points from  $p$ - $p$  coincidence counting with a deuterium target  $[process (a)]$ . Figure 3 refers to process  $(b)$ , with data obtained partly by  $p\n-*n*$  coincidence counting and partly by counting neutrons alone, all from a deuterium target. Data on process (c), as well as the experimental description of the work and a detailed discussion, are to be given later.

The curves thus obtained give some information on nucleon-nucleon scattering. A first step in their analysis is to obtain from them whatever information is possible on phase shifts of the diferent partial waves.

By Fourier analysis of the asymmetric part of the cross section limited to the minimum number of harmonics necessary to fit the data, we find

$$
(P\sigma)_{p-p} = 1.348 \sin 2\theta + 0.242 \sin 4\theta + 0.116 \sin 6\theta, \qquad (3)
$$

$$
(P\sigma)_{n-p} = -0.016 \sin\theta + 0.958 \sin 2\theta + 0.324 \sin 3\theta + 0.366 \sin 4\theta.
$$
 (4)

The solid curves in Figs. 2 and 3 represent these equations. If the sixth harmonic term is omitted from Eq. (3) the fit is still fairly good; however, the fourth harmonic term is necessary in both Eqs. (3) and (4) to fit the data satisfactorily.

From the Fourier analysis it is possible to infer some conclusions on the phase shifts, as is to be shown in the following letter by Dr. B.D. Fried.

This work was performed under the auspices of the U. S. Atomic Energy Commission.

<sup>1</sup> Chamberlain, Segrè, Tripp, Wiegand, and Ypsilantis, Phys<br>Rev. **93,** 1430 (1954). We use the same notation as in this paper<br><sup>2</sup> Marshall, Magle, and Skolnik have worked on the<br>same subject. We thank them for communicat

before publication. '

 Chamberlain, Segre, and Wiegand, Phys. Rev. 83, 923 (1951); Chamberlain, Pettengill, Segre, and Wiegand, Phys. Rev. 93, 1424 (1954).

<sup>4</sup> Kelly, Leith, Segrè, and Wiegand, Phys. Rev. 79, 96 (1950).

## Phase Shifts for High-Energy Nucleon-Nucleon Scattering

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EVERAL attempts have recently been made to fit nucleon-nucleon scattering data using a small number of phase shifts.<sup>1</sup> In particular, Thaler and Bengston have shown that with seven phase shifts—'S, <sup>3</sup>S,  ${}^{1}P$ ,  ${}^{3}P$  and a single  ${}^{3}D$ —it is possible to get a good fit to both the  $p-p$  and  $n-p$  differential cross sections at an- energy of 260 Mev. It is of interest to see whether the recent experiments by Chamberlain  $et$   $al.^{2}$  on the scattering of polarized protons at a similar energy are consistent with the Thaler and Bengston phase shifts.

If partial waves with  $L>2$  are excluded and Coulomb effects are neglected, then, for an incident beam of completely polarized protons,

$$
(d\sigma/d\Omega)_{p-p} = k^{-2} [A_0 P_0(\cos\theta) + A_2 P_2(\cos\theta) + A_4 P_4(\cos\theta) + B_1 \cos\phi \sin\theta P_1(\cos\theta)], \quad (1)
$$

where k is the wave number of the relative motion,  $\theta$ and  $\phi$  are the usual center-of-mass polar angles, and the initial momentum and polarization of the incident protons have been chosen along the  $z$  and  $y$  axes, respectively. Explicit expressions for  $A_0$ ,  $A_2$ , and  $A_4$ in terms of phase shifts are given by Thaler, Bengston, and Breit.<sup>3</sup> In the same notation

$$
B_1 = 9S(\delta_{11}, \delta_{12}) + 6S(\delta_{10}, \delta_{12}), \tag{2}
$$

where  $S(x,y) = -S(y,x) = \sin x \sin y \sin(x - y)$ , and  $\delta_{1}x$ are the  ${}^{3}P$  phase shifts.

In their analysis, Thaler and Bengston assumed that (a) the nuclear interaction is charge-independent; (b) the triplet phase shifts are all equal,

$$
\delta_{21} = \delta_{22} = \delta_{23} = {}^{3}K_{2}; \tag{3}
$$

(c) the mixing of  ${}^{3}S$  and  ${}^{3}D$  waves, such as would result from a tensor force or other noncentral interaction, can be neglected. The cross section for polarized protons scattered by neutrons is then

$$
\begin{aligned} \left(d\sigma/d\Omega\right)_{p-n} &= \frac{1}{4} \left(d\sigma/d\Omega\right)_{p-p} + \frac{1}{4} k^{-2} \left\{ \sum_{n=0}^{4} C_n P_n(\cos\theta) \right. \\ &\left. + \cos\phi \sin\theta \left[D_0 P_0(\cos\theta) + D_2 P_2(\cos\theta)\right] \right\}. \end{aligned} \tag{4}
$$

Again, the  $C_n$  are given by TBB,<sup>3</sup> while

$$
D_0 = 5S(^{3}K_{0}, \delta_{12}) - 3S(^{3}K_{0}, \delta_{11}) - 2S(^{3}K_{0}, \delta_{10}),
$$
  
\nd  
\n
$$
D_2 = 5[5S(^{3}K_{2}, \delta_{12}) - 3S(^{3}K_{2}, \delta_{11}) - 2S(^{3}K_{2}, \delta_{10})],
$$
\n(5)

where  ${}^3K_0$  is the  ${}^3S$  phase shift.

and

An examination of the qualitative features of Eqs. (1) and (4) shows that they are incompatible with the data on the scattering of polarized protons as presented in Figs. 2 and 3 of the preceding letter. The following points may be made: (1<sup>o</sup>) According to Fig. 3,  $(P\sigma)_{p-n}$ contains an appreciable amount of fourth harmonic, whereas Eq. (4) allows none;  $(2^{\circ})$  The amplitudes of whereas Eq. (4) allows none; (2°) The amplitudes of<br>the second harmonic components of  $(P\sigma)_{p-p}$  and  $(P\sigma)_{p-n}$  have a ratio of 1.4 instead of the ratio of 4.00 the second harmonic components of  $(P\sigma)_{p-p}$  and  $(P\sigma)_{p-n}$  have a ratio of 1.4 instead of the ratio of 4.00 predicted by Eqs. (1) and (4); (3°) The  $(P\sigma)_{p-p}$  data cannot be fitted with a second harmonic term alone. cannot be fitted with a second harmonic term alone.

