Space Charge Formation and the Townsend Mechanism of Spark Breakdown in Gases

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The Townsend criterion for electrical breakdown in gases is based upon a singularity in the Townsend expression for the current as a function of voltage. The conventional derivation of this equation involves the assumption that space charges are absent. A more precise expression of this criterion, involving the effects of space charge, removes the singularity and describes a current that is finite at every finite point. Beyond a certain current, however, the derivative of the current with respect to voltage becomes negative, indicating that the system is incapable of withstanding a voltage larger than that at which the negative characteristic develops. In most cases, the breakdown voltage so established should not differ appreciably from that given by the ordinary Townsend equation.

The second Townsend coefficient is normally determined from the curvature found, as the sparking separation is approached, in plots of the logarithm of the current *versus* electrode separation at constant electric field. The extent of this curvature can be influenced somewhat by distortion of the electric field by space charges if the initial photocurrent from the cathode is sufficiently large.

I. INTRODUCTION

According to the Townsend theory of electrical breakdown in gases, the steady-state current flowing in a uniform-field gap is given by

$$i=i_0e^{\alpha\delta}/[1-(\omega/\alpha)(e^{\alpha\delta}-1)], \quad (1)$$

where i_0 is the initial photocurrent, δ is the electrode separation, and α and (ω/α) are the primary and secondary ionization coefficients, respectively.[1,2] Because both ω and α are increasing functions of the electric field, E, the equation describes a current which increases, as E increases, toward an unbounded value reached when

$$(\omega/\alpha)(e^{\alpha\delta}-1)=1. \quad (2)$$

This condition is taken to define implicitly the breakdown (threshold) electric field.

The derivation of Eq. (1) involves the assumption that the densities of positive and negative charges in the gap are sufficiently low that the electric field is not distorted. Under such conditions, α will be constant throughout the gap. However, as the sparking threshold is approached, this situation clearly no longer obtains. In fact, the formation of field-distorting space charge must be an essential feature of the mechanism of gas breakdown, rather than "following after" the spark in some poorly defined way as has often been assumed. Actually, this fact was pointed out many years ago by von Engel and Steenbeck.[3] In order to account for the results of recent measurements of formative time lags in gas breakdown, other investigators have also recognized, in a qualitative manner, the importance of positive space charge in the initiation of the spark in slightly overvolted gaps.[4,5]

It is the purpose of this paper to show that the buildup of positive space charge in a gas can be described by the appropriate modification of the Townsend equation. It is demonstrated that this buildup occurs as the voltage approaches the threshold voltage predicted by Eq. (1), providing i_0 is not too large. Varney, White, Loeb, and Posin[6] have done a similar calculation for the case where $(\omega/\alpha)=0$ (no secondary mechanism), and our study is essentially an extension of theirs.

II. THE SPACE CHARGE EQUATION

In the steady state the currents of positive and negative charges in the spark gap (cathode at origin, anode at δ) must satisfy the following equations:

$$\begin{aligned}\nabla\cdot\mathbf{i}_- &= \alpha\mathbf{i}_-,\\ \nabla\cdot\mathbf{i}_+ &= -\alpha\mathbf{i}_-,\\ \mathbf{i} &= \mathbf{i}_+ + \mathbf{i}_- = \text{constant}.\end{aligned} \quad (3)$$

Furthermore, they are related to the charge densities n_+ and n_- by the relations

$$\begin{aligned}\mathbf{i}_- &= n_-e\mu_-\mathbf{E},\\ \mathbf{i}_+ &= n_+e\mu_+\mathbf{E},\end{aligned} \quad (4)$$

where μ_- and μ_+ are the *magnitudes* of the charge mobilities and e is the magnitude of the electronic charge. Finally, n_+ and n_- determine the divergence of the electric field:

$$\nabla\cdot\mathbf{E}=4\pi e(n_+-n_-). \quad (5)$$

We will specialize these equations to a situation in which the field is directed along the x axis, the properties of the system being independent of y and z.

The generalized secondary mechanism, characterized

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[1] See, for example, L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939), Chap. IX.

[2] Dutton, Haydon, Llewellyn Jones, and Davidson, Brit. J. Appl. Phys. 4, 170 (1953).

