VII. CONCLUSIONS

The neutron component of the cosmic radiation in water at mountain altitudes has been studied. From an analysis of the experimental results it can be concluded that neutron energy and spatial distributions undergo a transition in the first 30-cm layer of water adjacent to the surface. Below the transition region the thermal neutron intensity remains almost constant over a 20-cm interval of water. The neutron production rate and the thermal neutron flux in water at an altitude of 10 600 feet are 4.6×10^{-5} neutron g^{-1} sec^{-1} and about 4.3×10^{-4} thermal neutron cm^{-2} sec^{-1}, respectively.

VIII. ACKNOWLEDGMENTS

The author wishes to express his sincere appreciation to Professor S. A. Korff for suggesting the problem and for giving helpful suggestions during the course of the investigation. The author is indebted to Mr. A. C. Neuberg for his continued assistance in seeing the experiment through to its completion.

The author wishes to thank Professor Byron Cohn and Professor Mario Iona for their splendid cooperation during his stay at the Inter-University High Altitude Laboratory; and the Department of Mountain Parks of the City of Denver for their permission to use Echo Lake as the site for the experiment.

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High-Energy, Large-Angle Distribution of Pair Electrons Produced by Bremsstrahlung*

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The results of a calculation of the number of high-energy pair particles emitted into large angles in a pair production process initiated by bremsstrahlung are tabulated. A very approximate estimate of the effect of nuclear screening is given.

THE considerable amount of work being done at present with high-energy bremsstrahlung from electron synchrotrons and betatrons makes it useful to have some information on pair production at high energies and large angles. The computations below should have some use for estimating background and jamming levels, and may also be of interest in suggesting a test of some details of the pair production theory at high energies.

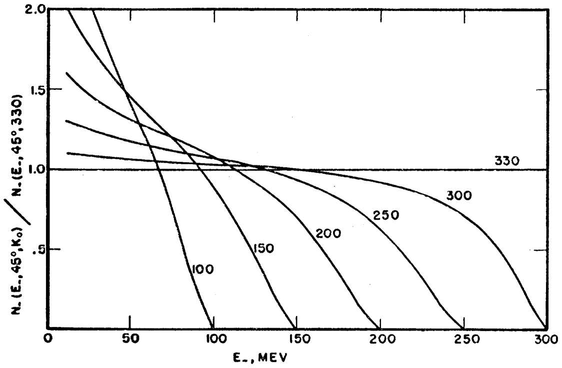

FIG. 1. Curves for extention of data in Table I to other values of k_o. Curves are labeled by k_o in Mev. These curves were calculated for $\theta_-=45°$, but may be used for other angles between 22.5° and 90° with an accuracy of a few percent.

The expression obtained by Hough[1] for the angular distribution of high-energy electrons (or positrons) emitted into large angles in a pair production process has been integrated over the bremsstrahlung spectrum. The spectrum used is the zero angle formula obtained by Schiff,[2] normalized to 1 erg/cm² beam energy. Our notation is that of Heitler,[3] *viz.*, E_-, p_-, θ_- are, respectively, the electron total energy, momentum $\times c$, and angle relative to the photon. The same symbols with subscript $+$ refer to the positron. μ is the electron rest energy, and k_0 is the maximum energy of the bremsstrahlung spectrum. $N_-(E_-,\theta_-,k_0)$ is the number of electrons per hydrogen nucleus per erg/cm² beam energy per Mev electron energy per steradian.

$$I_-(E_-,\theta_-,k_0)=\int_{E_-}^{k_0} N_-(E_-',\theta_-,k_0)dE_-'$$

is the number of electrons whose energy exceeds E_-, per hydrogen nucleus per erg/cm² beam energy per steradian.

Hough's expression results from an approximate integration of the Bethe-Heitler differential cross section over positron angles, and is valid for energies E_+, $E_-\gg\mu$ and angles $\theta_-\gg\mu/E_-$. Outer screening is neglected. For electron energies near the maximum energy k_0 of the bremsstrahlung spectrum, the number of electrons N_- receives significant contributions from

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* Assisted by the joint program of the U. S. Office of Naval Research and the U. S. Atomic Energy Commission, and by a grant from the National Science Foundation.

