energy-level diagram for O^{17} is included in Fig. 2, which also shows the 870.5±2.0 kev gamma ray reported by Thomas and Lauritsen.[10]

The authors wish to express their appreciation for the assistance they received in the course of this work from the various members of the ONR Generator Group. We are indebted to Dr. M. M. Elkind, Mr. Walter J. Fader, and Mr. C. M. Braams for their help in obtaining preliminary data. We wish especially to thank those who read the photographic plates, particularly Mr. W. A. Tripp, Miss Janet Frothingham, and Miss Sylvia Darrow.

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Proton-Proton Scattering at 5.77 Mev*

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Absolute differential scattering cross sections for proton-proton scattering at laboratory energies of 5.77 and 5.86 Mev (±1 percent) have been obtained with an accuracy of about one percent at many angles (23° to 110° c.m.) by two independent experiments, one employing nuclear emulsion plates as detectors and the other employing proportional counters. Reduced to the same energy, the average indicated S-wave nuclear phase shift is in excellent agreement with other data for this energy region. However, at small scattering angles, important for determining a P-wave phase shift, the measured cross sections differed originally by 3 to 5 percent, the nuclear emulsions method indicating a P-wave shift of −0.08±0.05 degree, and the original counter data indicating −0.34±0.05 degree. After the beam collimation was improved and the energy spectrum of the incident proton beam was examined, check runs with the counters at six scattering angles failed to indicate the large P-wave effect, giving −0.08±0.07 degree, in agreement with the emulsion data. It therefore appears likely that the P-wave shift at this energy is small (less than 0.1 degree) and negative, in agreement with other determinations in this energy region.

I. INTRODUCTION

Results of the proton-proton scattering experiments carried out over a period of several years at the University of Illinois Cyclotron Laboratory will be reported here. While some of the data have already been published in preliminary form,[1,2] this report gives a complete analysis of the experiments, together with some additional information.

Two separate methods were used to measure the differential scattering cross sections. The two experiments were almost completely independent, only the device for measuring the charge being common to both. One experiment employed the scattering chamber constructed by Rodgers, Meagher, and Leiter[3–5] in which the scattered protons were recorded in nuclear emulsions. The second method used a scattering chamber in which scattered protons were detected by proportional counters used in coincidence to eliminate the effect from background radiation produced by the cyclotron.

A number of studies of proton-proton scattering in the low-energy region (below about 10 Mev) have been made. Reference will be made here only to some fairly recent theoretical analyses which contain references to early theoretical and experimental work.[6–11] Reference will also be made to some very recent experimental papers,[12–17] most of which also contain bibliographies.

II. NUCLEAR EMULSION EXPERIMENT[18]

The scattering chamber, used previously by Rodgers,[3] Leiter,[4] Meagher,[5] and concurrently by Kreger,[19] is constructed so that particles scattered over a wide range of angles are recorded simultaneously on six

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[1] E. J. Zimmerman and P. G. Kruger, Phys. Rev. 83, 218 (1951).
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[17] Worthington, McGruer, and Findley, Phys. Rev. 90, 899 (1953).
[18] The work of this section was aided in part by a grant from the Research Corporation. This section is part of a thesis submitted by R. O. Kerman in partial fulfillment of the requirements for the degree of Doctor of Philosophy at the University of Illinois, 1953.
[19] Kreger, Jentschke, and Kruger, Phys. Rev. 93, 837 (1954).

50-micron Ilford nuclear plates mounted around the periphery of the cylindrical chamber. The slit system is such that all particles striking a certain region of the nuclear plate have been scattered through nearly the same laboratory angle. Consequently the incident angle and range of the particles in the emulsion need not be measured; to obtain cross sections it is only necessary to count all the tracks on a given swath of the plate and to measure precisely all the different dimensions which enter into the geometry.

The construction and operation of the chamber, the details of the various annular slits used, and the method used to measure the total charge are all described by Kreger.[19] The technique by which the uncorrected cross sections are calculated from the nuclear emulsion data is discussed by Leiter.[4]

A. Experimental Procedure

Hydrogen gas was admitted to the chamber through a heated palladium thimble to a pressure of about 5 cm Hg. Liquid nitrogen traps were not used because of the findings of Rouvina[12] concerning their effect on the cross sections. The low-angle annular slit was used in recording protons scattered through a laboratory angle of from 11.5° to 30°, while the high-angle slit was used for the range 21° to 55°. In addition, a background run was taken, and also a run using a "closed slit"[19] (identical with the high-angle annular slit except that the two slit edges were just closed) to check effects due to slit penetration. A drawing of the annular slits and a table of their geometrical dimensions are given in Fig. 3 and Table I, respectively, of reference 19.

The number of incident protons was determined by collecting the undeflected particles in a Faraday cup to which was attached a bank of polystyrene capacitors.[20] A vibrating reed electrometer[21,22] was used to measure the capacitor voltage.

The energy of the incident protons was determined (in connection with a different experiment[19]) by measuring their range in the emulsion of an Ilford C2 nuclear plate, and was found to be 5.80 Mev$\pm$0.7 percent probable error with an actual spread in energy corresponding to a Gaussian distribution with a half-width of about 30 kev. The energy was decreased by 0.03 Mev to allow for the energy loss in the hydrogen gas before scattering occurs. In order to enter the scattering chamber, the proton beam from the cyclotron must be bent through an angle of 37° in an analyzer magnet whose field was stabilized by means of a proton nuclear magnetic resonance device. The proton magnetic resonance frequency was measured for each data run and was used to indicate any small changes in the proton energy.

[20] Manufactured by John E. Fast and Company, Chicago, Illinois.
[21] Manufactured by Applied Physics Corporation, Pasadena, California.
[22] Palevsky, Swank, and Grenchik, Rev. Sci. Instr. **18**, 298 (1947).

B. Determination of Corrected Cross Sections

Four corrections to the measured cross sections are required before comparison with the theory can be made.

1. Penetration Correction

The closed slit run was made to measure experimentally the correction for effects caused by slit penetration in the annular slit. The measured correction agreed with that obtained from the theoretical expression;[3,19] both are listed in Table I. Values of the penetration correction used in the present experiment are given in column 4 of Table II.

An additional check on the validity of the penetration correction was carried out by counting five angles of one of the low angle data runs using two different minimum track lengths. After making the background and penetration corrections, both cross sections should be the same. The cross section obtained by using the smaller minimum track length came out $\frac{1}{4}$ percent higher on the average. This difference was probably not significant, but it did suggest the possibility that some of the short tracks were due to protons which had been scattered out of the emulsion before coming to rest. A recount of these angles using a higher microscope magnification failed to show such an effect. In the error analysis the probable error in the penetration correction was taken to be 15 percent in order to take into account the $\frac{1}{4}$ percent difference.

