Ratio of $\bar{\Gamma}_n^0/\bar{D}$ for Slow Neutron Resonances*

R. S. CARTER, J. A. HARVEY, D. J. HUGHES, AND V. E. PILCHER†
Brookhaven National Laboratory, Upton, New York
(Received June 28, 1954)

A survey has been made of resonance parameters of the heavy elements which have been measured recently. The variation of the ratio $\bar{\Gamma}_n^0/\bar{D}$ with atomic weight has been compared with theoretical predictions. A maximum was found in the ratio $\bar{\Gamma}_n^0/\bar{D}$ at atomic weight of approximately 160 in fair agreement with the more recent theories of neutron scattering by nuclei. The experimental data, however, are not sufficiently accurate to make a choice between the new theories which have been proposed.

THE extreme compound nucleus models proposed by Feshbach, Peaslee, and Weisskopf,[1] Feshbach and Weisskopf,[2] and Teichmann and Wigner[3] predicted that the ratio $\bar{\Gamma}_n^0/\bar{D}$ would be constant, independent of atomic weight. In the black-nucleus model[1] $\bar{\Gamma}_n^0/\bar{D}$ equals $2k_0/\pi K$, where $\bar{\Gamma}_n^0$ is the average reduced neutron width, $\bar{D}$ is the average level spacing for the spin state, k_0 is the wave number of a 1-ev neutron, and K is the wave number of the neutron inside the nucleus. The reduced neutron width of a resonance, Γ_n^0, is defined by $\Gamma_n^0=\Gamma_n/\sqrt{E_0}$, where Γ_n is the neutron width of the resonance and E_0 is the kinetic energy of the incident neutron at resonance expressed in electron volts. The value of $\bar{\Gamma}_n^0/\bar{D}$ depends on the nuclear well depth, V_0, and is 1.4×10^{-4} or 1.0×10^{-4} for well depths of 19 or 42 Mev, respectively. Recently, however, theories have been proposed by Feshbach, Porter, and Weisskopf,[4] Bohr and Mottelson,[5] and Thomas[6] which predict a variation of the $\bar{\Gamma}_n^0/\bar{D}$ ratio with atomic weight. The cloudy crystal-ball model[4] assumes a complex nuclear potential well with $V=V_0(1+i\xi)$ and a nuclear radius equal to $r_0A^{\frac{1}{3}}(r_0=1.45\times10^{-13}$ cm). A well depth of 42 Mev was found by Adair[7] to best fit the low-energy neutron cross-section data. For this well depth, the model predicts maxima in $\bar{\Gamma}_n^0/\bar{D}$ for S-wave neutrons at atomic weights 11, 55, and 155. A fit of the theory to measured total neutron cross sections in the Mev energy region[8] requires ξ to have the value 0.03,[4] for $V_0=42$ Mev. With this choice of parameters the peak in the ratio $\bar{\Gamma}_n^0/\bar{D}$ at atomic weight 155 is 6 times the black-nucleus value. Between maxima the value is about $\frac{1}{6}$ the black-nucleus value. In the cloudy crystal-ball model, $\bar{\Gamma}_n^0/\bar{D}$ is independent of the distribution of the reduced neutron widths and is equal to $(\sum\Gamma_n^0)/\Delta E$, where ΔE is the energy interval over which the Γ_n^0's for a single spin state are summed.

During the past year the energy resolution of the instruments available for the measurement of low-energy neutron resonances has been greatly improved. It is now possible to obtain Breit-Wigner[9] resonance parameters ($g\Gamma_n$, E_0, Γ, and σ_0) from transmission measurements for many resonances of the heavier nuclei. The quantity $g\Gamma_n$ is the neutron width multiplied by the statistical weight factor g, E_0 is the energy of the resonance, Γ is the total width, and σ_0 is the peak cross section. The factor g equals $\frac{1}{2}[1\pm1/(2I+1)]$; for the zero spin target nuclei ($I=0$), g equals unity; for target nuclei of high spin, g is approximately $\frac{1}{2}$. Since $g\Gamma_n=\sigma_0\Gamma/4\pi\lambda\!\!\!^{-}{}_0^2$, where $2\pi\lambda\!\!\!^{-}{}_0$ is the de Broglie wavelength of a neutron of energy E_0, $\sigma_0\Gamma$ is a direct measure of $g\Gamma_n$. The product $\sigma_0\Gamma$ is obtained directly from the area above a transmission curve for a thin sample. When it is necessary to use thick samples the quantity $\sigma_0\Gamma^2$ is obtained. It is then necessary to know Γ to obtain $g\Gamma_n$. A detailed discussion of the area method is given in the paper by Seidl *et al.* and Melkonian *et al.* (references *a* and *k* of Table I).

