$\theta$ and of some parameter $\tau$ which determines the spread of the distribution function. Upon recalling that diffusion is a random walk process, we recognize that the distribution function $RW(\theta,\tau)$ will obey the standard diffusion differential equation if we associate $\tau$ with time, or more appropriately, with the dimensionless quantity $tD/a^2$, where $D$ is the diffusion coefficient, $a$ the radius of the spherical surface upon which diffusion is imagined to be occurring. Thus, we have

$$\frac{\partial}{\partial\tau}RW(\theta,\tau)=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\sin\theta\frac{\partial}{\partial\theta}RW(\theta,\tau). \tag{9}$$

The solution of this differential equation, subject to the boundary condition that $RW(\theta,\tau)$ approaches a delta function about $\theta=0$ as $\tau\rightarrow 0$, is

$$RW(\theta,\tau)=\sum_{n=0}^{\infty}\left\{\int_{-1}^{1}P_n^2(\cos\theta)d\cos\theta\right\}^{-1}\times e^{-n(n+1)\tau}P_n(\cos\theta). \tag{10}$$

We thereby obtain

$$\int_{-1}^{+1}RW(\theta,\tau)P_n(\cos\theta)d\cos\theta=e^{-n(n+1)\tau}, \tag{11}$$

or

$$\langle P_n(\cos\theta)\rangle_{RW}=e^{-n(n+1)\tau}, \tag{12}$$

where $\langle\ \rangle_{RW}$ denotes an average with respect to the random walk function. We now eliminate the parameter $\tau$ by observing that

$$J_s(T)=J_s(0)\langle P_1(\cos\theta)\rangle_{RW}, \tag{13}$$

which equation, combined with (12) leads to

$$\langle P_n(\cos\theta)\rangle_{RW}=\{J_s(T)/J_s(0)\}^{n(n+1)/2}. \tag{14}$$

Upon combining Eqs. (8) and (14), we obtain our final equation,

$$\langle S_n(\alpha_1,\alpha_2,\alpha_3)\rangle_{RW}=\{J_s(T)/J_s(0)\}^{n(n+1)/2}S_n(\bar{\alpha}_1,\bar{\alpha}_2,\bar{\alpha}_3). \tag{15}$$

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# Angular Distribution of 12- and 16-Mev Gamma Rays from the Proton Bombardment of a Thin Boron Target*

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The angular distribution of 12- and 16-Mev gamma rays from proton bombardment of thin boron targets has been obtained for proton energies ranging from 0.6 to 2.0 Mev. The angular distribution may be represented by the equation $w(\theta)=1+A\cos\theta+B\cos^2\theta$, indicating interference between at least two levels of opposite parity. An analysis of the energy dependence of the coefficients $A$ and $B$ for the 16-Mev gamma rays indicates the interference to be between more than two levels.

## INTRODUCTION

GAMMA-RAY resonances in the yield from the proton bombardment of boron have been observed by various investigators at 163,[1] 680,[2,3] 1388, 2650,[4] and 3550 kev,[5] all except the first having very large widths. The excited states of $C^{12}$ may decay directly to the ground state with the emission of gamma rays of energy equal to the excitation energy ($Q=15.949$ Mev) or to the 4.43-Mev excited state and then to the ground state with the emission of cascade radiation. These gamma-rays are called, in the following, the "16-Mev," "12-Mev," and "4.43-Mev" radiations. Earlier work[6] has shown a $\cos\theta$ term in the angular distribution of the gamma radiation from this reaction over the energy region from 300 to 1100 kev. This indicates interference between at least two states of opposite parity. Since the spin and parity are known for the $C^{12}$ level corresponding to the 163-kev resonance, this investigation of the angular distribution of the 16-Mev gamma rays was undertaken so that the spins and parities of the interfering levels could be determined. Although the theoretical analysis of the angular distribution of the 12-Mev gamma rays is more complex, the experimental determination of this angular distribution was also undertaken since the spin and parity of the 4.43-Mev excited state of $C^{12}$ are known.