2. Background Correction

A background run was made to measure the correction needed to account for slit-scattered or impurity-scattered protons which reached the photographic plates. The run was exactly like a low-angle data run, except that no gas was admitted to the chamber. The run was started after a length of time equal to the time required to admit the hydrogen, and was continued for twice the length of a regular data run. The background correction was measured to be less than 0.1 percent except at the three lowest angles, where corrections up to 1.3 percent were required. The tracks comprising the background at the lowest angles were considerably shorter than would be expected from contaminant scattering and were evidently due to penetration through or scattering from one of the anti-scattering baffles in the collimation system. The background correction is recorded in Table II.

To check the high-angle background, two plates in one of the high-angle data runs were covered with aluminum foil sufficiently thick to almost stop protons scattered by protons; protons scattered from heavy nuclei would manifest themselves by their long track lengths. The number of such tracks was negligible, and

TABLE I. Comparison of theoretically and experimentally determined penetration corrections. Values are the percentage ratios of the number of penetration particles to the number of particles passing through the annular slit opening, for a given minimum track length used in counting.

Laboratory angle	23°	25°	30°	35°	40°	45°	50°
Experimental correction (%)	0.50±0.05	0.65±0.06	1.06±0.07	1.09±0.07	1.05±0.06	0.96±0.06	0.58±0.04
Theoretical correction (%)	0.50	0.74	1.10	1.24	1.08	0.89	0.59

no background correction was made for the high angle run.

3. *Second Order Geometry Correction*

The finite size of the proton beam, its divergence, and the variation of angle and cross section over the swath area were neglected in deriving the formula used for calculating the uncorrected cross sections. These were taken into account by considering the scattering from an element in the scattering volume to an element of area in the nuclear plate swath and integrating over the scattering volume and the swath area, as was done by Critchfield and Dodder[23] for a different geometry. The result[24] was an expression consisting of three parts: the first represented a correction to the expression for the solid angle; the second part represented the correction necessary because the average scattering angle was slightly different from the angle determined by a line from the center of the scattering volume to the center of the swath area; and the third part took into account the change in cross section over the angular interval accepted by the detector. The geometry corrections, never greater than 0.9 percent, are listed in Table II.

4. *Energy Correction*

The scattering chamber is so constructed that the position of the scattering volume changes with scattering angle. For example, a particle scattered through 55° (lab) must travel about 14 cm farther along the axis of the chamber before being scattered than a particle scattered through 11.5°. Thus, the energy at the scattering volume changes with scattering angle. In addition, it was found that the incident proton energy was slightly smaller for one of the data runs than for the others. It was therefore necessary to correct the cross sections obtained from that run to the same energy as those obtained from the other runs; and it was also desired to correct the cross sections for the slight energy dependence on the scattering angle. Accordingly, a number of curves of cross section *vs* energy for various laboratory angles from 10° to 45° were plotted for energies up to 7.5 Mev using published results. The slope of each curve was measured graphically at 5.75

TABLE II. Corrections and final cross sections for plates at 5.77 Mev.[a]

Angle (c.m.)	Angular[d] resolution	No. of counts[b]	Penetrat.[e] correction %	Background correction[c] %	Geom. correct., %	Cross section (c.m.) mb/steradian	App. S-wave phase shift
23.00°	±32′	24 427	1.11	0.16	0.13	118.0 ±0.8	55.23±0.48°
			1.89	1.28			
23.87	±32′	11 390	1.27	0.31	0.33	110.0 ±0.9	55.18±0.48
			1.66	0.81			
26.00	±34′	19 558	1.11	0.09	0.63	95.91±0.70	54.75±0.40
			1.96	0.21			
27.85	±37′	10 540	1.70	0.07	0.64	91.73±0.79	55.67±0.40
30.00	±40′	22 651	1.07	0.06	0.67	87.30±0.64	55.68±0.35
31.84	±44′	10 185	1.91	0.05	0.77	83.53±0.73	55.10±0.37
35.00	±46′	22 766	1.43	0.05	0.88	81.98±0.58	55.17±0.32
			2.17	0.05			
40.00	±49′	21 564	1.53	0.05	0.92	82.82±0.60	55.57±0.32
			2.01	0.05			
45.00	±53′	12 391	1.71	0	0.86	81.12±0.67	54.55±0.35
50.00	±56′	31 417	1.0	0	0.78	83.28±0.64	54.98±0.34
54.76	±57′	22 614	1.48	0	0.68	85.00±0.62	55.23±0.36
60.00	±58′	35 480	0.82	0	0.58	85.70±0.65	55.17±0.35
69.70	±58′	12 188	1.23	0	0.38	86.86±0.71	55.08±0.37
79.68	±59′	11 606	1.08	0	0.16	86.73±0.70	54.73±0.38
84.73	±59′	10 733	1.52	0	0.05	86.51±0.72	54.57±0.38
89.68	±60′	10 576	0.87	0	−0.09	87.18±0.70	54.90±0.37
94.71	±63′	11 617	1.08	0	−0.18	87.28±0.68	54.87±0.37
99.68	±52′	10 564	0.66	0	−0.28	87.87±0.70	55.18±0.37
109.69	±52′	10 454	0.41	0	−0.49	86.01±0.69	54.72±0.37

[a] Errors listed are probable errors.
[b] The value listed is the total of all runs counted at that angle.
[c] Two values are listed where two different minimum track lengths were used.
[d] The angular resolution is in the laboratory system.
[e] Values of the penetration correction in Table II differ from those in Table I because different minimum track lengths were used.

[23] C. L. Critchfield and D. C. Dodder, Phys. Rev. **75**, 419 (1949).
[24] R. O. Kerman, Ph.D. thesis, University of Illinois, 1953 (unpublished).