We have made a critical survey of resonance parameters for nuclei above atomic weight 100 which have been measured in the past year. The compilation of the evaluated parameters $\bar{\Gamma}_n^0/\bar{D}$, $\bar{D}$, and $\bar{\Gamma}_n^0$ is given in Table I. The table is based on data from the Brookhaven fast chopper, supplemented by data from the Brookhaven crystal spectrometer, the Argonne fast chopper, and the Columbia pulsed cyclotron. Parameters have been tabulated only when more than 3 resonances have been measured per element. The number of resonances measured per element varies from 4 to 30. The average spacing for each spin state, $\bar{D}$, is determined at low energies where it is believed that no resonances have been missed. For example, 16 resonances were found in $_{69}Tm^{169}$ up to 160 ev, but only the first 9 resonances up to 65 ev were used to determine the observed level spacing. For zero spin-target nuclei, $\bar{D}$ is just the observed level spacing; for nonzero spin-target nuclei, $\bar{D}$ is taken to be twice the observed level spacing. In the case of $_{63}Eu$ and of $_{75}Re$ where the resonances have not been assigned to the individual isotopes, the

---

* Work carried out under contract with U. S. Atomic Energy Commission.
† Doctoral candidate from North Carolina State College, Raleigh, North Carolina.
[1] Feshbach, Peaslee, and Weisskopf, Phys. Rev. **71**, 145 (1947).
[2] H. Feshbach and V. F. Weisskopf, Phys. Rev. **76**, 1550 (1949).
[3] T. Teichmann and E. P. Wigner, Phys. Rev. **87**, 123 (1952).
[4] Feshbach, Porter, and Weisskopf, Phys. Rev. **90**, 166 (1953); their extensive treatment of this model for a 42-Mev well depth will be published in the *Physical Review*.
[5] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskab. Selskab Mat.-fys. Medd. **27**, 159 (1953).
[6] R. G. Thomas (private communication).
[7] R. K. Adair, Phys. Rev. **94**, 737 (1954).
[8] H. H. Barschall, Phys. Rev. **86**, 431 (1952).
[9] G. Breit and E. P. Wigner, Phys. Rev. **49**, 519 (1936).

level spacing was assumed to be the same for the 2 isotopes. In the original formulation, $\bar{\Gamma}_n{}^0/\bar{D}=(\sum\Gamma_n{}^0)/\Delta E$, the summation is taken over a single spin state. If, however, $\bar{\Gamma}_n{}^0/\bar{D}$ is assumed to be the same for both spin states, it can be shown that $\bar{\Gamma}_n{}^0/\bar{D}=(\sum g\Gamma_n{}^0)/\Delta E$ where the summation is taken over both spin states. The summation can extend to a higher energy than was used to calculate $\bar{D}$ because small resonances which might be missed at higher energies contribute very little to the sum of $g\Gamma_n{}^0$'s. For elements containing two isotopes it can also be shown that if $\bar{\Gamma}_n{}^0/\bar{D}$ has the same value for each isotope and spin state, then $\bar{\Gamma}_n{}^0/\bar{D}=(\sum fg\Gamma_n{}^0)/\Delta E$ where f is the fractional abundance of the isotope. It was necessary to make this assumption to determine $\bar{\Gamma}_n{}^0/\bar{D}$ for ${}_{63}$Eu and ${}_{75}$Re. In the case of ${}_{50}$Sn where only one or two resonances are present in each isotope below 180 volts, the ratio $\bar{\Gamma}_n{}^0/\bar{D}$ given in Table I is the average for the isotopes. The probable errors on parameters listed in Table I are calculated from the deviation of the individual values from the mean.

TABLE I. Compilation of $\bar{\Gamma}_n{}^0/\bar{D}$ ratio for slow neutron resonances.

Isotope	$(\bar{\Gamma}_n{}^0/\bar{D})\times10^4$	$\bar{D}$ per spin state per isotope (ev)	$\bar{\Gamma}_n{}^0$ (10^{-3} ev)	Reference
${}_{47}\mathrm{Ag}^{107}$	0.5 ± 0.2	40 ± 10	2.0 ± 0.9	a
${}_{47}\mathrm{Ag}^{109}$	1.0 ± 0.3	31 ± 6	3.1 ± 0.9	a
${}_{49}\mathrm{In}^{113}$	0.55 ± 0.12	15 ± 2	0.8 ± 0.2	b, c
${}_{49}\mathrm{In}^{115}$	0.20 ± 0.05	16 ± 3	0.32 ± 0.10	b, c
${}_{50}\mathrm{Sn}^{112,6,7,8,9}$	0.20 ± 0.10	200 ± 100	4 ± 2	c
${}_{51}\mathrm{Sb}^{121}$	0.45 ± 0.12	28 ± 5	1.3 ± 0.4	d
${}_{51}\mathrm{Sb}^{123}$	0.6 ± 0.2	55 ± 15	3.3 ± 1.4	d
${}_{53}\mathrm{I}^{127}$	1.2 ± 0.4	25 ± 5	3.0 ± 1.0	a
${}_{55}\mathrm{Cs}^{133}$	0.7 ± 0.2	50 ± 10	3.5 ± 1.2	c, e
${}_{63}\mathrm{Eu}^{151,3}$	1.9 ± 0.2	2.4 ± 0.3	0.46 ± 0.08	f, g
${}_{65}\mathrm{Tb}^{159}$	1.8 ± 0.4	10.5 ± 1.0	1.9 ± 0.5	g
${}_{67}\mathrm{Ho}^{165}$	2.9 ± 0.5	12.0 ± 1.0	3.5 ± 0.6	h, i
${}_{69}\mathrm{Tm}^{169}$	1.8 ± 0.3	15 ± 2	2.7 ± 0.5	h, i
${}_{71}\mathrm{Lu}^{175}$	1.7 ± 0.2	8 ± 2	1.4 ± 0.5	h, i
${}_{72}\mathrm{Hf}^{177}$	2.7 ± 0.7	9.0 ± 1.0	2.4 ± 0.7	j
${}_{72}\mathrm{Hf}^{179}$	1.6 ± 0.4	11 ± 2	1.8 ± 0.6	j
${}_{73}\mathrm{Ta}^{181}$	1.0 ± 0.4	12 ± 3	1.2 ± 0.5	k, l
${}_{75}\mathrm{Re}^{185,7}$	0.9 ± 0.3	7 ± 3	0.6 ± 0.3	k
${}_{79}\mathrm{Au}^{197}$	0.8 ± 0.2	60 ± 10	4.8 ± 1.3	a
${}_{90}\mathrm{Th}^{232}$	0.8 ± 0.2	22 ± 4	1.8 ± 0.5	a

