can be obtained for $\chi$, the center-of-mass scattering angle, by using the proton range and assuming a pion energy of 113 Mev to calculate $\chi$. This is done for the 13 events where $\chi < 30°$.

## III. SCANNING

The scanning technique used here is described in detail in reference 5. The entire volume of emulsion is covered by scanning overlapping strips 115 microns wide defined by a whipple disk reticle. The scanner examines all single proton beginnings for an incoming and scattered pion. All 3-pronged events of this type were recorded. The difficulty in finding small angle scatterings is the short recoil proton (9.2 microns for $\chi = 12.5°$). Because of this, the scanners were required to search for and record all short protons longer than 7 microns and perpendicular to the beam direction within 30°. This added condition reduced the scanning rate a factor two. Scanning efficiency for these small angle scatterings can be checked strip by strip since an average of 9 of these short background protons

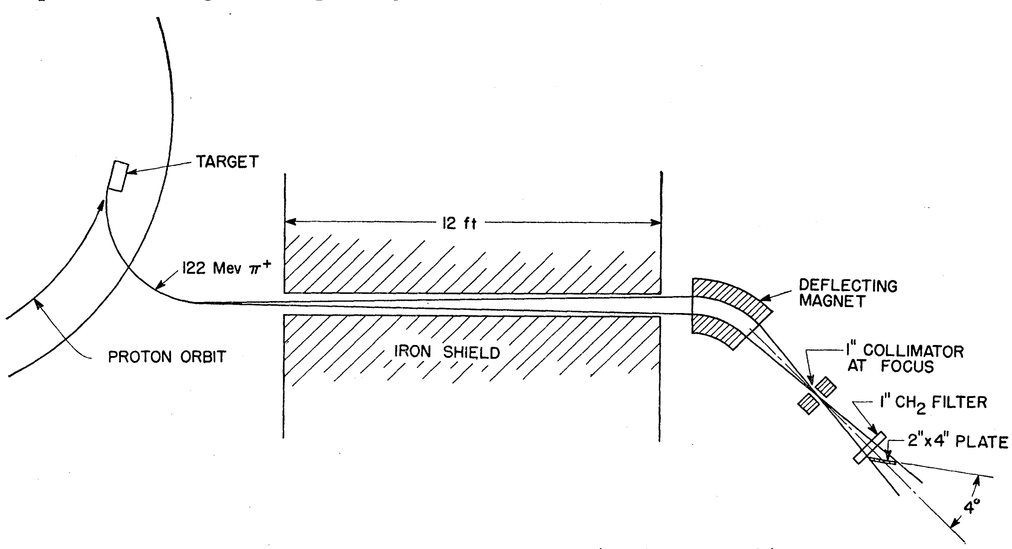


FIG. 2. Experimental setup for exposure (not drawn to scale).

[6] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).

occurred in each strip. Over 3000 were recorded in all. About 50 strips distributed fairly uniformly over the area were spot checked. Consistently no more than 15 percent of the short protons were missed. As an independent check a scanning test was constructed using short protons and scattering events especially selected for being difficult to see. The testee was given a list of toothline coordinates[7] defining the selected strips. He then scanned the selected strips no slower than his usual rate. Scores on this test were 85 percent, 88 percent, and 89 percent. Since a short proton which has two pion tracks leading to it is easier to notice, these scanning efficiency determinations should give a lower limit for scatterings where the recoil proton is greater than 9 microns long in the plane of the emulsion. On the basis of the above evidence it is assumed that the region $12.5° < \chi < 15°$ and $\phi < 40°$ has been scanned with at least 90 percent efficiency. $\phi$ is the angle between the plane of the event and the plane of the emulsion. The event with the smallest proton range in the plane of the emulsion (12.5 microns) that was found is shown in Fig. 3. The true proton range is 30 microns corresponding to $\chi = 19°$ and $\phi = 64°$. The next region where high-scanning efficiency is assumed is $15° < \chi < 20°$ and $\phi < 65°$. The other regions used are 20° to 130°, no limit on $\phi$; and 130° to 160°, $\phi < 60°$. These restrictions on large angle scatterings are in a region where the recoil protons are lighter tracks (~60 Mev) and are within 6° of the beam direction. In this case they might tend to be considered as two pion beam tracks which accidentally overlap, and thus not be given proper attention. Events within 15 microns (after shrinkage) of each surface might be missed and are difficult to measure. This region is also excluded. Finally, events are rejected which either are not coplanar within the accuracies of measurement[5] or do not satisfy the angular correlation dictated by energy-momentum conservation. The total number of events surviving all these restrictions is 333. Nonhydrogen collisions which are quasi-elastic and give no other visible prongs contribute about a 4 percent background. The technique of how this background determination is made is discussed in detail in reference 5. In calculating cross sections the scanning efficiency is assumed to be 96±4 percent. This correction is exactly offset by the 4 percent background correction.

