Isomeric levels with long lifetimes are rare in even-even nuclei; only about three are known and these are highly excited states.[28] All proposed nuclear models predict, in agreement with this fact, that the low-lying levels in such nuclides differ by only one or two units of spin. On the other hand, numerous odd-odd isomers are known.[28] Nordheim's[24] coupling rules, as mentioned before, favor a spin of 0 or a high spin for the ground state of Ho^{164}. If the high spin is obtained, then an isomeric level with spin about 1 would be required to permit the 36.7-min decay. Low-lying isomeric states in odd-odd nuclei are, in fact, predicted by the strong-coupling collective model of Bohr and Mottelson.[19] The parity of the isomeric state would be the same as that of the ground state in this case.

Of interest with regard to these considerations is the work of Butement[29] on Ho^{166}. This observer reports an activity with half-life greater than 30 years, in addition to the well known 27-hour decay. This indicates the probability of an isomeric pair similar to that hypothesized here for Ho^{164}. A long-lived activity decaying to the 90- and the 73-kev γ-ray emitting states was searched for in the present work with a source which had been irradiated for nine hours in the betatron. No evidence was found to support such a long-lived decay. However, it is felt that the inefficient method of activation here employed does not preclude its existence. Butement, in the course of his work, irradiated Ho with fast neutrons but found no long-lived activity which could not be attributed to neutron capture and Ho^{166}.

In conclusion, the 72.8- and 90.5-kev levels shown in the decay scheme are thought to be reasonably well established, as well as their spins and parities. The 110-kev state in Dy^{164} is probably correct but not absolutely forced by the present work. For instance, the existence of a level at about 36.5 kev would give rise to two gammas of almost the same energy. It would be difficult to completely rule out this possibility by the data described above, but the relative intensities in the γ-γ coincidence experiment are more consistent with the scheme proposed here. Furthermore, such a low-lying state (36.5 kev) has no precedent in the regular behavior of the many other even-even nuclei. The 36.7-min isomeric transition shown is not at all well established and must be considered to be highly tentative.

[28] M. Goldhaber and R. D. Hill, Revs. Modern Phys. **24**, 179 (1952).

[29] F. D. S. Butement, Proc. Phys. Soc. (London) **A65**, 254 (1952).

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Interaction of Heavy Primary Cosmic Rays in Lead*

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The energy spectrum of heavy primary cosmic-ray nuclei was measured and found to be in agreement with the assumption that no primaries with energy below the geomagnetic cutoff enter the earth. The mean free path of the primary nuclei in lead was measured. If the transparency of nuclear matter is taken into account, the present results for lead and the measurements by others for lighter elements are in good agreement. If the expression $R_A=R_0\times A^{\frac{1}{3}}$ is used for the radius of the nucleus, the data indicate for R_0 the value 1.3×10^{-13} cm. It is also shown that the ratio of the abundances of the charge groups $Z=12$ to 17 and $Z=18$ to 28 is $\sim$2.1.

I. INTRODUCTION

An "emulsion cloud chamber" consisting of lead plates and photographic emulsions was flown[1] to an altitude of approximately 100 000 feet for about 10 hours. The box was made of aluminum, inside dimensions 4 in.$\times$6 in.$\times5\frac{3}{4}$ in. In its walls $\frac{1}{8}$-in. grooves were cut so that 4 in.$\times$6 in.$\times\frac{1}{8}$ in. lead plates could slide in. The grooves were spaced accurately to within 0.002 in. and the lead plates were machined to the same accuracy.

Number G5 photographic emulsions, 100μ thick, mounted on 4-in.$\times$6-in. specially cut glass plates, were inserted below each lead plate. The position of a single particle going from plate to plate in the stack could be predicted to within 50–100μ. The total number of plates was 22. (In cases of distorted emulsions it was found that the correct angle to be used for the prediction of the position is that given by the part of the track near the air surface. Apparently, in the drying stage, the surface layers dry uniformly, and thus preserve the true angle.)

An ordinary Leitz binocular research microscope

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[1] The balloon flight was conducted by the Aero-Medical Field Laboratory, Hollaman Air Force Base, New Mexico (41.7°N geomagnetic latitude) on July 13, 1952.

clamped down on a steel base was used for following the tracks. Two depth gauges were connected to the microscope stage so that the x and y motions could be measured accurately to within 2 microns.