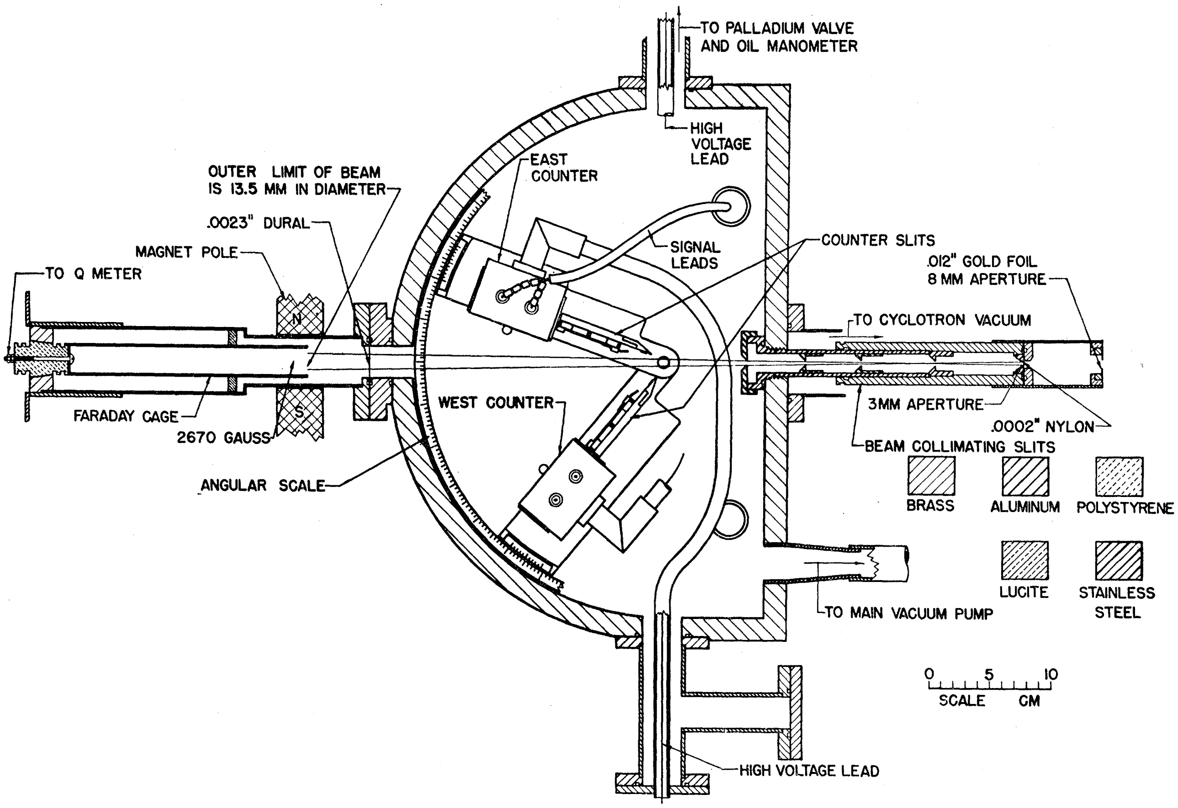

FIG. 1. Horizontal view of counter scattering chamber. Depth perpendicular to drawing is about 10 cm.

Mev. Finally, a plot of $\partial\sigma/\partial E$ *vs* angle was made; this curve was the basis of all energy corrections.

The corrected center-of-mass cross sections together with the estimated probable errors are listed in Table II. These errors were calculated in the usual way and include those due to statistics, uncertainty in measurements of angles, geometrical dimensions, total charge, and gas pressure and temperature, as well as a 0.4 percent probable error in counting tracks and an assumed 15 percent error in each correction. The average angular resolution in the laboratory system as defined by Kreger *et al.*[19] is recorded in column 2 of Table II. The corrected center-of-mass cross sections are plotted against the center-of-mass scattering angle in Fig. 5.

III. PROPORTIONAL COUNTER EXPERIMENTS[25]

A. Scattering Chamber

Concurrently with the use of nuclear plates to detect scattered protons, a scattering chamber for use with gas-filled counters was completed, and is shown in Fig. 1. It was placed adjacent to the photographic scattering chamber at the end of the exit tube making an angle of 51° with the particles emerging from the cyclotron.[3] After being deflected in the analyzer magnet, the proton beam passed through a round 8-mm aperture in an insulated gold foil used for monitoring the intensity of the analyzed beam, and entered the scattering chamber through a collimating slit system and a 0.0002-in. nylon foil. The slit geometry in a horizontal plane is shown in Fig. 2. The first and last of the collimating slits defined the angular spread of the beam in the chamber; the others prevented protons scattered from the slit-holder walls from entering the scattering volume. The proton beam left the rear of the chamber through a 0.0023-in. Dural foil and entered the Faraday cup, which was maintained at 10^{-5} mm Hg by a separate vacuum system. The Faraday cup construction and the charge measurement technique were similar to those used with the photographic chamber.

Scattered protons were detected on either side of the incident beam by one of two double proportional counters movable in the horizontal plane (the East and West counters). These counters were mirror images, and Fig. 3 shows the construction of one of them. The single high-voltage electrode containing the two separate counting volumes was designed so that all particles entering the counter would have equal ionization paths. The electrode was supported by three Kovar-glass seals which were modified from commercial seals by clipping off the stem and spotting glass over the exposed metallic portion of the seal. The high-

[25] Part of the material in this section is from a thesis submitted by E. J. Zimmerman in partial fulfillment for the requirements for the Ph.D. at the University of Illinois, 1951.

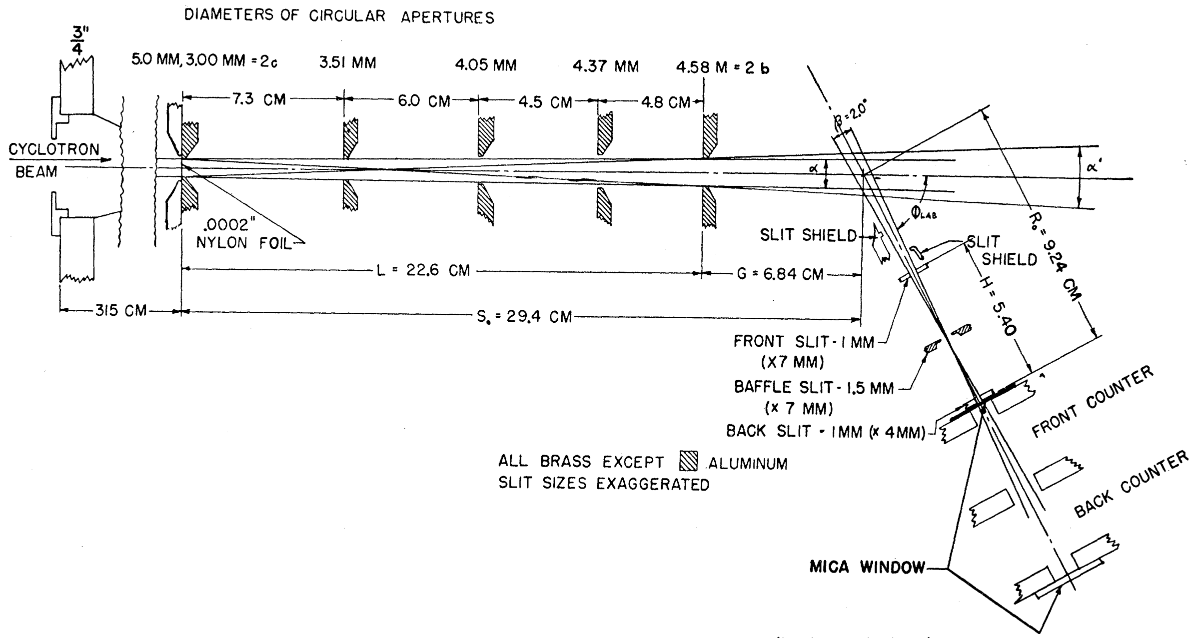

FIG. 2. Collimating slit system and counter slit system (horizontal plane).

voltage lead was enclosed in a re-entrant flexible copper tube attached to the side of the counter. These precautions were necessary to prevent sparking in the low-pressure hydrogen atmosphere of the chamber. Pulses from the counters were taken at ground level from the 0.002 in. platinum center wires and fed through shielded cables to conventional preamplifiers shock mounted directly below the scattering chamber.