## EXPERIMENTAL PROCEDURE

Protons were accelerated in the University of Kentucky 3-Mev electrostatic accelerator to energies

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† Now at Virginia Polytechnic Institute, Blacksburg, Virginia.
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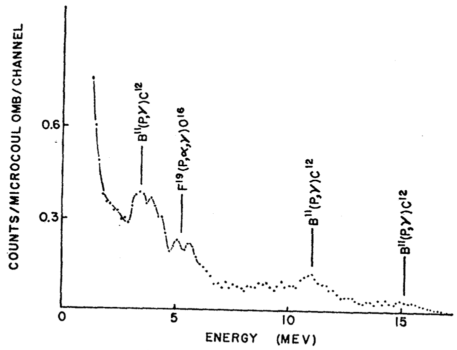


FIG. 1. Gamma-ray spectrum from the proton bombardment of boron at 0.873 Mev. The gamma rays from fluorine are caused by fluorine contamination of the target backing.

ranging from 600 to 2000 kev, with an energy resolution of 0.25 percent. Beam currents of from 1 to 5 microamperes were used.

The gamma-ray detector was a NaI(Tl) crystal $1\frac{1}{2}$ inches in diameter and 1 inch thick mounted on the face of a selected Dumont 6292 photomultiplier tube. The crystal, surrounded by magnesium oxide,[7] was housed in a thin-wall aluminum case and slipped over the end of the photomultiplier tube. The detector had an energy resolution of 10 percent at 0.661 Mev. The front face of the crystal was 7 cm from the target, and the detector could be rotated about an axis through the target and perpendicular to the beam direction to 135° on either side of the beam direction. The photomultiplier tube was magnetically shielded and operated at a potential of 750 volts supplied by series-connected 45-volt batteries. The pulses from the photomultiplier tube were sorted by a twenty-channel pulse-height analyzer.

The angular distribution data were obtained using a thin boron film deposited on tantalum backing by the diborane process.[3] The target was made available by the Van de Graaff group at the California Institute of Technology.

By use of the twenty-channel pulse-height analyzer it was possible to obtain the gamma-ray spectrum at each of seven angles (225°, 270°, 315°, 0°, 45°, 90°, and 135°) and at proton bombarding energies of 600, 700, 800, 1000, 1200, 1400, 1600, 1800, and 2000 kev. The projection of the beam through the target is defined as 0°. Figure 1 shows a spectrum taken at 0° for a proton bombarding energy of 873 kev.

Inasmuch as the degree of anisotropy in the distribution was not large it was most important that all extraneous sources of anisotropy be eliminated. The symmetry of the target-detector geometry was carefully checked by mechanical measurements and using radioactive sources.

## EXPERIMENTAL RESULTS

The twenty-channel pulse-height analyzer was adjusted such that the twenty channels covered the energy region from approximately 8 to 17 Mev. This adjustment was maintained throughout the experiment. The upper seven channels covered the region above 14 Mev. After absorption and finite solid angle corrections, the variation in the total count from these seven channels as a function of angle and energy yielded the angular distribution of the 16-Mev gamma rays.

The lower seven channels of the analyzer covered the the energy region between 8 and 11 Mev, the plateau seen just below the 12-Mev pair peak in the spectrum of Fig. 1. The total count in these seven channels was caused by both 16- and 12-Mev radiation, and it was necessary to separate the two contributions. Campbell and Boyle[8] have theoretically determined the energy resolution up to 18 Mev of NaI(Tl) scintillation counters. Their results are in good agreement with experiment. Using their results, the contribution from the 16-Mev gamma rays to the portion of the spectrum between 8 and 11 Mev was obtained and subtracted from the total count in the lower seven channels. The remainder was attributed to 12-Mev gamma radiation, and from its variation with angle and energy the angular distribution was obtained.