One obvious conclusion which may be drawn from these observations is that partial waves with  $L>2$  are necessary, as is certainly to be expected at this energy necessary, as is certainly to be expected at this energy  $(k^{-1} = 5.1 \times 10^{-14} \text{ cm})$ . In particular, since only triple partial waves can contribute to the polarization, the fourth harmonic in  $(P\sigma)_{p-p}$  requires at least a <sup>3</sup>F term. However, it may be noted that the discrepancies  $(1^{\circ})$ and  $(2^{\circ})$  above could be resolved without using partial waves with  $L>2$ . The absence of a second harmonic term in  $(P\sigma)_{p-n} - \frac{1}{4}(P\sigma)_{p-p}$  and the lack of a fourth

harmonic term in  $(P\sigma)_{p-n}$  are both consequences of the special assumptions (b) and (c) mentioned above. If the requirement  $(3^{\circ})$  and the assumption that  ${}^{3}S$  and  $D$  do not mix are dropped, then the missing second and fourth harmonic terms will appear (the former arising from interference effects between the  ${}^3S(Y_0^0)$ and  ${}^{3}D(Y_{2}^{\pm 1})$  partial waves, the latter from interference between different  ${}^{3}D$  partial waves). Thus, it may be possible to fit the data of Figs. 2 and 3 of the preceding letter and also the ordinary  $n-p$  and  $p-p$  differential cross sections (for unpolarized protons) by using the most general set of phase shifts consistent with  $L < 2$ , together with a single  ${}^{3}F$  phase shift  $\lceil$  to give the sin  $4\theta$ term in  $(P\sigma)_{p-p}$ .

The number of parameters available in this scheme is 13 (12 real phase shifts plus one real "mixing parameter" for the  $J=1$ ;  $L=0$ , 2 part of the scattering matrix'), while a Fourier analysis of the data will provide 18 Fourier coefficients —2 from  $(P\sigma)_{p-p}$ , 5 from.  $(Po)_{p-n}$ , and 4 and 7 from the unpolarized  $p-p$  and  $n-p$  cross sections, respectively). Of course, even aside from the experimental uncertainties in these Fourier coefficients, it is not likely that a unique set of phase shifts can be found, for as can be seen from Eqs. (2) and (5), the Fourier coefficients are by no means single-valued functions of the phase shifts. Calculations aimed at finding a set of phase shifts consistent with the present knowledge of the cross sections are now in progress.

I am indebted to Drs. Segre, Chamberlain, and Wiegand and to T. Ypsilantis and R. Tripp for many informative discussions on polarization experiments. This work was performed under the auspices of the Atomic Energy Commission.

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<sup>1</sup>A. Garren, Phys. Rev. 92, 213, 1587 (1953) and R. M. Thaler<br>and J. Bengston, Phys. Rev. 94, 679 (1954).<br><sup>2</sup> Chamberlain, Donaldson, Segrè, Tripp, Wiegand, and Ypsi-<br>lantis, preceding letter [Phys. Rev. 95, 850 (1954).]

258 (1953).

## $\pi$ -Meson Production in  $\pi$ -Nucleon Collisions at  $1.5$  Bev\*

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~ ~ ~ ~ collisions at 1.5 Bev lead to the production of a S has been pointed out previously,<sup>1</sup> most  $\pi^-$ single additional  $\pi$  meson. Since the last note more data have been obtained. (About 150  $\pi$ - $\phi$  interactions have been examined.) In particular, the reaction

$$
\pi^- + p \rightarrow \pi^+ + \pi^- + n
$$

shows characteristics very similar to those of the



FIG. 1. Angular distribution in the c.m. system of all the  $\pi$ 's from the reactions  $\pi^-+p\rightarrow \pi^-+p+\pi^0$  and  $\pi^-+p\rightarrow \pi^-+n+\pi^+$ .

production of a single  $\pi^0$ . The neutron usually goes backward in the center-of-mass system with an average momentum of 500 Mev/c. The  $\pi^+$  and  $\pi^-$  emerge from the reaction with an average angle of  $135^{\circ}$  between them in the c.m. system. The angular distribution of the  $\pi^+ + \pi^-$  is very similar to the angular distribution of the  $\pi$ <sup>-+ $\pi$ ° from the reaction  $\pi$ <sup>-+</sup>p- $\pi$ <sup>-+ $\pi$ °+p.</sup></sup>

The center-of-mass angular distribution of all the mesons from the above two reactions is given in Fig. 1. There is evidence of peaks in the forward and backward direction in the c.m. system. The forward peak contains mostly fast mesons, and the backward peak relatively slower mesons in the c.m. system, as shown by Fig. <sup>2</sup> which gives the angular distributions of mesons of



FIG. 2. Angular distributions of mesons of momenta greater than and less than 350 Mev/ $c$  in the c.m. system.