[3] A. von Engel and M. Steenbeck, *Elektrische Gasentladungen* (Julius Springer, Berlin, 1932), Vol. II, p. 50.

[4] L. H. Fisher and B. Bederson, Phys. Rev. **81**, 109 (1951).

[5] G. A. Kachickas and L. H. Fisher, Phys. Rev. **88**, 878 (1952).

[6] Varney, White, Loeb, and Posin, Phys. Rev. **48**, 818 (1935).

by (ω/α), is introduced by the expression,

$$i_-(0)=i_0+\int_0^\delta \omega(x)i_-(x)dx. \tag{6}$$

Combination of this with Eq. (3) and the introduction of a new independent variable,

$$u=\int_0^x \alpha dx,\quad du=\alpha dx, \tag{7}$$

yield the generalized Townsend equation,

$$i_-(x)=i_0e^u\Big/\left[1-\int_0^{\bar{u}}(\omega/\alpha)e^u du\right], \tag{8}$$

$$\bar{u}=\int_0^\delta \alpha dx.$$

If we assume, as is usually done, that (ω/α) is approximately independent of the electric field, then we have

$$i_-(x)=i_0e^u/[1-(\omega/\alpha)(\exp\bar{u}-1)]. \tag{9}$$

Equations (4), (5), and (9) may now be combined to give

$$\alpha E dE=\frac{[(4\pi i_0/\mu_+)(\exp\bar{u}-\exp u)du]}{[1-(\omega/\alpha)(\exp\bar{u}-1)]}. \tag{10}$$

In arriving at Eq. (10), $1/\mu_-$ has been considered to be negligible compared with $1/\mu_+$.

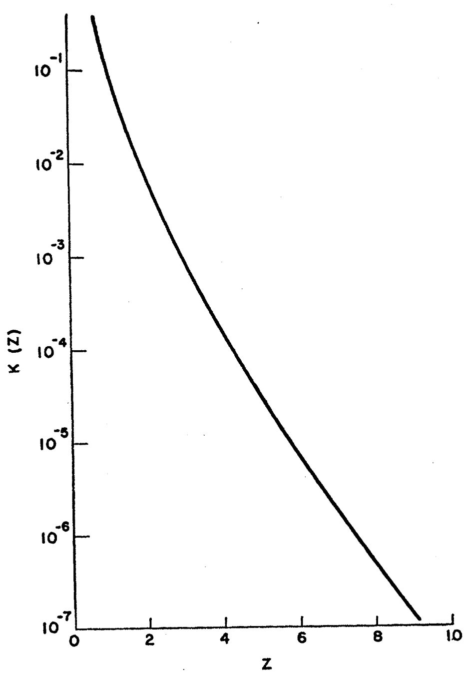

FIG. 1. Calculated curve of $K(z)$ *vs* z.

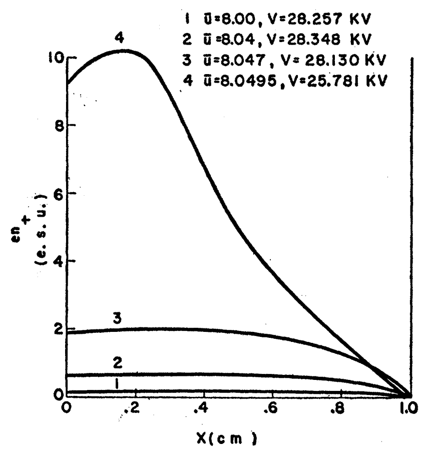

FIG. 2. Distribution of positive ions across the spark gap for various voltages near threshold.