The counter boxes were sealed with 1.5 mg/cm² mica foils and filled with pure methane at 35 cm Hg.[26]

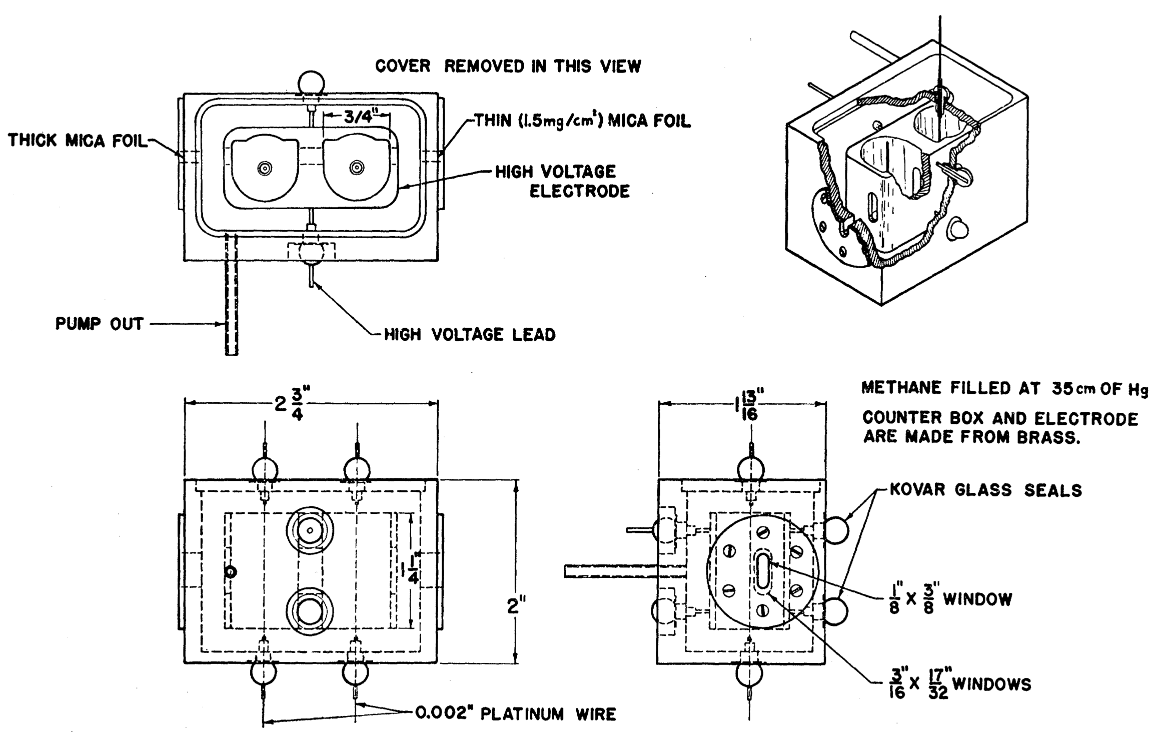

FIG. 3. Construction of the double proportional counter.

[26] The sealing and filling were done by Radiation Counter Laboratories, Chicago, Illinois.

Extensive tests were made to determine the counting characteristics. The cross section at a fixed scattering angle was independent of counter voltage from 1400 to 2100 volts; normal operating range was 1700 to 1900 volts. The gas multiplication varied exponentially with counter voltage over the same range. In the normal operating range, the gas multiplication was measured to be about 250 to 900. Because of the relatively low methane counter gas pressure and small counting volumes, only a small part of the range of the scattered protons was spent in the counters. Consequently, protons of different energies could not be very readily distinguished. There was some energy discrimination, however, since the low-energy protons scattered at 55° (lab) had nearly 2.5 times the pulse heights of those scattered at 13°, in good agreement with the Bragg curve for protons.

Linear amplifiers similar to the Los Alamos Model 100[27] were used to amplify the preamplifier signals to a level of about 75 volts. Pulses originating in the front and rear counters were fed to a stable coincidence circuit having a resolving time of 80 microseconds. Thus, only particles coming from the direction of the scattering volume were counted. In addition, coincidence signals from the front counter were pulse-height analyzed so that pulses due to protons of energy far removed from that of properly scattered protons could be rejected. By analyzing the counter pulses in this manner, background was reduced to a negligible amount; and since the singles counting rate never exceeded 10 counts per second, no correction was needed for the resolving time of the coincidence circuit. Test runs with the proton beam stopped in front of the chamber, or with no hydrogen in the chamber, showed that differences (about 0.2 percent) between the singles and coincidence counting rates could be traced to transient sparks and arcs in the cyclotron control room, and possibly to neutron recoils in the methane counter gas. This type of background was small, and no correction was made for it.

Typical pulse amplitude analyses are shown in Fig. 4. In computing the "proton count" for each scattering run, both very large and very small pulses were discarded. Perhaps the largest source of error in the experiment was due to personal factors involved in deciding where to cut off the pulse distribution; the average estimated uncertainty in the proton count from this cause was 0.8 percent. This uncertainty, estimated for each run, was combined with the statistical standard deviation for that run, and the result used as the standard deviation in the proton count.

As shown in Fig. 2, the scattering volume was defined by two rectangular slits in front of each counter. The dimensions of the various slits were measured with a comparator, and their spacing with a micrometer.

[27] W. C. Elmore and M. Sands, *Electronics* (McGraw-Hill Book Company, Inc., New York, 1949), p. 166.