This arrangement was primarily used in studying the interactions of the heavy primaries. The top plate was scanned for primaries of charge $Z \geq 6$. The particles were followed through the entire "cloud chamber" until they left the stack or else suffered a nuclear interaction. In the latter case, the event was examined in more detail. Five hundred and eight primaries were traced, traversing a total amount of about 16 meters of lead. Of these, about 300 left the stack without interacting, and about 200 suffered nuclear interactions.

II. THE ENERGY SPECTRUM AND THE MAGNETIC CUTOFF

The energy of the interacting particles was estimated in the following way. The interaction of a heavy primary with the target nucleus gives rise to an ordinary star in the rest system of the incident particle. Such a star contains visible prongs which are, in general: a recoil, evaporation tracks (α particles and protons), "grey" tracks (protons of up to 200 Mev), and mesons, if sufficient energy is available. In the laboratory system the evaporation tracks will form a shower traveling in the forward direction. The average evaporation energy, T_α, of α particles from nuclei of charge $Z \sim 12$ is taken from Perkins'[2] work as ~ 10 Mev. The root-mean-square angle in the laboratory system of the α particles with the direction of motion of the primary is given[3] by:

$$\langle \Theta^2 \rangle^{\frac{1}{2}} = [\langle T_\alpha \rangle M / 3p^2]^{\frac{1}{2}} = 0.056/p \text{ radians},$$

where M is the proton mass and p the momentum per nucleon in Bev/c of the incident particle. (We call this the "opening angle" of the α shower.) Since no evaporation α particles are emitted with energy over 30 Mev,[2] the primary momentum cannot be underestimated by more than a factor of two.[3] In several cases the shower contained a heavy fragment (the "recoil" in the rest system) which stopped in the stack. In these cases it was possible to determine the energy accurately by the known range-energy relations, and to compare it to the energy obtained by the α-shower method. The agreement is quite good, as shown in Table I.

Since the cross section does not depend strongly upon the energy in the energy range considered here,[4] the sample of particles which suffered nuclear interactions is a random one, and therefore represents the energy spectrum of the heavy primaries.

The energy at the top of the atmosphere was obtained by adding to the value of the energy deduced from the analysis of the interaction, the energy lost in the residual atmosphere and in the apparatus up to the point of interaction, assuming that the particle is a primary particle. This assumption is indeed not justified, since probably about 40 percent of the ions observed in the stack (see Sec. IV) are heavy fragments of heavier primaries. Therefore, the energy loss in the atmosphere, and hence the energy at the top of the atmosphere, will occasionally be underestimated. Figure 1 shows the differential energy spectrum obtained. In about 20 percent of the cases, the energy is below the assumed geomagnetic cutoff (1.5 Bev/nucleon). However, using the fragmentation probabilities as given in Sec. IV, and the energy spectrum obtained beyond[5] $E = 1.5$ Bev per nucleon, one can predict that indeed about 20 percent of the particles will have to be found with energy below the cutoff. Our data, therefore, agree with the assumption that all primaries at latitude 41.7°N enter the earth with energy above 1.5 Bev per nucleon.

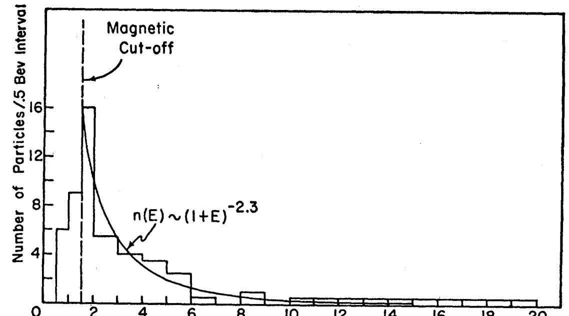

FIG. 1. Differential energy spectrum at the top of the atmosphere obtained by the α-shower method from opening angle measurements.

The spectrum obtained can be represented quite well by

$$n(E)dE \propto (1+E)^{-2.3}dE,$$

(where E is the kinetic energy in Bev/nucleon) and agrees with the integral spectrum obtained by Kaplon *et al.*[3]

TABLE I. Comparison between the energy obtained with the α-shower method and the residual-range method.