The absorption of 16-Mev gamma rays caused by the 10-mil tantalum target backing was calculated theoretically[9] and found to be in excellent agreement with experimental results. An absorption correction of

TABLE I. Coefficients in the expression $W(\theta)=1+A\cos\theta+B\cos^2\theta$ for the yield of 16-Mev gamma rays from the proton bombardment of boron.

| $E_p$ (Mev) | $A$ | $B$ |
|---|---|---|
| 0.600 | $-0.01\pm0.01$ | $0.09\pm0.02$ |
| 0.700 | $-0.04\pm0.01$ | $0.15\pm0.03$ |
| 0.800 | $-0.02\pm0.01$ | $0.16\pm0.02$ |
| 1.000 | $-0.05\pm0.01$ | $0.13\pm0.02$ |
| 1.200 | $0.01\pm0.01$ | $0.20\pm0.02$ |
| 1.400 | $0.05\pm0.02$ | $0.26\pm0.04$ |
| 1.600 | $-0.01\pm0.01$ | $0.27\pm0.02$ |
| 1.800 | $0.02\pm0.01$ | $0.26\pm0.02$ |
| 2.000 | $0.05\pm0.02$ | $0.16\pm0.04$ |

TABLE II. Coefficients in the expression $W(\theta)=1+A\cos\theta+B\cos^2\theta$ for the yield of 12-Mev gamma rays from the proton bombardment of boron.

| $E_p$ (Mev) | $A$ | $B$ |
|---|---|---|
| 0.600 | $0.18\pm0.01$ | $0.14\pm0.02$ |
| 0.700 | $0.12\pm0.02$ | $0.01\pm0.04$ |
| 0.800 | $0.19\pm0.02$ | $0.18\pm0.03$ |
| 1.000 | $0.39\pm0.03$ | $0.33\pm0.05$ |
| 1.200 | $0.50\pm0.03$ | $0.44\pm0.05$ |
| 1.400 | $0.48\pm0.04$ | $0.45\pm0.08$ |
| 1.600 | $0.60\pm0.05$ | $0.33\pm0.09$ |
| 2.000 | $0.47\pm0.03$ | $0.76\pm0.06$ |

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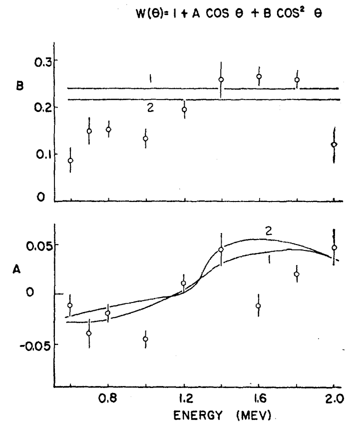


FIG. 2. Energy dependence of the coefficients $A$ and $B$ in the expression $W(\theta)=1+A\times\cos\theta+B\cos^2\theta$ for the yield of 16-Mev gamma rays from the proton bombardment of boron. Curves 1 and 2 show the poor agreement obtained under the assumption that the two interfering levels are 163 kev (2+) and 1388 kev (1−) or 1388 kev (1−) and 3550 kev (2+), respectively.

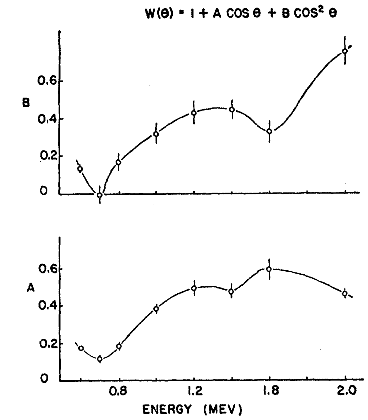


FIG. 3. Energy dependence of the coefficients $A$ and $B$ in the expression $W(\theta)=1+A\times\cos\theta+B\cos^2\theta$ for the yield of 12-Mev gamma rays from the proton bombardment of boron.

3.4, 2.4, and 2.4 percent was applied to the data at 0°, 45°, and 315°, respectively.

