room temperature conductivity in an almost reversible way. At low heat-treatment temperatures there is reason to expect that the *n*-type impurities would segregate around edge dislocations because of the strain-energy interaction between edge dislocations and impurity atoms. If we heat treat a sample at a sufficiently high temperature, these impurities are thermally agitated, escape the strain potential energy of the dislocation, and distribute themselves uniformly throughout the crystal. We find that the conductivity of the sample goes down because of the compensation of these *n*-type impurities. The *p*-type impurities, probably substitutional, are considered to be immobile. If we now anneal the crystal at a temperature below 900°C, the thermal energy of the impurities is not sufficient to keep them from being trapped at the dislocations and we find that the conductivity goes back to its original value and slightly above. The aforementioned cycle is not closed because it is not possible to heat the sample to the range close to 1100°C without evaporating some of the impurities. If there were no evaporation of impurities at all, the flow of impurities in and out of edge dislocations would be a completely reversible process.

One of the striking similarities between the condensation of impurities and interstitial germanium at the dislocations in germanium and the condensation of impurities at dislocations in silicon is the fact that the critical temperature for condensation at the dislocation, which is 900°C in silicon, corresponds to the temperature at which plastic deformation can be first observed in silicon. Similarly, the temperature at which interstitial germanium and impurities like copper[4] condense at dislocation lines in germanium is approximately 500°C, which happens to be the temperature at which plastic deformation in germanium becomes noticeable. This correlation points up the possibility mentioned by Seitz[5] that the influence of impurity atoms in locking dislocations in germanium and silicon may have a dominant influence on the plastic properties of these semiconductors.

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Electron-Electron Scattering in Alkali Metals*

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The calculation of the electron-electron scattering cross section in a degenerate Fermi gas which is reported here was prompted by recent interest in the electron-electron collision mean free path in alkali metals. Reference is made here to recent developments in the interpretation[1] of electron spin resonance line widths,[2] work on electromagnetic absorptivity and anomalous skin effect in metals,[3] and thermal conductivity. In order that electron-electron collisions have an effect in these cases, the mean free path must be comparable to the ordinary electron-phonon mean free path which enters the expression for the resistivity. The result is that this condition is not met.

Table I. Electron-electron scattering cross section in sodium.

Energy of relative motion	Total cross section
0.253 rydberg = Fermi energy	$\sigma_F = 17\pi a_0^2$
0	$\sigma_0 = 60\pi a_0^2$

The electron-electron interaction in the metal is of the screened Coulomb type. The character of the electronic interactions in metals has been discussed at length by Pines.[4] We approximate the interaction by a potential of the form $V=(1/r)\exp(-r/r_0)$ (atomic units). Here r_0 is the screening radius for which values for various metals have been given in reference 4, and r is the separation of the interacting particles. We employ the one-electron approximation and use plane waves as the wave functions for the single-particle states. The effect of the periodic potential of the metallic crystal is accounted for by the introduction of an effective mass.

The validity criterion[5] for the Born approximation is not met in the present case since the electron energies are not large. The Born approximation is likely to be invalid for all problems involving the scattering of metallic electrons of a few volts energy unless the effective charge of the potential is very small. In the present case, the Born approximation overestimates the scattering by a factor 5.

The scattering must therefore be calculated by the partial wave method. Only *s*-wave scattering contributes appreciably to the cross section. Overhauser[6] has estimated the *s*-wave phase shift using a somewhat different potential. His result yields a smaller cross section than that obtained here. The calculation of the *s*-wave phase shift is carried out here by numerically integrating the wave equation for the relative motion within the range of the potential. The results for sodium are presented in Table I, where a_0 indicates the Bohr radius.

The mean free path l, for electron-electron collisions, can then be estimated by

$$l \approx (1/N\sigma_F)(E_F/kT)^2,$$

where N is the density of conduction electrons and E_F is the Fermi energy. The two factors of E_F/kT account for the availability of initial and final states which are restricted by the Fermi distribution function in the metal. The mean free path for sodium is then ~4.5 microns at 300°K and ~2.5 cm at 4°K. The electron-electron mean free path is thus longer than the electron-phonon mean free path throughout this range of temperatures.