Primary	Shower particles	Energy in Bev/nucleon Opening angle method	Residual range method
Fe^{26} [a]	$O^8+3\alpha+p$'s	0.69	0.40
Ca^{20}	$Al^{13}+3\alpha+p$	0.23	0.20
Mg^{12}	$N^7+\alpha+p$'s	0.14	0.23
Cr^{24}	$S^{16}+3\alpha+p$'s	0.33	0.57

[a] Reported by Kaplon *et al.* (see reference 3).

[2] D. H. Perkins, Phil. Mag. **41**, 138 (1950).
[3] Kaplon, Peters, Reynolds, and Ritson, Phys. Rev. **85**, 295 (1952).
[4] M. F. Kaplon (private communication).

[5] The shape of the spectrum beyond $E = 1.5$ Bev per nucleon will not be affected appreciably by the correction due to the fragmentation.

III. THE MEAN FREE PATH AND ITS RELATION TO THE SIZE OF NUCLEI

An interaction was defined in the present work as a collision which results in removal of one or more charge units from the primary particle.

There are examples of collisions in which no apparent change in the charge was observed.[6] Such collisions could not be detected by us, but they are quite rare, hence they do not affect our results.

The mean free path of an incident nucleus A in a target of mass number A' is

$$\lambda_A(A') = A'/[N\sigma_A(A')]\ \text{g/cm}^2, \qquad (1)$$

where N is Avogadro's number, and $\sigma_A(A')$ is the cross section of particle A in the target A'. Assuming a geometric cross section (namely, 100 percent "black" nuclei), and taking radii of nuclei to be $R_A = R_0 \times A^{\frac{1}{3}}$, we obtain

$$\sigma_A(A') = \pi(R_A + R_{A'})^2 = \pi R_0^2 (A^{\frac{1}{3}} + A'^{\frac{1}{3}})^2. \qquad (2)$$

If the target is composed of more than one element, the mean free path will be

$$\lambda(\text{cm}) = 1/(\textstyle\sum_i \sigma_i n_i) = \{\pi R_0^2 N \sum_i m_i (A_i^{\frac{1}{3}} + A^{\frac{1}{3}})/A_i\}^{-1},$$

where m_i is the density in g/cm^3 of the element A_i of the target. The average atomic number of the target, $\bar{A}$, will depend upon A and is given by:

$$m(\bar{A}^{\frac{1}{3}} + A^{\frac{1}{3}})^2/\bar{A} = \textstyle\sum_i m_i (A_i^{\frac{1}{3}} + A^{\frac{1}{3}})^2/A_i,$$

where m is the density of the target, $m = \sum_i m_i$.

The mean free path of cosmic-radiation heavy primaries in glass was measured by Bradt and Peters[5] and in brass by Kaplon *et al.*[3] Both measurements indicated that the observed mean free path is longer than the expected one, obtained by using expression (2) above, and by using for R_0 the value 1.45×10^{-13} cm, which was widely accepted in 1951–1952. Recent experiments[7] indicate that the value of R_0 is about 1.2×10^{-13} cm, 20 percent smaller than it was believed to be.

In Fig. 2 the mean free paths we obtained are plotted *vs* the mass number A of the incident particles. The present experimental points fall on the theoretical curve obtained by using $R_0 = 1.2\times10^{-13}$ cm, and assuming geometric cross sections. The interpertation of these results is not so clear, since it is known that nuclei are not completely "black." In the following calculations we have attempted to estimate the expected mean free path, taking into account the finite free path of a nucleon inside the nucleus. For a 100 percent "black" nucleus the cross section may be written as

$$\sigma = \int_0^{R_1+R_2} 2\pi b\,db = \pi(R_1+R_2)^2.$$

Assuming the nucleus to be a uniform sphere, one may introduce the transparency of nuclear matter by writing the cross section as

$$\sigma = 2\pi \int_0^{R_1+R_2} [1 - P(b)] b\,db,$$

where $P(b)$ is the probability that for the impact parameter, b, none of the nucleons of A_2 will interact with any of the nucleons of A_1 (see Fig. 3). Such probability is