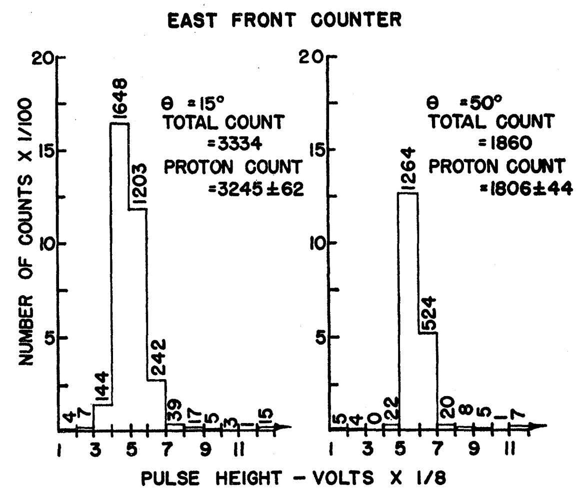

FIG. 4. Typical pulse amplitude histograms. Front East counter analyzed in coincidence with back East counter, for two scattering angles.

The geometrical constants are defined and stated in Table III.

The East and West counters were mounted on arms which could be rotated from outside the chamber. The scattering angle could be read to 0.05° by means of an angular scale and verniers mounted on the counter arms. The angular misalignment of the entire chamber

TABLE III. Notation and geometrical constants for counter experiment.

Symbol	
$\sigma_{\text{c.m.}}, \sigma_L$	Differential scattering cross section in center-of-mass, laboratory system.
$\theta_{\text{c.m.}}, \theta_L$	Scattering angle in center-of-mass, laboratory system
N_s	Number of scattered protons observed at θ_L
ω	Solid angle subtended by back counter slit at the center of the scattering volume $=fd/R_0^2$
τ	Gas target thickness for counter slit system at 90° to the incident beam of protons $=2wR_0/H$
H_1	Difference in heights of oil columns in manometer
m	Temperature coefficient of density of Octoil-S; $d_t=d_0[1-m(t-25)]$
V	Vibrating Reed Electrometer output voltage, difference before and after a run
R_s	Vibrating Reed Electrometer calibration
T	Absolute temperature in scattering chamber
H	Distance between counter slits
R_0	Distance from scattering volume to back counter slit
$2w$	Width of front counter slit
d	Width of back counter slit
f	Height of back counter slit
E	Energy of incident protons
$E(\theta_L)$	Energy of proton scattered through a laboratory angle θ_L

Quantity		East counter	West counter
H	(cm)	5.397 ±0.003	5.398 ±0.003
R_0	(cm)	9.238 ±0.004	9.250 ±0.003
$2w$	(cm)	0.9418±0.00014	0.9570±0.00014
d	(mm)	0.9566±0.00014	0.9658±0.00014
f	(mm)	4.032 ±0.004	4.099 ±0.004
τ	(cm)	0.1612±0.0003	0.1640±0.0003
$\omega\times10^4$	(sterad)	4.520 ±0.008	4.627 ±0.008

was reduced to 0.02±0.05 degree by moving the chamber until the cross section measured at a fixed angle was the same on each side of the incident beam. To further reduce the effects of such a misalignment, the measured cross sections obtained from each counter at each angle were averaged.

B. Experimental Procedure

Immediately preceding a series of scattering runs, the chamber was filled with hydrogen to a pressure of about 5 cm Hg (measured with a calibrated[28] Octoil-S manometer). The hydrogen was admitted through a heated palladium thimble and flowed through the chamber at a rate sufficient to completely change the gas every three or four hours. The scattering gas temperature and pressure were read several times during each individual run. For laboratory angles less than 45°, a particular run was terminated when 2500 scattered protons were recorded (about 20 minutes); above 45° only 1500 scattered protons were counted per run.

Protons were counted alternately in the East and West counters, and the order in which the scattering angle was changed was varied from day to day. One hundred usable scattering runs were made during which 10 000 scattered protons were detected by each counter at each scattering angle.

The energy of the incident protons was measured in connection with a previous experiment.[29] The range of the protons in air was measured using a scintillation counter technique and the proton energy was found to be 5.93 Mev±1 percent, with a 65-kev spread about the mean energy. The energy of the protons was degraded by their passing through the 0.0002-in. nylon foil and 29.5 cm of gas in the chamber before reaching the scattering volume. The energy loss was calculated from available data[30,31] and was found to be 70±10 kev, so that the energy at scattering was 5.86 Mev±1 percent.

During the data runs, as in the photographic plates experiment, both the cyclotron magnet and analyzer magnet fields were stabilized by use of a proton-moment automaton control.

Most of the data were taken using liquid nitrogen cold traps which were connected to the chamber. However, during the analysis of these data, it was learned that Rouvina,[12] using photographic plates as detectors, found that the use of liquid nitrogen traps during the experiment gave incorrect results, apparently because of the less efficient pumping action of the traps with hydrogen in the chamber as compared to background runs taken with no gas in the chamber. While investigations of contaminant scattering (described below) had convinced us of the absence of any contaminant vapors in our scattering chamber, it was thought desirable to check for a liquid nitrogen effect. Accordingly, a series of runs were made at several small scattering angles (where contaminant errors are largest) in an effort to compare cross sections taken with liquid nitrogen with those taken without liquid nitrogen. No appreciable effect was found. Since these data were in agreement with those taken in the initial series of runs, they are included in Table V.

TABLE IV. Corrections and energy reduction data for the counter cross sections. The coincidence and finite geometry corrections were used in obtaining the final counter cross sections, while the derivatives listed in the last three columns were used to reduce these data to an equivalent energy of 5.77 Mev.

$\theta_{\text{c.m.}}$	Coincidence correction East counter %	Coincidence correction West counter %	Finite geometry correction (%)	$-\frac{\Delta\sigma}{\Delta E}$ $\frac{\text{mb}}{\text{Mev}}$	$\frac{E\Delta\delta_0}{\Delta E}$ (deg)	$\sigma\frac{\Delta\delta_0}{\Delta\sigma}$ (deg)
24	0.20	0.16	1.8	25	50	44
26	0.20	0.16	1.4	20	43	37
28	0.20	0.16	1.2	15.5	38	35
30	0.20	0.16	0.9	14.5	36	33
40	0.23	0.19	0.4	11.5	31.5	33
45	0.27	0.22	0.3	11.5	31	33
50	0.31	0.25	0.2	11.7	32	34
60	0.43	0.35	0.2	12.0	33	35
70	0.63	0.51	0.1	12.7	34	35.5
80	0.98	0.80	0.1	13	34	36
90	1.62	1.32	···	13	34	36
100	3.26	2.68	···			
110	7.12	5.84				

C. Calculation of Corrected Cross Sections

Upon inserting values for the fundamental constants and constants of the apparatus, the formula for the center-of-mass cross section in terms of the laboratory scattering angle takes the form:

$$\sigma_{\text{c.m.}}(\theta_L)=3.0198\times10^{-33}\left(\frac{1}{\tau\omega}\right)\left[\frac{T}{1-m(t-25)}\right]\left(\frac{N_s\tan\theta_L}{H_1VR_s}\right),$$

with notation and constants tabulated in Table III. Two corrections were made to the experimental cross sections: a "missed coincidences" correction, and a correction for finite geometry. The former was necessary because some protons which had correctly passed through the counter slit system were so scattered in the mica foil that they failed to enter the rear counter. This effect was studied with a collimated beam of protons as a function of proton energy, and the measured correction was then expressed in terms of the laboratory scattering angle by means of the relation $\cos^2\theta_L=E/E(\theta_L)$. That this effect was actually due to scattering in the mica foils was verified by observing that the ratio of the measured correction for the East Counter

[28] See reference 3, p. 658.