Now to integrate Eq. (10), it is necessary to assume a functional form for the dependence of α upon E. For many gases, it has been demonstrated that α can be sufficiently approximated by an expression of the form,

$$\alpha/P=Ae^{-BP/E}, \tag{11}$$

in the greater part of the region of E/P significant to discharge phenomena. In this expression, P is the gas pressure, and A and B are constants characteristic of the gas. To be sure, Eq. (11) is not the only functional relation which can be applied accurately to this limited range of E/P. We use it here not only because it appears to have some theoretical significance,[7] but also because it contributes to the ease of computation. Thus, it is possible to show that

$$K(1/\eta)=K(1/\eta_0)+\frac{[(4\pi i_0/\beta\mu_+)(u\exp\bar{u}-\exp u+1)]}{[1-(\omega/\alpha)(\exp\bar{u}-1)]}, \tag{12}$$

where

$$\beta=AP(BP)^2,$$

$$\eta=E/BP,\quad \eta_0=E_0/BP,\quad E_0=E(0),$$

$$K(z)=\tfrac{1}{2}[e^{-z}/z^2-e^{-z}/z-\mathrm{Ei}(-z)],$$

$$-\mathrm{Ei}(-z)=\int_z^\infty (e^{-t}/t)dt.$$

The function $K(z)$ is plotted in Fig. 1.

We have then for the electric field

$$E=BP/K^{-1}\left\{K_0+\frac{[(4\pi i_0/\beta\mu_+)(u\exp\bar{u}-e^u+1)]}{[1-(\omega/\alpha)(\exp\bar{u}-1)]}\right\}. \tag{13}$$

Here K^{-1} is the inverse of the K function, and K_0

[7] T. Kihara, Revs. Modern Phys. 24, 45 (1952).

$=K(1/\eta_0)$. The restrictions on the solutions are

$$\int_0^\delta Edx=\int_0^{\bar{u}}(E/\alpha)du=V,\quad V=\text{voltage};$$
$$\int_0^\delta dx=\int_0^{\bar{u}}(1/\alpha)du=\delta,\quad \delta=\text{electrode spacing}. \tag{14}$$

These restrictions serve to fix K_0 and $\bar{u}$; that is, they are two equations, relating K_0 and $\bar{u}$, that must be solved simultaneously. This may be done by introducing Eq. (13) into each of the two equations; a numerical procedure must be used to obtain the required pair, $(K_0,\bar{u})$. All of the functions of interest, $E(x)$, $\alpha(x)$, $n_+(x)$, $i_-(x)$, etc., can then be calculated.

Equation (13) shows clearly the role of space charges in a Townsend breakdown process. It gives $E(x)$ as essentially equal to E_0 when

$$\left|\frac{4\pi i_0}{\beta\mu_+}\cdot\frac{u\exp\bar{u}-\exp u+1}{1-(\omega/\alpha)(\exp\bar{u}-1)}\right|\ll K_0.$$

Insertion of physically reasonable numbers shows that in most cases the inequality is violated only when the generalized Townsend threshold, defined by

$$(\omega/\alpha)(\exp\bar{u}-1)=1,$$

is approached. Numerical illustrations of the influence of space charges on the Townsend current are presented in the following sections.

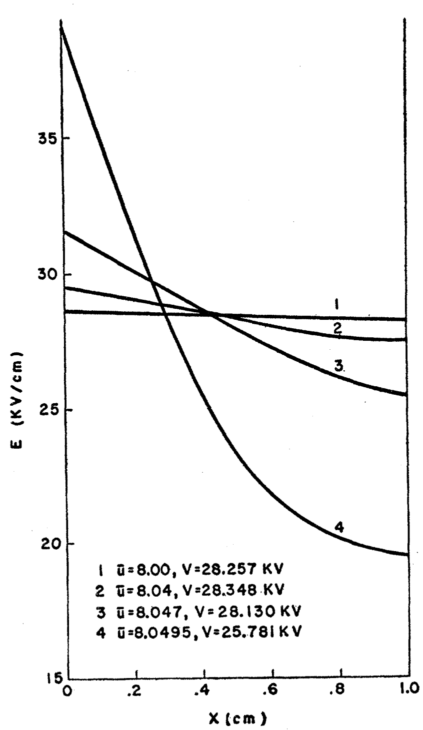

FIG. 3. Distribution of the electric field across the spark gap for various voltages near threshold.