$$P(b) = \exp[-X(b)/\lambda],$$

where $X(b)$ is the integrated path length of all the nucleons of A_2 inside A_1 for the given impact parameter b, and λ is the mean free path of a nucleon in nuclear matter. We have used for λ the value[8]

$$\lambda = 5\times10^{-13}\ \text{cm},$$

which was derived from the analysis of nucleon-nuclei scattering experiments at machine energies. Lambda seems to be reasonably constant in the energy range from 0.2 to 1.5 Bev. The function $X(b)$ is given by

$$X(b) = \int_{b-R_1}^{R_2} \int_{-\Theta_0}^{\Theta_0} 4\rho r_2 (R_2 - r_2)^{\frac{1}{2}} \times (R_1^2 - r_2^2 - b^2 + 2r_2 b \cos\Theta)^{\frac{1}{2}} dr_2 d\Theta,$$

where Θ_0 is determined by

$$\cos\Theta_0 = (b^2 + r_2^2 - R_1^2)/(2br_2),$$

and R_1 and R_2 are the radii of the target and the incident nucleus, respectively. For a uniform distribution of nucleons, ρ, the density of nuclear matter, is $(4\pi R_0^3/3)^{-1}$. $X(b)$ was tabulated by numerical integration. It is a function of R_1 and R_2, hence of A_1 and A_2. For $A_2 = 25$ and $A_1 = 207$, 64, and 31, the

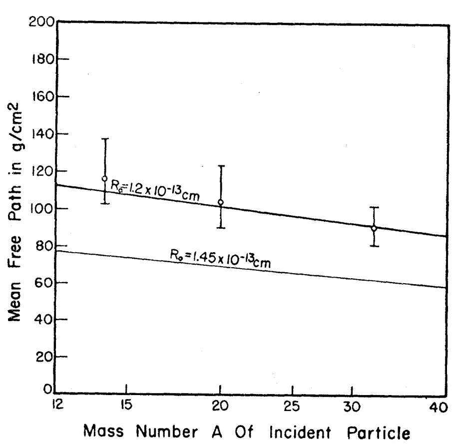

FIG. 2. The interaction mean free path in lead *vs* the mass number of the incident particles.

[6] H. Bradt and B. Peters, Phys. Rev. **77**, 54 (1950); H. Bradt and B. Peters, Phys. Rev. **75**, 1779 (1949); and J. Noon (private communication).

[7] F. Bittner and H. Feshbach, Phys. Rev. **92**, 837 (1953).

[8] T. Taylor, Laboratory of Nuclear Studies, Cornell University (private communication).

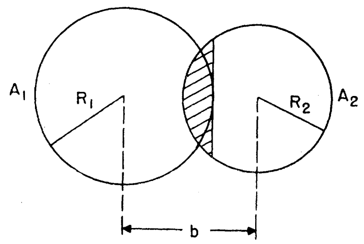

FIG. 3. Model used for calculating the mean free path and the fragmentation probability.

reduction in the cross section due to the transparency as introduced above comes out to be 16 percent, 23.2 percent, and 25.1 percent, respectively.

In Fig. 4 we plotted the calculated cross sections and the observed ones for incident primaries of $\bar{A}=25$, *vs* the mass number of the target nuclei. With this model of a transparent nucleus, the best fit to the experimental data is obtained if one chooses $R_0=1.3\times10^{-13}$ cm. This value of R_0 for a real "grey" nucleus is perhaps more significant than the value $R_0=1.2\times10^{-13}$ cm which would give the best fit to the data if one idealizes nuclei to be 100 percént "black."

Two additional experimental points have been obtained recently at the University of Rochester.[9] These are the mean free path in emulsion ($\bar{A}$ emulsion≈ 38) of: Medium Group

$$(6\leqslant Z\leqslant 10),\quad \bar{A}_M=14,\quad \lambda_M=59.6\pm6\ \text{g/cm}^2,$$

Heavy Group

$$(10<Z),\qquad \bar{A}_H=32,\quad \lambda_H=36.5\pm4.8\ \text{g/cm}^2.$$

If a correction is made for the difference in A_{Av} of the incident particle, λ_H comes to be in agreement with the predicted value (circled point in Fig. 4), but λ_M seems to point to a much smaller value of R_0, namely $R_0=1.0\times10^{-13}$ cm, if the 100 percent "black" nucleus is assumed. However, taking into account the transparency in calculating λ_M and using $R_0=1.3\times10^{-13}$ cm, we have obtained $\lambda_M=52.5$ g/cm^2, in agreement with the observed value. The reduction in the cross section in this case is 31.5 percent.