[29] Taylor, Jentschke, Remley, Eby, and Kruger, Phys. Rev. **84**, 1034 (1951).

[30] Aron, Hoffman, and Williams, University of California Radiation Laboratory Report No. 121, revised, 1948 (unpublished).

[31] M. S. Livingston and H. Bethe, Revs. Modern Phys. **9**, 245 (1937).

to that for the West Counter was the same as the ratio of the thicknesses of the corresponding foils. The uncertainty of the measured corrections was as much as 15 percent in some cases, but the correction itself was small except for large scattering angles. The corrections are listed in Table IV.

The other correction was one for the second order geometry effect due to the finite size of the proton beam and counter slits. The method of Critchfield and Dodder[24] was applied and carried out to third order terms.[32] These corrections are also given in Table IV; they are largest at the smallest scattering angles (1.8 percent) and decrease with increasing angle.

The possibility that multiple scattering and contaminant scattering were occurring was also checked. Earlier tests by Meagher[5] had indicated that multiple scattering effects in this chamber were unimportant for pressures up to at least 5 cm Hg. To check this, the scattering cross section (at 20° lab) of protons on unpurified tank oxygen was measured as a function of pressure up to 3 cm Hg. Multiple scattering effects in oxygen at 3 cm Hg should be about 15 times as great as in hydrogen at 5 cm; and since the proton-oxygen cross sections were observed to be independent of pressure, no correction was applied to the proton-proton cross sections.

To investigate contaminant scattering, the chamber was pumped down to its base pressure, then closed off from the pumps for 36 hours. At the end of this time a "scattering" run was started, and the accumulated gas produced less than 0.2 percent background at all scattering angles. No background correction was made.

After correcting the cross section for each run as described above, the final corrected East and West cross sections at each angle were obtained by a weighted averaging over all the individual cross sections at that angle. Since no consistent difference between the East and West cross sections were observed, these too were averaged.

In order to compare the results of the counter and plates experiments, a correction must be applied to one or both sets of data because the incident proton energy was different for each experiment. The comparison can be carried out in two different ways. One might reduce the cross sections from the two experiments to the same energy using published data to calculate $(\partial\sigma/\partial E)\theta$ as described in Sec. II. Or, since the ultimate comparison of the data is to be made in terms of a partial wave analysis, the data given in Jackson and Blatt[8] can be used to calculate the apparent S-wave phase shift: their values of $E(\partial\delta_0/\partial E)_{\sigma,\theta}$ and $\sigma(\partial\delta_0/\partial\sigma)_{E,\theta}$ can then be used to correct this phase shift over the energy interval ΔE. The results of the two methods are consistent within 0.1 percent in all cases. For comparison, the counter data were reduced to the energy used in the plates experiment, 5.77 Mev. Values of the derivatives used for this reduction are shown in Table IV, and the resulting cross sections at 5.77 Mev are given in Table V. The uncertainties attached to these cross sections include, in addition to the statistical error defined earlier, those incurred in the measurement of the counter solid angle, scattering angle, total charge, gas temperature, and pressure.

TABLE V. Final cross sections of counter experiments reduced to 5.77 Mev.

Values of the functions $P_1(E,\theta)$	$\theta_{c.m.}$ (deg)	$\sigma_{c.m.}$ (mb/sterad) Original runs (%)	$\sigma_{c.m.}$ (mb/sterad) Check runs (%)	Apparent S-wave phase shift (deg) Original runs	Apparent S-wave phase shift (deg) Check runs
−5.64	24	114.8 ±2	109.9 ±2	57.46±0.75	55.76±0.75
−4.71	26	102.2 ±1	99.26±1	57.19±0.51	56.02±0.52
−4.05	28		93.07±0.9		56.35±0.50
−3.44	30	89.89±0.7	87.88±0.9	56.70±0.43	55.90±0.47
−1.66	40	84.9 ±0.9		56.40±0.48	
−1.19	45		85.24±0.9		56.19±0.43
−0.84	50	84.8 ±1.0		55.57±0.48	
−0.42	60	87.6 ±1.0		55.89±0.49	
−0.15	70	88.0 ±1.0		55.53±0.50	
−0.04	80	89.3 ±0.8		55.80±0.46	
−0.00	90	88.5 ±0.8	88.05±0.8	55.34±0.46	55.17±0.46
−0.04	100	88.4 ±0.8		55.38±0.46	
−0.15	110	88.8 ±0.8		55.86±0.44	
−0.04	Av 80 and 100	88.8 ±0.6		55.59±0.33	
−0.15	Av 70 and 110	88.4 ±0.7		55.70±0.33	

D. Check Runs

When it became apparent that the counter data were to a certain extent in conflict with the data taken from the photographic plates, in other laboratories as well as our own, an intensive effort was made to locate the source of the discrepancies. Because the missed coincidences correction was angle-dependent, an effort was made to avoid it by operating the proportional counters in the scattering chamber without windows. As expected, however, when the counters were operated in very pure hydrogen at about 5 cm pressure, the pulses were quite small and the spread of pulse amplitudes was so large that no reliable proton count could be obtained. The counters could be stabilized by the addition of small amounts of methane, but this would have necessitated a different experiment to obtain the proton-proton cross sections. It was therefore decided to work with the original counters. The missed coincidences correction was remeasured, and was found to agree with the previous measurement.

The possibility of scattering from the collimation and counter slits was also investigated since it would have produced apparent cross sections too large at small scattering angles. A polonium alpha source was mounted at the scattering volume (with the chamber evacuated) and two sets of runs were made: one with the counter slit system in place and one with the slit system replaced by a single aluminum slit sufficiently thick to just stop the alpha particles. A sufficient number of counts was recorded so that as few as 3 percent low-energy scattered alpha particles could have been easily

[32] G. R. Briggs, Ph.D. thesis, University of Illinois, 1953 (unpublished).