[1] P. V. C. Hough, Phys. Rev. **74**, 80 (1948), his formula 2.

[2] L. I. Schiff, Phys. Rev. **83**, 252 (1951).

[3] W. Heitler, *Quantum Theory of Radiation* (Oxford University Press, London, 1949), second edition, p. 196.

TABLE I. $N_-(E_-,\theta_-,k_o)$, the number of electrons per hydrogen nucleus per erg/cm^2 beam energy per Mev electron energy per steradian. $k_o=330$ Mev. N_- is given by $n\times 10^{-p}$, where n is the four-digit number at the left of each column and p the two-digit number at the right. The exponent p is given only when it changes. To obtain N_- for other elements, multiply by Z^2.

	θ_-																			
E_-	22.5°		30°		37.5°		45°		52.5°		60°		67.5°		75°		82.5°		90°	
(Mev)	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p
10.11	0.296	25	0.946	26	0.390	26	0.188	26	0.101	26	0.587	27	0.362	27	0.234	27	0.156	27	0.108	27
20.11	0.439	26	0.140		0.575	27	0.276	27	0.148	27	0.857	28	0.526	28	0.338	28	0.225	28	0.154	28
30.11	0.135		0.430	27	0.176		0.846	28	0.452	28	0.261		0.160		0.103		0.681	29	0.465	29
40.11	0.568	27	0.181		0.740	28	0.355		0.189		0.109		0.669	29	0.428	29	0.284		0.193	
51.10	0.269		0.855	28	0.350		0.168		0.895	29	0.516	29	0.316		0.202		0.133		0.907	30
61.83	0.146		0.465		0.190		0.912	29	0.487		0.281		0.171		0.109		0.723	30	0.491	
72.55	0.869	28	0.276		0.113		0.541		0.288		0.166		0.102		0.648	30	0.428		0.290	
83.28	0.549		0.175		0.714	29	0.342		0.182		0.105		0.640	30	0.408		0.269		0.183	
94.01	0.364		0.116		0.473		0.226		0.121		0.695	30	0.424		0.270		0.178		0.121	
104.73	0.251		0.797	29	0.326		0.156		0.830	30	0.478		0.291		0.186		0.122		0.827	31
115.46	0.178		0.565		0.231		0.111		0.589		0.339		0.206		0.131		0.865	31	0.585	
126.19	0.129		0.412		0.168		0.804	30	0.428		0.246		0.150		0.954	31	0.628		0.424	
136.92	0.961	29	0.306		0.125		0.597		0.318		0.183		0.111		0.707		0.465		0.314	
147.64	0.726		0.231		0.943	30	0.450		0.240		0.138		0.838	31	0.533		0.350		0.236	
158.37	0.556		0.177		0.722		0.345		0.183		0.105		0.641		0.407		0.267		0.180	
169.10	0.431		0.137		0.559		0.267		0.142		0.814	31	0.495		0.314		0.206		0.139	
179.82	0.337		0.107		0.436		0.208		0.111		0.636		0.386		0.245		0.161		0.108	
190.55	0.265		0.841	30	0.343		0.164		0.870	31	0.499		0.303		0.192		0.126		0.846	32
201.28	0.209		0.664		0.271		0.129		0.686		0.393		0.239		0.151		0.990	32	0.665	
212.00	0.166		0.526		0.214		0.102		0.541		0.310		0.188		0.119		0.780		0.524	
222.73	0.131		0.415		0.169		0.805	31	0.427		0.244		0.148		0.938	32	0.614		0.412	
233.46	0.103		0.327		0.133		0.632		0.335		0.192		0.116		0.735		0.481		0.322	
244.18	0.805	30	0.255		0.103		0.492		0.260		0.149		0.902	32	0.571		0.373		0.250	
254.91	0.619		0.196		0.794	31	0.377		0.200		0.114		0.691		0.437		0.286		0.191	
265.64	0.466		0.147		0.595		0.283		0.150		0.855	32	0.517		0.327		0.214		0.143	
276.37	0.339		0.107		0.432		0.205		0.108		0.619		0.374		0.237		0.155		0.104	
287.09	0.233		0.731	31	0.296		0.140		0.742	32	0.424		0.256		0.162		0.106		0.708	33
297.82	0.146		0.457		0.185		0.876	32	0.463		0.264		0.160		0.101		0.659	33	0.441	
308.55	0.753	31	0.236		0.953	32	0.452		0.239		0.136		0.825	33	0.521	33	0.340		0.227	
319.27	0.233		0.729	32	0.294		0.140		0.737	33	0.421	33	0.254		0.161		0.105		0.701	34