In the case of the 12-Mev gamma rays, the total count in the chosen seven channels is reduced because of absorption in the target backing at the angles 315°, 0°, and 45° and is increased because of the degradation in energy of some of the 16-Mev gamma rays. The correction for the latter was found to be negligible. The calculated correction for the former is 2 percent. Since the uncertainty in the contribution from the 16-Mev gamma rays is of this order of magnitude, this correction was not actually applied.

A least-squares fit of the equation $W(\theta)=1+A\cos\theta+B\cos^2\theta$ to the data was made following the method of Rose.[10] Solid angle corrections were made and the angular distributions transformed to center-of-mass coordinates. Values of the coefficients for various bombarding energies are shown in Tables I and II.

## DISCUSSION

A comparison of the coefficients with the values obtained by other investigators shows good agreement with the work of Gove and Paul,[4] Jenkins,[11] and Cross.[12] The results do not agree with those of Glattli and Stoll[13] at low bombarding energies. The energy dependence of the coefficients is clearly indicated in Figs. 2 and 3.

A theoretical analysis of the angular distributions of the 16-Mev radiation was made following the method of Devons and Hine[14] with the assumption of two-level interference. Curves 1 of Fig. 2 indicate the theoretical variation of the coefficients with energy under the assumption[3] that the interfering levels are 163 kev with spin 2 and even parity (2+) and 1388 kev with spin one and odd parity (1−). The agreement is not within the experimental errors. Curves 2 in Fig. 2 indicate the theoretical results for the assumption of interference between the 1388-kev level (1−) and the 3550-kev level (2+). Various choices of the values of the parameters $t$ and $\delta$, in the notation of Devons and Hine, did not materially alter this disagreement.

Calculations were made for other combinations of the 1388, 2650, and 3550-kev levels for two-level interference. The results are shown in Table III. None of the calculations agreed within experimental errors. It is, therefore, concluded that more than two interfering levels are involved in the problem.

TABLE III. Calculated values of the coefficients in the expression $W(\theta)=1+A\cos\theta+B\cos^2\theta$ for the yield of gamma rays from the proton bombardment of boron. Interference between two levels is assumed, and various choices of spin and parity are made for the levels corresponding to bombarding energies of 1.388, 2.650, and 3.550 Mev.

| $E_p$ (Mev) | 1.388 (1−) 2.650 (2+)[a] | | 1.388 (2+)[a] 2.650 (1−) | | 1.388 (2+) 3.550 (1−) | |
|---|---|---|---|---|---|---|
| | $A$ | $B$ | $A$ | $B$ | $A$ | $B$ |
| 0.600 | −0.045 | 0.625 | −0.060 | 0.625 | −0.100 | 0.625 |
| 0.700 | −0.035 | 0.625 | −0.048 | 0.625 | −0.073 | 0.625 |
| 0.800 | −0.023 | 0.625 | −0.022 | 0.625 | −0.054 | 0.625 |
| 1.000 | −0.012 | 0.625 | −0.006 | 0.625 | −0.021 | 0.625 |
| 1.200 | −0.005 | 0.625 | 0.000 | 0.625 | 0.000 | 0.625 |
| 1.400 | 0.001 | 0.625 | 0.003 | 0.625 | 0.011 | 0.625 |
| 1.600 | 0.003 | 0.625 | 0.005 | 0.625 | 0.023 | 0.625 |
| 1.800 | 0.003 | 0.625 | 0.008 | 0.625 | 0.032 | 0.625 |
| 2.000 | 0.002 | 0.625 | 0.012 | 0.625 | 0.040 | 0.625 |

[a] See reference 4.

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*Note added in proof.*—Grant, Flack, Rutherglen, and Deuchars [Proc. Roy. Soc. (London) **A417**, 751 (1954)] have obtained angular distribution data in the proton energy region from 175 to 680 kev. They conclude that the 163-kev resonance has $J=2+$ and the 1388-kev resonance has $J=1-$.

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