At 8°K, the electron-phonon mean free path is about 0.08 cm as determined by the experiments of MacDonald and Mendelssohn.[7] If we extrapolate this result by means of the Bloch-Gruneisen T^5 law, we find that at 4°K, the electron-phonon and electron-electron free paths are comparable. The concept of an electron-phonon free path is used here for purposes of comparison of the importance of the two effects. 4°K is a temperature at which the lattice scattering is negligible compared to the residual resistance. These remarks tend to show that electron-electron scattering effects are less important that lattice or impurity scattering throughout all temperatures.

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Two Types of Band Emission Curves of Copper in the Soft X-Ray Region

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EXPERIMENTAL curves showing the energy distribution of the higher electronic levels in solid metals may be obtained from the soft x-ray emission spectra of metals. The methods which have been used may be described as follows: (a) the voltages applied to the anticathode are kept constant and the generally soft x-rays emitted are analyzed by means of a diffraction grating, and (b) the voltages applied to the anticathode are varied gradually, the soft x-rays thus emitted are detected with a photoelectric device, and the resulting voltage *vs* photocurrent curves are differentiated graphically.

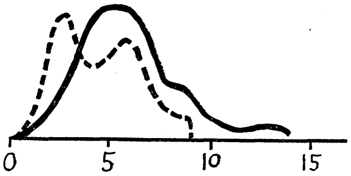


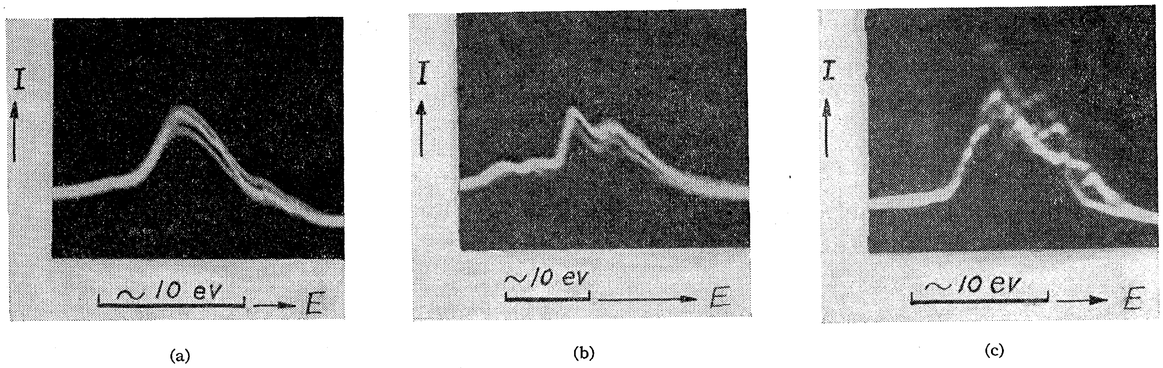
FIG. 2. The full curve shows the typical result obtained by previous investigators and the dotted one shows the theoretical curve of Rudberg and Slater (reference 6).

The former method has recently been improved and used by several investigators,[1] while the latter method has not been used for nearly thirty years.

We have improved this latter method. In our method electronic differentiation devices are used instead of an inaccurate graphical one, and the voltages applied to the anticathode are supplied from the saw-tooth wave oscillator of 100 cps. In this way spectral intensity curves of the soft x-ray may be shown directly on a cathode-ray oscillograph.[2,3] As the period of the repetition of the saw-tooth wave is fairly short, occasional changes due to adhesion and liberation of gas molecules do not cause any harmful effects.

Some of the results obtained for $M_{\mathrm{II,III}}$ emission curves of pure copper are shown in Fig. 1. Two types of spectrograms were found. The one shown in (a) resembles those of a number of workers,[4,5] while the one shown in (b) has not been hitherto found. The full curve in Fig. 2 shows the typical result obtained by previous investigators and the dotted one shows the result obtained by Rudberg and Slater[6] theoretically; the latter curve is quite similar to our curve (b).

Usually, a curve like that shown in (a) was observed on the oscillogram. However, the curve frequently changes its shape and after a short transient stage, as



(a) (b) (c)

FIG. 1. Two types of the $M_{\mathrm{II,III}}$ band emission curves of pure copper. The curve (a) resembles those of previous investigators, and (b) resembles Rudberg and Slater's theoretical curve. The transient time from (a) to (b) or (b) to (a) was very short. These two curves were observed simultaneously in photograph (c). Exposure: 1/25 sec.