events involving positrons whose energy is too small to satisfy the requirement $E_+ \gg \mu$. The error in our estimates from this source is about 5 percent if E_- is less than k_0 by 10 Mev, and decreases steadily as the difference between k_0 and E_- increases.

Table I lists the results for $k_0=330$ Mev, and these are extended to other values of k_0 in Fig. 1. The angular dependence is $(\cot\frac{1}{2}\theta_-/\sin\frac{1}{2}\theta_-)^2$ to within a few percent, and the energy variation is, very roughly, $E_-^{-3.5}$. The error in these estimates from violation of the conditions $E_- \gg \mu$, $\theta_- \gg \mu/E_-$ is less than 3 percent in the most unfavorable case ($E_-=5$ Mev, $\theta_-=22.5°$), and decreases rapidly for larger E_- or θ_-. Table II presents the number of electrons whose energy exceeds a given energy. k_0 is 330 Mev, and Fig. 2 extends the results to other values of k_0. These estimates were obtained by graphical integration of the results in Table I.

Failure of the Born approximation for moderate and high Z elements and the effect of recoil corrections on the pair cross section for light elements,[4] both of which may be noticeable at high energies and large angles, are neglected in these results. A very approximate account of inner screening, which, because of the finite size of the nuclear charge distribution, should begin markedly reducing N_- whenever $[2(p_-/\hbar)$

TABLE II. $I_-(E_-,\theta_-,k_o)=\int_{E_-}^{k_o} N_-(E_-',\theta_-,k_o)dE_-'$, the number of electrons whose energy exceeds E_-, per hydrogen nucleus per erg/cm^2 beam energy per steradian. k_o is 330 Mev. I_- is given by $n\times 10^{-p}$, where n is the four-digit number at the left of each column and p the two-digit number at the right. The exponent p is given only when it changes. To obtain I_- for other elements, multiply by Z^2.