In conclusion we may say that, taking into account the transparency of nuclear matter, and using $R_0=1.3\times10^{-13}$ cm, the calculated mean free paths agree with the observed ones for incident particles of $A=14$ to 32 and targets of mass numbers $A'=23$ to 207.

IV. RELATIVE ABUNDANCE OF COSMIC-RAY PRIMARIES

In order to extrapolate the relative abundance of the elements observed in the stack (under 25 g/cm^2 on the average from the top of the atmosphere) to the top of the atmosphere, the probability of fragmentation by collision with air nuclei had to be estimated. The fragmentation probability will depend upon (1) the size of the target nucleus A_1 and of the incident nucleus A_2, and (2) the energy of the incident nucleus. In general, the collision will involve "cutting" of a part of the incident particle (removal of the nucleons which are in the overlapping region, shaded area in Fig. 3) and subsequent evaporation of nucleons from the residual nucleus. In the extreme relativistic case, $E\gg Mc^2$, the side-scattering of the nucleons during the collision may be neglected. Therefore, the evaporation energy will come from the friction energy (breaking of the bonds between neighboring nucleons) and the change in the surface energy.[10] For an impact parameter b, the fractional number of nucleons contained in the overlapped region and lost in the "cutting" is

$$f_1=\tfrac{1}{4}[3(h/R_2)^2-(h/R_2)^3],$$

where h is the amount of overlap: $h=R_1+R_2-b$ if $R_1-b\leqslant R_2$, and $h=2R_2$ if $R_1-b\geqslant R_2$. The probability for such a collision is

$$P(h/R_2)=\frac{2[(R_1/R_2)+1-\frac{1}{2}(h/R_2)]h/R_2}{[(R_1/R_2)+1]^2}.$$

The friction energy is given by

$$U_F=1.5R_2^2[2(h/R_2)-(h/R_2)^2]\ \text{Mev},$$

and the surface energy is

$$U_S=4.12\pi R_2^2\{1-\tfrac{1}{4}(h/R_2)^2-[1-\tfrac{3}{4}(h/R_2)^2+\tfrac{1}{4}(h/R_2)^3]^{\frac{2}{3}}\}\ \text{Mev}.$$

The total excitation energy of A_2 is $U=U_F+U_S$. The average number of nucleons emitted in the evaporation process is[11]

$$N(U)=6.4\times10^{-2}U-2.4,\qquad U>100\ \text{Mev}$$
$$N(U)=4.5\times10^{-2}U\qquad U<100\ \text{Mev}.$$

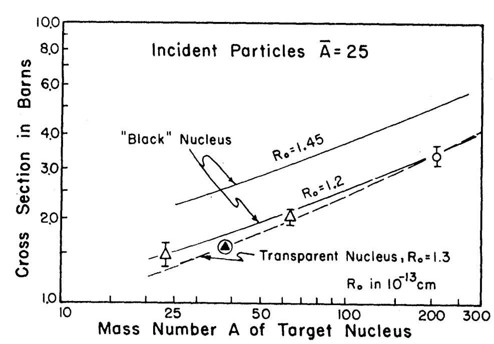

FIG. 4. The cross section for incident particles of $\bar{A}=25$ *vs* the mass number of the target used. Circle: present experiment; triangle: results of the Rochester group; solid line: calculated geometric cross section; and broken line: cross section calculated by using the transparent nucleus model.

[9] J. Noon (private communication).

[10] We have followed here the calculations of W. Heitler and Ch. Terreaux, Proc. Phys. Soc. (London) **A66**, 929 (1953).

[11] K. J. Le Couter, Proc. Phys. Soc. (London) **A63**, 259 (1950).

