and consider a specific amount of power being fed into the cavity, we can calculate the frequency for which η becomes unity. If we take $Q \sim 10^3 \lambda^{\frac{1}{2}}$, where λ is given in centimeters, and consider the case in which 10^{-4} watts is fed into the cavity, then η becomes unity for $\lambda \sim 10^{-2}$ cm. The electron wave packet half-width *b* which minimizes the mean square deviation in velocity for the same frequency dependence for the loaded *Q*, which is the one pertinent to the present discussion. this wavelength and the above assumptions is of the order of 10^{-4} cm. Thus, the minimum mean square deviation of the velocity becomes comparable to the modulation in velocity, for the particular conditions assumed, when the wavelength is of the order of a tenth of a millimeter.

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Penetration of 6-Mev Gamma Rays in Water

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The penetration of 6-Mev gamma rays has been studied out to 190 cm in water. The dose rate has been measured with an anthracene scintillation detector as a function of the distance from the N^{16} source. The results agree closely out to 160 cm with the distribution calculated according to the theory of gamma-ray penetration as developed by Spencer and Fano.

A MEASUREMENT has been made of the broad beam penetration of high-energy (~ 6 Mev) gamma rays from N¹⁶ in water. Very little experimental data have been heretofore available on broad beam penetration with simple geometries and none at all in this energy region. The results are also of interest as a check on the theoretical method of calculation of gamma-ray penetration as developed by Spencer and Fano.¹

The N¹⁶ source is obtained by circulating demineralized water through the high-flux region of the Materials Testing Reactor and then piping it into a flat cylindrical disk, 30 cm in diameter, which serves as the source. The disk is made of a tightly wound coil of $\frac{1}{4}$ -in. i.d. Saran tubing. It is located in a large body of water with a minimum of 4 ft of water in all directions from the coil. A detector is positioned so that it can be moved along the axis of the cylindrical disk. Variations in source intensity due to changes in water flow or reactor flux are compensated for by means of a monitor.

The detector and monitor are anthracene scintillation counters. The anthracene cylinders are $1\frac{1}{2}$ in. in diameter and 1 in. high and are optically attached to RCA 5819 photomultiplier tubes. The output current of the photomultiplier is read on a low drift ac electrometer. This current is a measure of the dose rate.

The following important reactions are expected to occur upon neutron irradiation in the reactor: $O^{16}(n,p)N^{16}$, $O^{17}(n,p)N^{17}$, and $O^{18}(n,\gamma)O^{19}$. The N¹⁶ production should be predominant. In order to verify

this and to check on the existence of any spurious activity from impurities, decay curves have been



¹L. V. Spencer and U. Fano, J. Research Natl. Bur. Standards 46, 446 (1951); Phys. Rev. 81, 464 (1951).

taken with the detecting crystal located 22.9 cm in water from the source coil.

The analysis shows a component with a 29-sec half-life (3 percent) and a component with a half-life of 7-8 sec (97 percent) at this distance in water. The half-life of O^{19} is 29 sec and that of N^{16} is 7.35 sec.² N¹⁷ does not emit any gamma radiation while O¹⁹ emits a gamma ray of 1.6 Mev. The energies of the N¹⁶ gamma rays have been previously measured and are given as 6.13 Mev and 7.10 Mev in the ratio of 12.5:1.² Thus, the source is essentially 6.13 Mev in energy with small components of 7.10 Mev and 1.6 Mev.

The experimental results are shown in Fig. 1. The detector reading corrected for background and compensated for changes in monitor reading is shown as a function of the distance along the cylindrical axis of the source coil. The solid curve is that calculated by the Spencer-Fano method utilizing the calculations provided by the joint NBS-NDA computational

² Nuclear Data, National Bureau of Standards Circular 499 (U. S. Government Printing Office, Washington, D. C., 1950). program.³ The calculations take into account the slightly non-uniform source distribution caused by the decay of N¹⁶ as the water flows through the coil. All three energy components have also been included, but in the region plotted the calculations differ by a maximum of 4 percent (and by less than 1 percent up to 100 cm) from the distribution obtained by assuming a monochromatic source of 6.13 Mev.

The theoretical curve is normalized to the experimental data at the point 11.4 cm from the coil. The experimental points agree very closely with the theoretical curve out to 160 cm. The trend toward disagreement beyond 160 cm is not considered to be conclusive because of a low signal-to-background ratio at these distances. From these results, it is felt that the Spencer-Fano theory is valid in the region investigated.

We are greatly indebted to Dr. George Mechlin and Dr. Warren Witzig for their invaluable assistance in the design and installation of the irradiation loop.

³ Goldstein, Wilkins, and Spencer, Phys. Rev. 89, 1150 (1953); NDA Memo 15C-20, 1953 (unpublished).

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Energy Spectrum of Turbulence for the Entire Range*

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The wave-number spectrum,

$$F(k) = \frac{(14\nu\langle u^2 \rangle)^{2/3}Ck^4 \exp\{-14\nu k^2 \langle u^2 \rangle / \langle \epsilon \rangle\}}{F(k)}$$

$$[14\nu \langle u^2 \rangle + \frac{2}{3}k^{13/2}C^{3/2}(1 - \exp\{-21\nu k^2 \langle u^2 \rangle / \langle \epsilon \rangle\})]^{2/3}$$

is derived from two limiting laws. The technique is very similar to that employed in blackbody radiation.

1. THE LIMITING LAWS

NONSIDER an incompressible fluid in a statistical steady state of homogeneous isotropic turbulence with mean energy per unit volume $\langle u^2 \rangle$, and mean dissipation per unit volume and unit time $\langle \epsilon \rangle$. F(k) will represent the wave-number energy spectrum so that

$$\langle u^2 \rangle = \int_0^\infty F(k) dk. \tag{1}$$

The dissipation is given by

$$\langle \epsilon \rangle = 2\nu \int_0^\infty k^2 F(k) dk,$$
 (2)

where ν is the kinematic viscosity. The spectrum F(k)is known for two limiting cases. When the Reynolds number is large (and $\langle \epsilon \rangle$ large),

$$F(k) \sim (\langle \epsilon \rangle)^{2/3} k^{-5/3}, \tag{3}$$

which is the Kolmogoroff-Onsager-von Weizäcker¹⁻³ limiting law. When the Reynolds is very small (and $\langle \epsilon \rangle$ small), then for large decay times

$$F(k) \sim Ck^4 \exp\{-2\nu k^2 t\},\tag{4}$$

which is the Batchelor-Karman-Lin^{4,5} limiting law. Since $t/7 = \langle u^2 \rangle / \langle \epsilon \rangle$ when (4) applies,

$$F(k) \sim Ck^4 \exp\{-14\nu k^2 \langle u^2 \rangle / \langle \epsilon \rangle\}.$$
 (5)

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