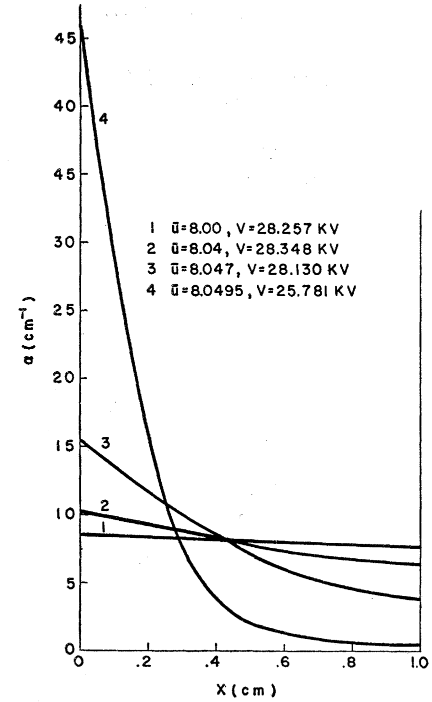

FIG. 4. Distribution of α across the spark gap for various voltages near threshold.

III. NUMERICAL ILLUSTRATIONS

A. Space Charge and the Townsend Threshold

Normally one can predict to within about 0.5 percent the sparking voltage of a gas by introducing Eq. (11), with experimentally determined constants, into Eq. (2); that is, by ignoring completely the possible influence of space charges. Furthermore, it has often been shown experimentally that, within certain limits, the magnitude of i_0 has little or no influence on the sparking potential. Therefore, we should be able to demonstrate by a numerical calculation that the introduction of the effects of space charges will lead to essentially the same threshold voltage as that predicted by the ordinary Townsend equation, provided i_0 is not too large.

In an effort to present a more meaningful illustration of the role played by space charges in a Townsend breakdown process, we have tried to choose the conditions of the problem so that our calculations will yield results close to those expected for a specific gas. Nitrogen was chosen because there is such a wealth of prebreakdown and breakdown data on this gas in the literature. The conduction data of Masch,[8] Posin,[9] and Dutton, Haydon, and Llewellyn Jones,[10] show that the

[8] K. Masch, Arch. Elektrotech. 22, 589 (1932).
[9] D. Q. Posin, Phys. Rev. 50, 650 (1936).
[10] Dutton, Haydon, and Llewellyn Jones, Proc. Roy. Soc. (London) A213, 203 (1952).

Townsend α can be expressed conveniently by the relation

$$\alpha/P=7.0e^{-260P/E}, \qquad (15)$$

provided E/P is less than about 120 (volts/cm) per mm Hg. Introduction of this result into Eq. (2) leads to an expression which describes the variation of the sparking potential with $P\delta$. By fitting this expression to the sparking data of Ehrenkrantz,[11] we obtain a value of 3.19×10^{-4} for (ω/α), in good agreement with the recent direct measurements of Dutton, Haydon, and Llewellyn Jones[10] in the region $40<E/P<45$.

The conditions we have chosen are: $P=700$ mm Hg, $\delta=1.0$ cm, $(\omega/\alpha)=3.19\times10^{-4}$, $B=260$ (volts/cm) per mm Hg, $A=7.0$ (cm$\times$mm Hg)$^{-1}$, $\mu_+=3.0\times760/P$ cm^2/volt sec, and $i_0=6.36\times10^{-11}$ amperes/cm^2. Our calculations, based on these values, show that space charges do not become important until the voltage is less than one percent below threshold; in this case the ordinary Townsend condition for breakdown is adequate.

However, the behavior of current and voltage sheds much light on the mechanism of the spark. Figures 2 through 5 detail the results of calculations for four voltages near threshold. The first three show the variation of n_+, E, and α across the gap. Note the increasing inhomogeneity as $\bar{u}$ increases. Figure 5 shows the plot of log$i$ *versus* V; the *approach* to threshold is essentially that described by the ordinary Townsend equation. A fundamental feature of the plot is the negative $i-V$ characteristic that develops beyond $i\approx10^{-5}$ ampere/cm^2; this illustrates the instability of the system beyond this point. Points on the negative characteristic represent