	θ_-																			
E_-	22.5°		30°		37.5°		45°		52.5°		60°		67.5°		75°		82.5°		90°	
(Mev)	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p	n	p
5.00	0.571	24	0.183	24	0.754	25	0.364	25	0.196	25	0.114	25	0.708	26	0.458	26	0.308	26	0.213	26
7.50	0.277		0.887	25	0.365		0.176		0.945	26	0.549	26	0.338		0.218		0.146		0.100	
10.00	0.168		0.537		0.221		0.106		0.570		0.330		0.203		0.131		0.871	27	0.598	27
12.50	0.111		0.354		0.145		0.698	26	0.374		0.217		0.133		0.855	27	0.569		0.390	
15.00	0.782	25	0.250		0.102		0.492		0.263		0.152		0.935	27	0.600		0.399		0.273	
17.50	0.579		0.185		0.757	26	0.364		0.195		0.113		0.690		0.442		0.294		0.201	
20.00	0.444		0.142		0.580		0.278		0.149		0.860	27	0.527		0.338		0.224		0.153	
22.50	0.350		0.111		0.456		0.219		0.117		0.676		0.414		0.265		0.176		0.120	
25.00	0.282		0.897	26	0.367		0.176		0.942	27	0.544		0.333		0.213		0.141		0.962	28
27.50	0.230		0.733		0.300		0.144		0.769		0.444		0.272		0.174		0.115		0.784	
30.00	0.191		0.609		0.249		0.120		0.638		0.368		0.225		0.144		0.954	28	0.649	
35.00	0.137		0.435		0.178		0.854	27	0.456		0.263		0.161		0.103		0.680		0.462	
40.00	0.102		0.323		0.132		0.634		0.338		0.195		0.119		0.762	28	0.503		0.342	
45.00	0.778	26	0.248		0.101		0.485		0.259		0.149		0.911	28	0.582		0.384		0.261	
50.00	0.610		0.194		0.793	27	0.380		0.203		0.117		0.713		0.455		0.300		0.204	
55.00	0.487		0.155		0.633		0.303		0.162		0.930	28	0.568		0.362		0.239		0.162	
60.00	0.394		0.125		0.513		0.245		0.131		0.753		0.460		0.293		0.193		0.131	
65.00	0.324		0.103		0.421		0.201		0.107		0.618		0.377		0.240		0.159		0.107	
70.00	0.269		0.855	27	0.350		0.167		0.891	28	0.513		0.313		0.199		0.131		0.889	29
80.00	0.190		0.604		0.247		0.118		0.630		0.362		0.221		0.141		0.926	29	0.626	
90.00	0.139		0.440		0.180		0.861	28	0.459		0.264		0.161		0.102		0.673		0.455	
100.00	0.103		0.328		0.134		0.642		0.342		0.196		0.120		0.761	29	0.501		0.338	

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[4] S. D. Drell, Phys. Rev. **87**, 753 (1952).

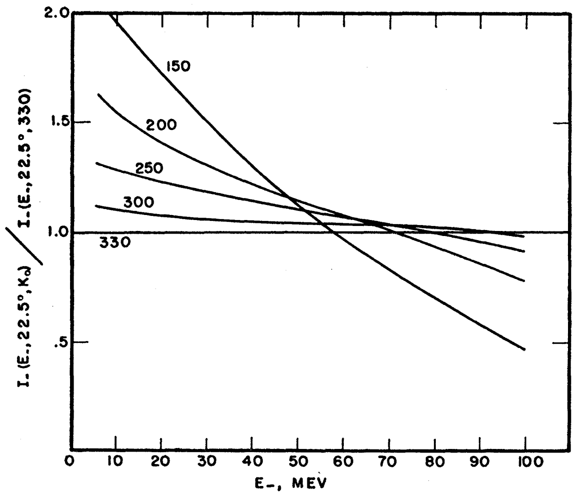

FIG. 2. Curves for extension of data in Table II to other values of k_o. Curves are labeled by k_o in Mev. These curves were calculated for $\theta_-=22.5°$, but may be used for all angles between 22.5° and 90° with an accuracy of a few percent.

$\times \sin\frac{1}{2}\theta_-]^{-1}$ (cm) is comparable with the nuclear radius, is given below.

For a spherically symmetric charge distribution, the nuclear form factor is, in the first Born approximation, a function only of the magnitude q of the nuclear recoil momentum. The form factor for a uniform charge distribution, assuming monochromatic photons, was calculated in the Born approximation by Hough,[5] who recommends the use of a value of q intermediate between $\bar{q}\equiv 2p_- \sin\frac{1}{2}\theta_-$ and $q_{\min}\equiv (k^2+p_-^2-2kp_- \times \cos\theta_-)^{\frac{1}{2}}-p_+$. ($q_{\min}$ is the smallest nuclear recoil kinematically possible for a given photon momentum k and given electron energy and angle.)