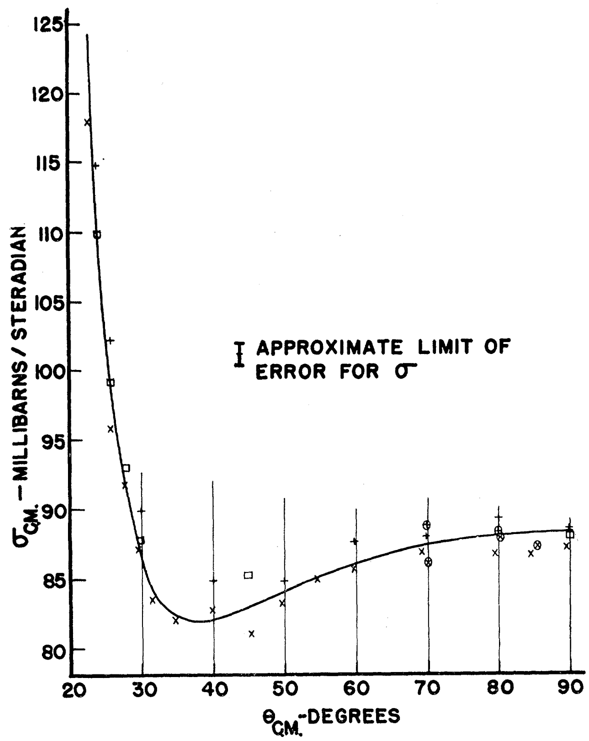

FIG. 5. Proton-proton center-of-mass cross section *vs* center-of-mass scattering angle at an energy of 5.77±0.05 Mev. × Nuclear plates; + original counter runs; □ check runs with counters; ○ means $\sigma(180-\theta_{c.m.})$ is plotted. Solid curve is calculated for pure S-wave scattering for $\delta_0=55.29°$. Approximate error in cross sections is indicated; for individual values see Tables II and V.

detected. No such low-energy particles were observed, and the two pulse amplitude analyses did not appear to be different. This was considered to be a significant test since the problem of slit-scattering is expected to be much more severe for the alpha particles than for the protons used in the experiments.

Further investigations were made by replacing one of the counters with a nuclear plate camera so that virtually all protons leaving the rear counter slit were recorded in the emulsion. It was hoped to obtain a measure of the slit scattering occurring with protons by comparing plates taken with and without the counter slit system in place, but no conclusive results were obtained because small-angle Coulomb scattering in the emulsion obscured the effect under investigation. However, these investigations did yield valuable information concerning the "purity" of the proton beam. A deuteron component in the beam would have increased the apparent low-angle cross sections because both the elastically scattered deuterons and protons would have given pulses in the counters indistinguishable from pulses due to properly scattered protons. In the many nuclear plates exposed to the cyclotron beam, not a single deuteron track (easily detected since its length would be twice as great as a proton track) was observed. The purity of the proton beam used in the original counter measurements was not investigated.

It was finally decided to repeat the counter experiment at several scattering angles. The incident proton energy was remeasured using a nuclear emulsion technique[19] and was found to be 5.84 Mev±1 percent at the scattering volume; otherwise the procedure of the original experiment was followed as closely as possible. The results of these check runs were reduced to an equivalent energy of 5.77 Mev and are given in Table V; they are plotted with the other data in Fig. 5.

The check runs were taken with an elliptical aperture $\frac{3}{8}$ in.×$\frac{9}{16}$ in. (and sufficiently thick to just stop 6-Mev protons) placed at the exit of the analyzer magnet chamber (about 3.6 meters from the scattering volume). The slit was not used for the original counter runs. It improved the energy homogeneity of the incident proton beam by reducing the number of particles entering the scattering volume which had been scattered in the beam collimation system. The energy measurement for the check runs showed that the energy spread in the incident proton beam was essentially the same as for the photographic plates experiment, and that there was no low-energy component in the beam, either proton or deuteron.

IV. DISCUSSION OF RESULTS

In Fig. 5 all measured cross sections are plotted as a function of scattering angle in the center-of-mass system. The solid curve is theoretically calculated for a pure S-wave phase shift of 55.29°. It is evident that there are some differences between the cross sections measured with the nuclear plates and those measured with the counters. Above about 50°, the original counter cross sections are consistently higher than the plate values by about 1.7 percent; as the scattering angle decreases, the differences increase to about 5 percent at the smallest angles measured. The check runs with counters failed to show the larger discrepancies at the small angles, but were still generally about 2 percent larger than those measured in the emulsion experiment. The consistent difference could be explained by assuming an error in one or both of the energies so that the actual energies in the two experiments differed by about 1.7 percent. Since the separate determinations of energy claim precisions of 1 percent (counters) and 0.7 percent (plates), this is not impossible, but in view of the agreement of the energy measurements between themselves, it does not seem likely. Considering the large number of geometrical measurements required and the uncertainties in the number of proton counts indicated by the pulse height histograms in the counter experiment, it is more likely that the differences are due to small systematic errors in one or both of the experiments. We believe the results based on the weighted averages should be accurate generally within about 1 percent.

Further discussion of the results is best done by the method of phase shift analysis described by Jackson and Blatt.[8] One calculates the nuclear phase shift on the assumption that only the S-wave contributes to the scattering. If there exists some contribution from states of higher angular momentum, this "apparent S-wave phase shift," δ_a, varies with the scattering angle. In the special case where only a small P-wave contributes, Jackson and Blatt showed that to a good approximation,

$$\delta_a = \delta_0 + P_1(E,\theta)\delta_1,$$

where δ_1 is the P-wave phase shift and δ_0 the true S-wave phase shift; the function $P_1(E,\theta)$ is tabulated in their Table V. Values of δ_a for our data are given with the final cross sections in Tables II and V. The uncertainties in the phase shifts are based on the errors in the cross sections and in the energy as given here and are calculated by the method given in Jackson and Blatt.[8] The apparent S-wave phase shifts are plotted as a function of P_1 for all our data in Figs. 6a and 6b. For each set of runs, the above equation was fitted to the phase shifts by a weighted least squares computation with the results tabulated in Table VI, and shown on Fig. 6.

The values of the true S-wave phase shift, δ_0, obtained from the two experiments reflect the consistent difference in the measured cross sections. The counter check runs are in excellent agreement with the original counter runs, but their average δ_0 is greater than δ_0 from the plates data by almost three times the sum of the separate probable errors. This again is evidence for a small systematic error. As can be easily seen from Fig. 6, δ_0 depends essentially on the measured values of the cross sections near $P_1=0$, that is, for $|P_1|<0.2$. For these values of P_1, the difference between δ_a and δ_0 is less than 0.02° in all cases. Hence it makes sense to average the thirteen phase shifts calculated from the cross sections measured in the angular range 70° to 110° (center of mass) corresponding to $|P_1|$-values less than 0.2. This gives a "direct" value of the S-wave phase shift, which, as indicated in Table VI, has been averaged with the values obtained from the separate experiments. The final value for the S-wave shift is 55.29°±0.30°, the error being calculated from the weighted deviations of the four separate values of δ_0 given in Table VI from their weighted average.