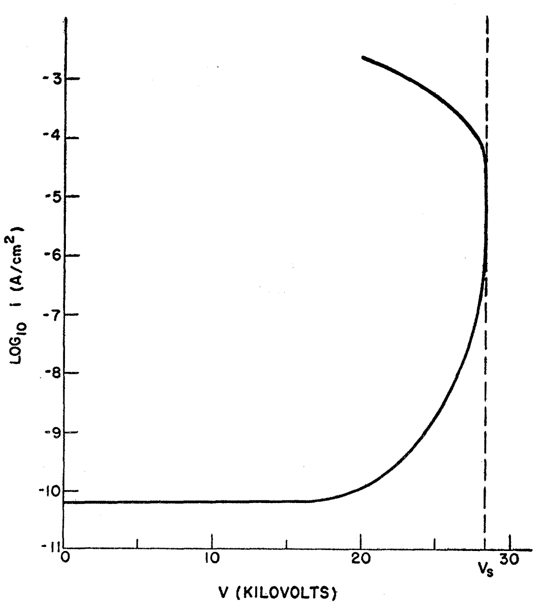

FIG. 5. Variation of the steady-state current with voltage.

[11] Florence Ehrenkrantz, Phys. Rev. **55**, 219 (1939).

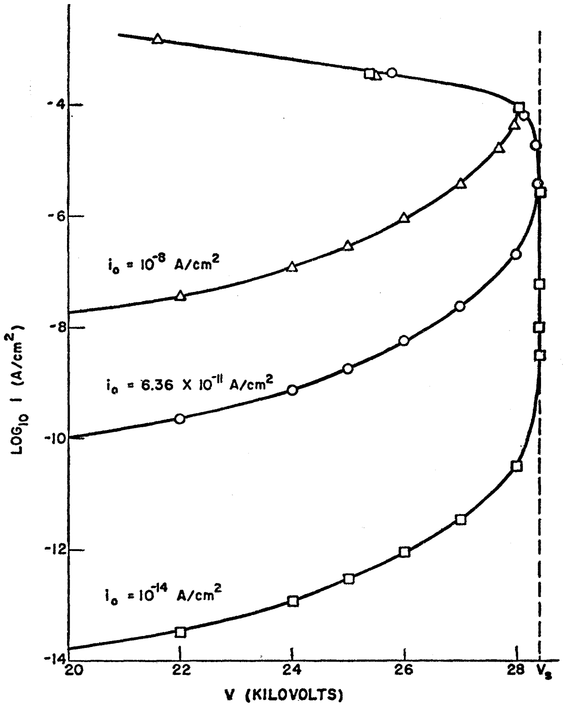

FIG. 6. The influence of the magnitude of i_0 on the sparking threshold.

states that are in principle steady states, but probably could not be realized experimentally.[12] The criterion for breakdown appropriate to this situation is that the slope of the log$i$ *versus* V curve become infinite, and not that the current itself reach some specified value as demanded by the ordinary Townsend equation. That is, if the external circuit is capable of supplying *at least* the current corresponding to the onset of the negative characteristic, the breakdown will not be influenced by the external circuit.

In order to demonstrate the influence of i_0 on the sparking threshold, we have carried out similar calculations with i_0 equal to 10^{-14} and 10^{-8} ampere/cm^2. The results, shown in Fig. 6, indicate that the influence of space charges on the threshold voltage is negligible unless i_0 is significantly larger than 10^{-8} ampere/cm^2.

It is important to mention that the negative $i-V$ characteristic can develop only if α increases with some power of E greater than one; otherwise the function is single-valued. This has been shown qualitatively by von Engel and Steenbeck.[3] Varney, White, Loeb, and Posin[6] also noted that such a dependence of α upon E could lead to the development of breakdown through space charge formation by α alone. However, the process was not discussed in terms of the negative characteristic.

[12] It should be pointed out that the existence of the negative characteristic, as calculated, much beyond the point of instability depends upon the applicability of the Townsend mechanism in this region. The space charge which gives rise to the negative characteristic may, under certain circumstances, result in the formation of a streamer, as has often been observed.