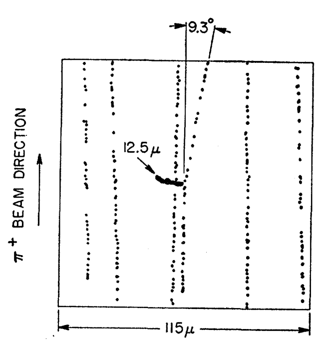


FIG. 3. Small angle ($\pi^+$,$p$) scattering showing other beam tracks in the same field of view. $\chi = 19°$. Plane of event is inclined 64° to plane of emulsion.

[7] J. Orear, Rev. Sci. Instr. **25**, 1023 (1954).

TABLE I. Data broken up into 9 equal units of solid angle (1.26 sterad each).

| $\Delta\chi$ | $\Delta N$ | $d\sigma/d\omega$(mb/sterad) |
|---|---|---|
| 12.5°– 40.2° | 28 | 4.31 |
| 40.2°– 55.7° | 34 | 5.24 |
| 55.7°– 68.7° | 29 | 4.46 |
| 68.7°– 80.6° | 19 | 2.93 |
| 80.6°– 92.1° | 31 | 4.77 |
| 92.1°–103.7° | 20 | 3.08 |
| 103.7°–115.9° | 45 | 6.94 |
| 115.9°–129.5° | 48 | 7.4 |
| 129.5°–159° | 79 | 12.2 |

## IV. ANALYSIS OF RESULTS

Table I gives a breakdown of the data into 9 equal units of solid angle. The average values of the differential cross section obtained from this data are shown in Fig. 1. Two events were found in the region of the "Coulomb dip" from 12.5° to 20°. This "point" is also represented in Fig. 1.

In calculating the phase shifts and their errors the data were broken into 3° intervals and the maximum likelihood method was used.[4] In this method the probability that the experiment would turn out the way it did is calculated assuming Eq. (1) is the true shape. The phase shifts which maximize this probability are solved for. We obtained the solutions $\alpha_3 = -(10.9\pm2)°$, $\alpha_{33} = (27\pm1.5)°$, and $\alpha_{31} = -(3.2\pm1)°$ for the destructive Coulomb interference and $\alpha_3 = (14\pm2)°$, $\alpha_{33} = -(26\pm1.5)°$, and $\alpha_{31} = (4.9\pm1)°$ for the constructive. The errors given are the statistical standard deviations assuming there is no $d$-wave. All possible variations of $a$, $b$, and $c$ within their errors allow the destructive phase shifts to fall within the following limits: $\alpha_3 = -10.9° \pm 3.5°$, $\alpha_{33} = 27° \pm 2.5°$, and $\alpha_{31} = -3.2° \pm 2.8°$. The only other possible phase shift solution is the Yang type which is $\alpha_3 = -10.9°$, $\alpha_{33} = 7.8°$, and $\alpha_{31} = 38°$, for the destructive case. A Steinberger-type solution with positive $\alpha_3$ large enough to give destructive interference is completely ruled out.

According to the maximum likelihood method the data fit much better in the destructive case than the constructive. The ratio of these two probabilities turned out to be about 4000 to 1 in favor of the destructive interference.

There are other ways of expressing this preference of the data for positive $\alpha_{33}$ which are perhaps easier to visualize. One simple approach is to use only the data in the region where Coulomb effects are negligible ($\chi > 40°$) to solve for the phase shifts. Then these phase shifts when used in Eq. (1) will predict the average number of events which should have been found in the region where the Coulomb effects are large ($\chi < 25°$). Six events were found in this region. According to the Poisson distribution the ratio of the probability of getting 6 events assuming positive $\alpha_{33}$ to the probability of getting 6 events assuming negative $\alpha_{33}$ is 340 to 1 in favor of the destructive interference. This same type of calculation when made using the best-fit phase shifts to *all* the data will give artificially suppressed odds. This is because the experimental "low value" of the cross section at small angles will exert a strong effect in pulling the constructive interference curve part way down to it. In this case the ratio is only 11 to 1. We do not consider this experiment by itself as conclusive proof that $\alpha_{33}$ is positive, but when considered along with all the other evidence pointing toward an attractive $p_{\frac{3}{2}}$ interaction, the "odds" in favor of a positive $\alpha_{33}$ become overwhelming.