TABLE VI. Phase shifts for the plates and counter experiments.

	S-wave phase shift (deg)	P-wave phase shift (deg)
Plates	54.96±0.11	−0.08±0.05
Original counter runs	55.57±0.12	−0.34±0.05
Check runs (counters)	55.63±0.25	−0.08±0.07
Average (plates and counters) for $\|P_1\|<0.2$	55.08±0.30	
Weighted average	55.29±0.30	

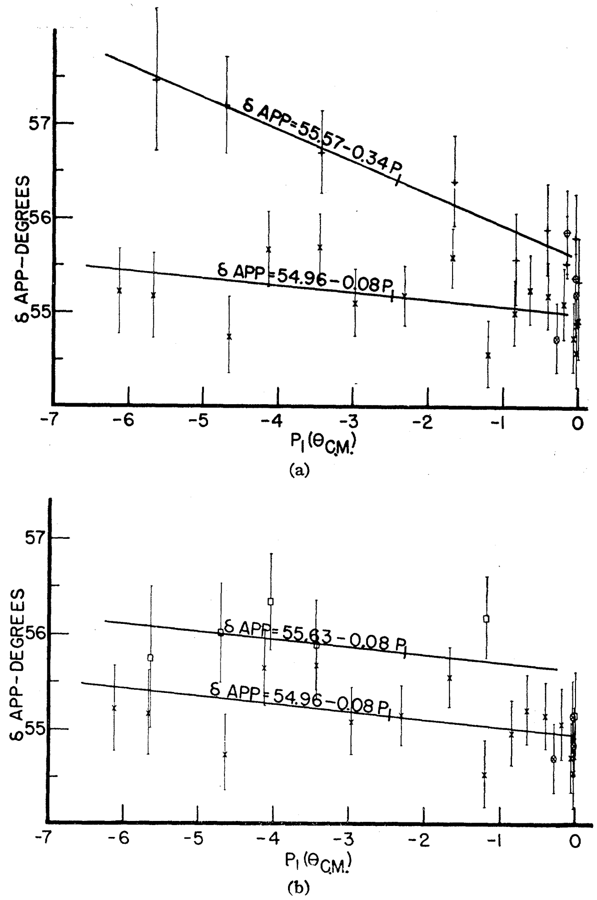

FIG. 6. Apparent S-wave phase shifts *vs* $P_1(\theta_{c.m.})$ for proton-proton scattering at 5.77 Mev. (a) Comparison of plates data with original counter data. (b) Comparison of plates data with those of counter check runs. Symbols are identified with experiments as in Fig. 4. Least-squares determinations of δ_1 are indicated. For individual values of δ_{app} see Tables II and V.

This value of the S-wave phase shift can be compared with those from other experiments by use of the K-function of Jackson and Blatt.[8] Analysis of a number of proton-proton scattering experiments in the low energy region[33] shows that K is a linear function of the energy according to the equation

$$K = 3.755 + 0.4603E.$$

From this equation, and at a laboratory energy of 5.77 Mev, we calculate $K=6.41$. The value of K calculated from our S-wave phase shift is 6.39±0.05.

The results of the P-wave analysis are more ambiguous. The original counter experiment indicated a fairly large P-wave shift (−0.34°), while the check runs as well as the plates experiment gave small P-wave shifts (−0.08°). In view of the agreement between the last two values we are inclined to believe that the indicated P-wave effect at 5.77 Mev is small and negative, and probably less than 0.1° in magnitude. The source of

[33] Bruce Cork, Phys. Rev. **80**, 321 (1950).

the discrepancy between the original counter measurements and the check runs (which amounts to a difference of from three to five percent in the low angle scattering cross sections) is not definitely known. There are, however, several possibilities. A small, low-energy proton group in the incident beam due to slit scattering somewhere in the collimation system could have caused an apparent increase in the counter cross sections at low angles; such an inhomogeneity may have gone undetected in the energy measurement for the original counter experiment. The elliptical slit used in the check runs was designed to reduce this type of scattering and, as a result, would have eliminated at least part of the discrepancy between the two sets of data.

There is also the possibility that the proton beam was contaminated with deuterons, as discussed in Sec. III D. However, it is improbable that the effect would have lasted over the period during which the original counter data were taken.

Some support for the first hypothesis is indicated from the results of other experiments performed at this laboratory.[19,34] There the two scattering chambers described earlier were used to measure the proton-alpha elastic scattering cross sections: the experimental procedures were almost identical to those used in the present work. The elliptical collimation slit was used at all times in the counter experiment and the primary beam was investigated and shown to be free from any group of low energy particles. It was found that the low angle scattering cross sections measured with the counters were in good agreement with those measures using nuclear emulsions.

At these energies, theory would lead one to expect that the higher angular momentum states should begin to contribute appreciably to the nuclear scattering. The extremely precise experiments performed at Wisconsin[11,17] gave almost indisputable evidence that a P-wave contribution does exist even at the somewhat lower energies used there. It has already been noticed that our data from the original counter runs give a P-wave phase shift in excellent agreement with the Wisconsin data extrapolated to 5.77 Mev[11] provided the Wisconsin data measurements at 4.2 Mev are heavily discounted. On the other hand, the Wisconsin data at 4.2 Mev imply less of a P-wave effect than occurs at even as low an energy as 2.4 Mev;[11] if the measurements at 4.2 Mev are weighed equally with the lower energy results, then the Wisconsin data extrapolated to our energy are consistent with a P-wave phase shift of about $-0.10°$. Taking the data at their face value, one would have to conclude that the P-wave scattering does indeed contribute about to the extent theoretically predicted (on the basis of a static potential) but that some other effect or effects, beginning at the unexpectedly low energy of 4 Mev, tend to mask the P-wave scattering. Most probably a final interpretation will have to await further precise experiments at higher energies and possible future advances in the theoretical treatment.[35,36]

[34] A. L. Atkins, Ph.D. thesis, University of Illinois, 1953 (unpublished).

V. ACKNOWLEDGMENTS

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[35] An analysis (see reference 36) of recent proton-proton scattering data at 9.7 Mev shows that the experimental results were not accurate enough to allow definite conclusions about the existence of a P-wave shift at this energy.

[36] H. H. Hall, Phys. Rev. 95, 424 (1954).