Reference to Fig. 3 shows that if p_+ is small, $\bar{q}\simeq q_{\min}$; on the other hand, if p_+ is large, the bulk of contributions to N_- come from the region $\theta_+\simeq 0$.[6] Since, then, $k-p_+\simeq p_-$, and $q\simeq 2p_- \sin\frac{1}{2}\theta_-=\bar{q}$.[7] Events for which $q>\bar{q}$ contribute negligibly to N_-. Hence, we may argue that the use of $q\simeq\bar{q}$, which is independent of k, and consequently leaves the inner screening calculation unaffected by integration over the bremsstrahlung spectrum, is sufficiently accurate for rough estimates. This is especially true in view of the fact that the Born approximation itself cannot be taken seriously when applied to moderate and high Z elements, where nuclear screening is of most importance.

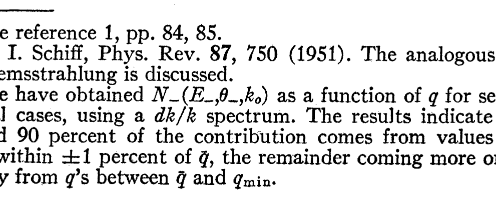

FIG. 3. Momentum diagram for a pair production event in which the electron is emitted with high energy into a large angle θ_-. The nuclear recoil energy is neglected. To illustrate for the case $q=q_{\min}$ (which corresponds to Heitler's $\phi_+=180°$), all vectors should be taken in the plane of the paper.

The Born approximation correction for screening by a uniform charge distribution is given in Fig. 4 for Cu and Al, assuming a radius $R=1.2\times 10^{-13}A^{\frac{1}{3}}$ cm. Yennie's phase shift calculations[8] for electron scattering from a uniform charge distribution show that the diffraction "zeros" in Fig. 4 are spurious, but that the Born approximation for Cu agrees fairly well with the exact phase shift calculation, except near these "zeros." The form factor is

$$F(q)=3[\sin(qR)-(qR)\cos(qR)]/(qR)^3,$$

and the correction is obtained by multiplying $|F(q)|^2$ into the results of Table I.

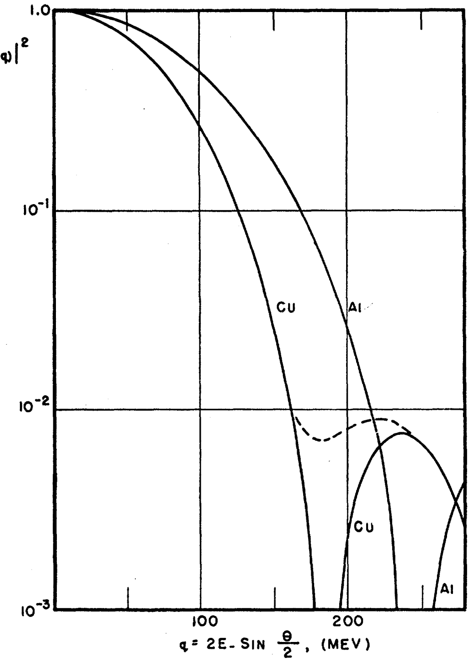

FIG. 4. $|F(q)|^2$, squared form factor for a uniform charge distribution, calculated in the Born approximation. $R=1.2\times 10^{-13}$ $A^{\frac{1}{3}}$ cm. The dashed curve is an estimate, based on Yennie's graphs, of the actual shape of $|F(q)|^2$ near the first "zero" in Cu.

Part of this calculation was made using the Illinois Digital Computer. I wish to thank Professor C. S. Robinson for suggesting and encouraging this work, and to acknowledge support by a General Electric Fellowship.

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[5] See reference 1, pp. 84, 85.

[6] L. I. Schiff, Phys. Rev. **87**, 750 (1951). The analogous case for bremsstrahlung is discussed.

[7] We have obtained $N_-(E_-,\theta_-,k_o)$ as a function of q for several typical cases, using a dk/k spectrum. The results indicate that around 90 percent of the contribution comes from values of q lying within ±1 percent of $\bar{q}$, the remainder coming more or less equally from q's between $\bar{q}$ and $q_{\min}$.

[8] Yennie, Wilson, and Ravenhall, Phys. Rev. **92**, 1325 (1953).