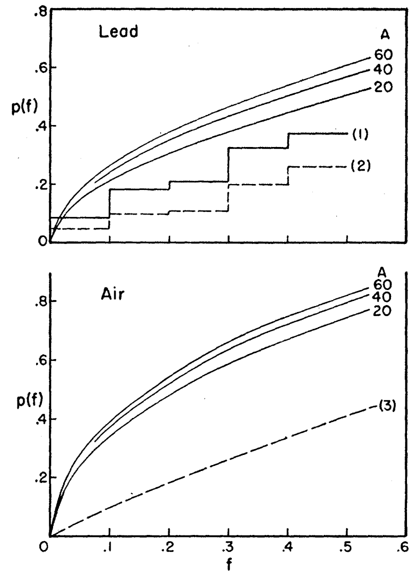

FIG. 5. The fragmentation probability for lead and air. $P(f)$, the probability of losing up to the fraction f of the number of nucleons in a collision, is plotted *vs* f.

The results of the above calculations are shown in Fig. 5. We have plotted $P(f)$, the integral probability of losing up to a fraction f of the number of nucleons, *vs* f for incident particles of $A=20$, 40, and 60, in lead and air targets. Histogram (1) (Fig. 5) represents 43 cases of particles interacting in lead, whose kinetic energy was $E\geqslant 1.3$ Bev/nucleon. Histogram (2) represents 92 cases of energy $E\geqslant 0.2$ Bev/nucleon. In both cases, the charge of the particles was $Z\geqslant 12$. One can see that indeed the higher the energy, the closer we get to the expected curve for the extreme relativistic case.

The expected curves for the extreme relativistic limit in air are about 40 percent higher than those for lead. We have, therefore, assumed that the fragmentation probability in air for the sample of particles which we observed will also be about 40 percent above the corresponding values in lead. Thus, curve (3) was drawn to be about 40 percent above the average of (1) and (2), and was assumed to represent the fragmentation probability in air.

The data were divided into two charge groups: group A, $Z=12$ to 17 and group B, $Z=18$ to 28. The natural abundance of groups A and B in the universe[12] is about the same. It is believed that the relative abundance of the higher charge group, group C, of charge $Z>28$, is much smaller. Let $A(x)$, $B(x)$, and $C(x)$ be the number of particles of groups A, B, and C, respectively, at the depth $x(\mathrm{g/cm^2})$ from the top of the atmosphere. Let P_{BA} be the probability that an interacting particle of group B will have as one of its interaction products, a particle of group A. P_{AA}, P_{BB}, and P_{CB} are defined similarly. Then we get

$$\text{(a)}\quad dA(x)=-dx[A(x)/\lambda_A-P_{BA}B(x)/\lambda_B-0.55P_{AA}A(x)/\lambda_A]$$

and

$$\text{(b)}\quad dB(x)=-dx[B(x)/\lambda_B-P_{CB}C(x)/\lambda_C-P_{BB}B(x)/\lambda_B].$$

Forty-five percent of group A is presumably Mg^{12}, which leaves the group upon interaction. Therefore the factor 0.55 was introduced in (a) above. P_{AA}, P_{BA}, and P_{BB} were taken from curve (3) of Fig. 5, namely

$$P_{AA}=P(0.2)=0.20,$$
$$P_{BA}=P(0.55)-P(0.3)=0.20,$$
$$P_{BB}=P(0.3)=0.25$$

$C(x)$ can be neglected; thus (a) and (b) can be solved immediately and the relative abundance is found to be $A(0)/B(0)=2.1$. It should be noted however, that approximately the same relative abundance is obtained (by chance) if we do not introduce any fragmentation correction at all and use simple exponential decay, namely

$$A(x)=A(0)\exp(-x/\lambda_A),$$

and

$$B(x)=B(0)\exp(-x/\lambda_B).$$

The author is greatly indebted to Professor G. Cocconi for his continuous help during the analysis of the data, and for many invaluable discussions of the results obtained.

Most of the scanning work was done by Mrs. P. Eliash, Mr. J. Arkin, and Mrs. S. Eisenberg. Their effort is greatly appreciated.

[12] R. A. Alpher and R. C. Herman, Revs. Modern Phys. **22**, 153 (1950).