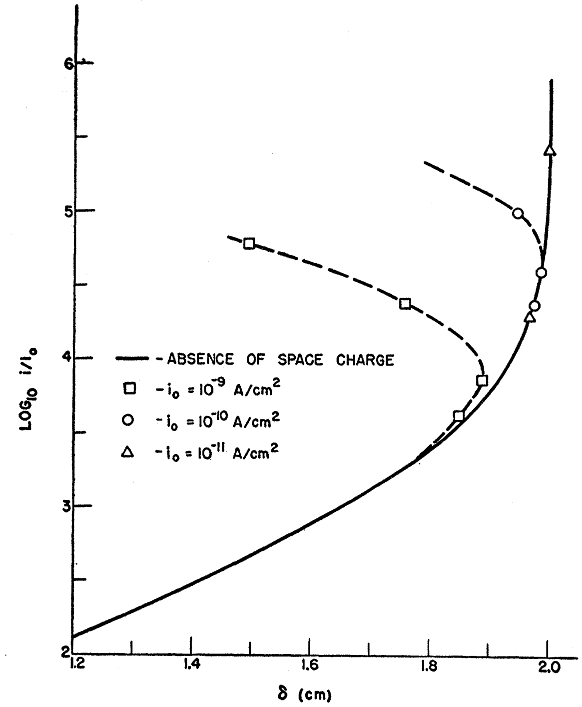

FIG. 7. The influence of space charges on the curvature of log$i$ *vs* δ plots at constant V/δ.

B. The Influence of Space Charge on Measurements of (ω/α)

There has been considerable discussion in the past concerning the possible influence of space charges on measurements of the secondary ionization coefficient, (ω/α). Equation (1) predicts a certain curvature in the plot of log$i$ *versus* δ at constant E/P as the sparking distance is approached. Experimental values of (ω/α) are usually obtained from the extent of this curvature. However, because space charges also become important as threshold is approached, their effects might complicate such measurements.

In 1936 Posin[9] found that it was impossible to evaluate accurately the secondary coefficient for N_2 from pre-breakdown current measurements unless i_0 was kept below about 10^{-13} ampere/cm². He attributed this to the effects of space charge. At about the same time, Varney, White, Loeb, and Posin[6] showed that such a field distortion can cause an increase in current more rapid than that predicted by the equation, $i=i_0e^{\alpha\delta}$, when i_0 is sufficiently large. However, secondary processes were omitted in their calculations.

In a recent paper, Dutton, Haydon, and Llewellyn Jones[10] have reported the results of a study of pre-breakdown currents in N_2, measured for various values of i_0. They found that the initial photocurrent could be varied from 6×10^{-15} amperes to 1.8×10^{-12} ampere[13] without any noticeable effect on the measured value of (ω/α). For these measurements, the gas pressure was 300 mm Hg, and the sparking distance 2.09 cm. For the highest value of i_0, the current became as high as 10^{-7} ampere as δ approached the sparking distance. This current is considerably in excess of that for which space charge distortion was commonly believed to be important.

In order to show numerically the effect of space charge distortion on the curvature of log$i$ as a function of δ, we must introduce Eq. (13) into the expression

$$\int_0^{\bar{u}} (E/\alpha)du \Big/ \int_0^{\bar{u}} (1/\alpha)du = V/\delta = \text{constant} \quad (16)$$

and again determine pairs $(K_0,\bar{u})$ that satisfy this condition. The current in the gap can then be calculated for various values of δ approaching threshold.

To illustrate, we have again chosen N_2, under approximately the same conditions used by Dutton, Haydon, and Llewellyn Jones[10] in their experimental measurements ($P=300$ mm Hg, $\delta_s=2.0$ cm, $V/\delta=1.245\times10^4$ volts/cm, and the same values of A, B, μ_+, and (ω/α) used in the previous calculation). The results are plotted in Fig. 7 for three values of i_0 (10^{-11}, 10^{-10}, and 10^{-9} ampere/cm²), and compared with the predictions of Eq. (1). Note again the negative characteristic that develops when space charges become important. These results indicate, however, that under the above conditions one should not observe any deviation of the curvature of $\log(i/i_0)$ from the Townsend prediction, until i_0 exceeds about 10^{-10} ampere/cm². That is, errors in measurement of (ω/α) should not appear until the initial photocurrent is large enough to cause a measurable decrease in the sparking distance. This current is far in excess of the largest value used by Dutton, Haydon, and Llewellyn Jones[10] in their experiments.

[13] Because approximately 0.5 cm² of the cathode was irradiated, this photocurrent is nearly the same as the current density.