[a] F. Seidl *et al.*, Phys. Rev. **95**, 476 (1954).
[b] V. L. Sailor and L. B. Borst, Phys. Rev. **87**, 161 (1952) and private communication.
[c] R. S. Carter and J. A. Harvey, Phys. Rev. **95**, 645 (1954) and private communication.
[d] L. M. Bollinger (private communication). The authors wish to thank Dr. Bollinger for making his results available prior to publication.
[e] H. H. Landon and V. L. Sailor, Phys. Rev. **93**, 1030 (1954).
[f] Sailor, Landon, and Foote, Phys. Rev. **93**, 1292 (1954) and private communication.
[g] V. E. Pilcher and R. S. Carter (private communication).
[h] Foote, Landon, and Sailor, Phys. Rev. **92**, 656 (1953).
[i] Pilcher, Carter, and Stolovy, Phys. Rev. **95**, 645 (1954).
[j] Hughes, Kato, and Levin, Phys. Rev. **92**, 1094 (1953) and private communication.
[k] Melkonian, Havens, and Rainwater, Phys. Rev. **92**, 702 (1953).
[l] R. L. Christensen, Phys. Rev. **92**, 1509 (1953).

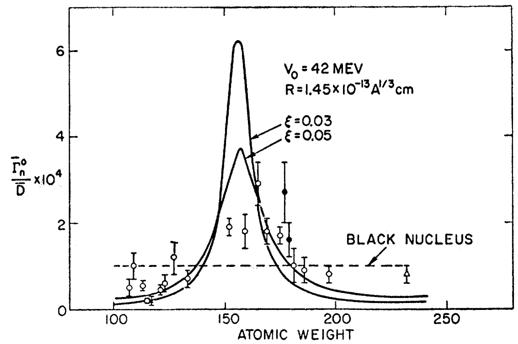

FIG. 1. Comparison of experimental values of $\bar{\Gamma}_n{}^0/\bar{D}$ *vs* atomic weight of the target nucleus with the theoretical predictions of black and cloudy crystal-ball models of the nuclear scattering. The symbols [○ odd Z-odd N, ● even Z-even N, △ even Z-odd N, and × the average for even Z-even N and even Z-odd N] refer to the compound nucleus. The errors represent the probable errors of the experimental points.

Figure 1 shows a plot of the experimental $\bar{\Gamma}_n{}^0/\bar{D}$ points as a function of atomic weight. The theoretical curves for a 42-Mev well depth are shown for the black nucleus model and for the cloudy crystal-ball model with $\xi=0.03$ and $\xi=0.05$.[10] The experimental points exhibit a maximum, in disagreement with the black nucleus picture. The experimental peak at about mass 160 is lower than the theoretical curve for $\xi=0.03$ which is the value of ξ giving the best fit for the total neutron cross sections in the Mev-region.[4] The experimental values off the peak are somewhat larger than predicted. Preliminary calculations by Choudhury[11] including the influence of the coupling to the nuclear rotational motion have shown that the maximum predicted in reference 4 at mass 155 would be depressed by a factor of about 2 and a secondary maximum would occur at approximately mass 120. In conclusion, the experimental data are in fair agreement with the recent theories[4–6] of neutron scattering; however, the present data are insufficient to select the best model. The authors wish to thank Dr. C. E. Porter, Dr. V. F. Weisskopf, Dr. H. Feshbach, Dr. B. R. Mottelson, and Dr. R. G. Thomas for many stimulating discussions.

[10] The two theoretical curves for $\xi=0.03$ and $\xi=0.05$ were obtained from Dr. C. E. Porter (private communication).

[11] D. C. Choudhury (private communication from B. R. Mottelson, and A. Bohr (July, 1954).