Some of the other evidence comes from the 65-Mev experiments at Columbia[2] and the 40-Mev experiments at Rochester[8] assuming charge independence. This is because the knowledge that $\alpha_{33}$ must become large as shown by the recent results from the Carnegie Institute of Technology[9] makes the Steinberger solution implausible. Also an attractive $p_{\frac{3}{2}}$ interaction is preferred theoretically in order to explain such large values of $\alpha_{33}$ as found in the 150 to 200-Mev region.[9] The values of the small angle scatterings below 30° are 17.9°, 19, 20, 23.4, 23.5, 23.6, 26.5, 27, 27.6, 28.9, 29.5, 29.6, and 29.7.

The earlier 110-Mev counter results of Anderson *et al.*[6] are fairly consistent with these plate results. Their values of $a$, $b$, and $c$ at 110 Mev are $3.6\pm0.7$, $-4.8\pm0.8$, and $7.5\pm1.9$ mb per steradian. Our values are $4.0\pm0.4$, $-3.5\pm0.7$, and $6.8\pm1.3$. At 120 Mev their phase shifts are $\alpha_3=-15°$, $\alpha_{33}=30°$, and $\alpha_{31}=4°$. Ours are $-10.9°$, 27°, and $-3.2°$ at 113 Mev.

The data shown in Table I gave an $M$ value of 11.8 when analyzed by the least squares method. $M$ is the least squares sum in units of the standard deviations of each point. According to statistics the mean value of $M$ should be 6 in the case of 9 experimental points and 3 parameters.[10] Since $M$ is $\chi^2$ distributed with $N=6$ in this case,[10] the probability that this experiment give $M\geqslant 11.8$ is 7 percent. Of course a small amount of $d$-wave might improve this fit, although the odds, 1 in 14, for having a fit as poor as this are not unreasonable. Inspection of Fig. 1 shows that the 9 experimental points alternate above and below the curve. We feel that this large $M$ value is just a statistical oddity. To support this view a second least squares analysis was made. This time the data was divided into 6 equal units of solid angle giving $M=3.2$, which is very close to the expected mean value of 3 (for the case of 6 points and 3 parameters). Statistics would have to be improved in order to be sure of seeing any $d$-wave.

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[8] J. Tinlot and A. Roberts, Phys. Rev. **95**, 137 (1954).

[9] Blaser, Ashkin, Feiner, Gorman, and Stern, Phys. Rev. **95**, 624 (1954).

[10] H. Cramer, *Mathematical Methods of Statistics* (Princeton University Press, Princeton, 1946), Chap. 37.

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# Method of Transition Probabilities in Quantum Mechanics and Quantum Statistics

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The concepts microscopic reversibility, detailed balancing, equal probabilities, equilibrium, $H$ theorem are often used in (coarse grained) quantum statistics. It is pointed out that each has a (fine grained) quantum-mechanical counterpart. These concepts are formulated, and their interrelations discussed, at an abstract mathematical level. The results are then interpreted by taking first the fine grained and then the coarse grained point of view. A type of transition matrix arises which does not seem to have been investigated so far, and some of its properties are discussed.

## 1. INTRODUCTION

TWO starting points for quantum statistical mechanics and the statistical foundation of thermodynamics are often used. In the first one appeals at the outset to some principle of equal *a priori* probabilities and random *a priori* phases without making any attempt to establish these from more fundamental considerations. These assumptions make possible the introduction of ensembles into the theory. The microcanonical ensemble enables one to prove an $H$ theorem for a perfectly isolated system in the following sense: if the ensemble is set up in accordance with an initial observation at a time $t=0$, when the value of $H$ for the ensemble is $H(0)$, then, at all later times $t$, $H(t)\leq H(0)$. The statement "$dH/dt\leq 0$ at all times $t$" is definitely stronger, and, as far as we know, it has never been established from this kind of argument. By identifying $H$ as a negative multiple of the entropy ($S$) one has here a restricted quantum statistical proof of the principle of the increase of entropy with time in a perfectly isolated system. The restriction resides in the fact that $S(t)\geq S(0)$ is weaker than $dS/dt\geq 0$. It

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