Letters to the Editor

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Superconductivity of a Charged Boson Gas

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(Received September 7, 1954)

IT is the purpose of this note to point out that there exists a relatively simple physical system which exhibits the essential equilibrium features of a superconductor,[1] namely a phase transition of the second kind at a critical temperature T_c and the occurrence of a Meissner-Ochsenfeldt effect below it. This system is the ideal gas of charged bosons. The existence of a transition point is well known.[2]

If one establishes the relation between the Fourier components of the current density, $\mathbf{i}(\mathbf{q})$ and of the vector potential, $\mathbf{A}(\mathbf{q})$, of an applied weak inhomogeneous magnetic field in the form

$$\mathbf{i}(\mathbf{q})=K(q^2)\mathbf{q}\times(\mathbf{q}\times\mathbf{A}(\mathbf{q})), \tag{1}$$

then, as was shown elsewhere,[3] a pole in $K(q^2)$ for $q=0$ means the occurrence of the Meissner-Ochsenfeldt effect. The particular case,

$$K(q^2)=1/c\lambda q^2, \tag{2}$$

is equivalent to the London[1] equation,

$$\mathrm{curl}(\lambda\mathbf{i}(x))=-(1/c)\mathbf{H}(x). \tag{3}$$

The thermal average current density for our model is easily evaluated by perturbation theory on the distribution function in a manner described before[3] and yields below the transition point:

$$K(q^2)=n_s\frac{e^2}{mc}\frac{1}{q^2}+K_0(q^2), \tag{4}$$

where n_s is the density of condensed bosons and $K_0(q^2)$ has a singularity of order $1/q^1$ only. This proves our assertion.

The fact that $K_0(q^2)$ still has a singularity shows that (3) is not valid but has to be replaced by an integral relationship. (This was already proposed by Pippard[4] for real superconductors.) In particular the penetration depth is not determined by n_s alone.

The very uniqueness of the phenomenon of Bose-Einstein condensation might be taken as a clue that this model is essentially the only one which exhibits the phenomenon of superconductivity. One would then have to show that in a metal at low temperatures charge-carrying bosons occur, e.g., because of the interaction of electrons with lattice vibrations.[5] In this connection it might be worthwhile investigating the possibility of existence of bound two-electron states. A more detailed discussion of this work will appear in a forthcoming paper.

I am greatly indebted to Dr. J. M. Blatt and Dr. S. T. Butler for stimulating discussion.

* Also supported by the Nuclear Research Foundation within the University of Sydney.

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Superfluidity of a Boson Gas

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(Received September 7, 1954)

WE have made a theoretical analysis of an idealized version of Andronikashvili's[1] experiment establishing the superfluid character of liquid helium below the λ point. A vessel is filled with liquid helium and set into rotation. The apparent moment of inertia is less than the classical value $I_0=\frac{1}{2}NMR^2$ (N = number of atoms, M = mass of atom, R = radius of vessel). The ratio between the observed moment of inertia and I_0 is by definition the "concentration of normal fluid."

We now make a calculation of the equilibrium value of the angular momentum L of an ideal Bose-Einstein gas as a function of the angular velocity ω. The result is shown in Fig. 1. Above the λ point, L is given by the dashed straight line, with slope equal to I_0. Below the λ point, L is a discontinuous function of ω, as shown by the full lines. The discontinuities occur whenever the angular velocity increases by an amount $\Delta\omega=\hbar^2/MR^2$, for then a new value of the angular momentum quantum number m gives rise to the lowest state of an atom in the vessel. Hence at this point a macroscopic number of atoms shifts from the lowest state with quantum number $m-1$ to the lowest state with quantum number m, and this accounts for the sudden jump in the equilibrium angular momentum L.

If equilibrium is reached, the observed moment of inertia is the mean slope of the solid curve, averaged over these quantum mechanical fluctuations; that is,