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Analysis of Magnetoresistance and Hall Coefficient in *p*-Type Indium-Antimonide and *p*-Type Germanium

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Existing theory on the variation of resistivity and Hall coefficient with magnetic field in semiconductors has been extended to the case of arbitrary concentrations of the electrons and holes. Equations are developed for the magnetoresistance $\Delta\rho/\rho_H$ and the Hall coefficient R_H, as functions of parameters such as temperature, magnetic-field strength, and impurity concentration. For *p*-type InSb, theory and experiment are in agreement on the following observations: (1) a shift to higher temperatures of the Hall coefficient crossover, magnetoresistance maxima, and Hall coefficient maxima with increasing magnetic field, (2) a decrease in magnitude of the Hall coefficient maxima with increasing magnetic field, (3) the occurrence of the largest effects in the transition region between extrinsic and intrinsic conductivity, (4) magnetoresistance maxima occurring at about the same temperature as Hall coefficient maxima.

For *p*-type germanium, however, the effects are not even qualitatively explained. The observed magnetic-field dependence of the Hall coefficient in the temperature region of the crossover is opposite to that predicted by the theory. Furthermore, no magnetoresistance maximum has been observed in the temperature region of the Hall coefficient maximum. Hence, it appears that a basic revision of existing treatments of *p*-type germanium is necessary.

INTRODUCTION

THERE has been an increasing amount of interest in the effects of magnetic field on the resistivity and Hall coefficient of semiconductors. There have been numerous contributors to the development of both the theoretical and experimental aspects. Recently, Johnson and Whitesell[1] have extended the theory by considering (1) scattering of charge carriers by impurity ions as well as by the lattice and (2) conduction by both electrons and holes of equal concentrations. Considering lattice scattering only, Madelung[2] has extended their work to include arbitrary concentrations of the electrons and holes, while Appel[3] has obtained analytical expressions for the weak and strong magnetic field of (1) above, as well as generalizing (2). The treatment presented here is also an extension of the earlier work. It is, however, presented in a somewhat different form, which we find readily adaptable to the analysis of our experimental results.

DEVELOPMENT OF EQUATIONS

The conductivity σ_H and Hall coefficient R_H of a two-carrier semiconductor in the presence of an external magnetic field can be written as[1]

$$\sigma_H=\frac{(A_1+A_2)^2+(B_1-B_2)^2}{A_1+A_2}, \tag{1}$$

$$R_H=\frac{-(B_1-B_2)}{H\{(A_1+A_2)^2+(B_1-B_2)^2\}}, \tag{2}$$

where subscripts 1 and 2 apply to electrons and holes, respectively. For the case of (1) mean free path independent of energy, (2) an arbitrary transverse magnetic field, (3) classical statistics, and (4) spherical energy surfaces,

$$A_j=n_je\mu_jK_1(\gamma_j), \quad j=1,2, \tag{3}$$

$$B_j=n_je\mu_j\gamma_j^{\frac{1}{2}}K_2(\gamma_j), \tag{4}$$

$$\gamma_j=1.77\mu_j^2H^2, \tag{5}$$

where n_j is the concentration of the electrons and holes and μ_j is the lattice mobility of the electrons and holes in zero magnetic field. All electrical quantities are in electromagnetic units. The K's can be expressed in terms of tabulated functions[4] as follows:

$$K_1(\gamma)=1-\gamma+\gamma^2e^{\gamma}\{-\mathrm{Ei}(-\gamma)\}, \tag{6}$$

$$K_2(\gamma)=\tfrac{1}{2}\pi^{\frac{1}{2}}(1-2\gamma)+\pi\gamma^{\frac{3}{2}}e^{\gamma}[1-\Phi(\gamma^{\frac{1}{2}})]. \tag{7}$$

For large values of γ, the asymptotic expressions are

$$K_1(\gamma)=\frac{2!}{\gamma}-\frac{3!}{\gamma^2}+\frac{4!}{\gamma^3}-\cdots, \tag{8}$$

$$K_2(\gamma)=\pi^{\frac{1}{2}}\left(\frac{1\cdot3}{2^2\gamma}-\frac{1\cdot3\cdot5}{2^3\gamma^2}+\frac{1\cdot3\cdot5\cdot7}{2^4\gamma^3}-\cdots\right). \tag{9}$$

For $H=0$,

$$A_j^0=n_je\mu_j, \tag{10}$$

and

$$\frac{\sigma_0-\sigma_H}{\sigma_0}=\frac{\Delta\rho}{\rho_H}=1-\frac{A_1+A_2}{A_1^0+A_2^0}-\frac{(B_1-B_2)^2}{(A_1+A_2)(A_1^0+A_2^0)}. \tag{11}$$

Substituting Eqs. (3), (4), and (10), in Eq. (11), and

[1] V. A. Johnson and W. J. Whitesell, Phys. Rev. **89**, 941 (1953).
[2] O. Madelung, Z. Naturforsch. **8a**, 791 (1953).
[3] J. Appel, Z. Naturforsch. **9a**, 167 (1954).
[4] See Jahnke-Emde, *Tables of Functions* (Dover Publications, New York, 1945).

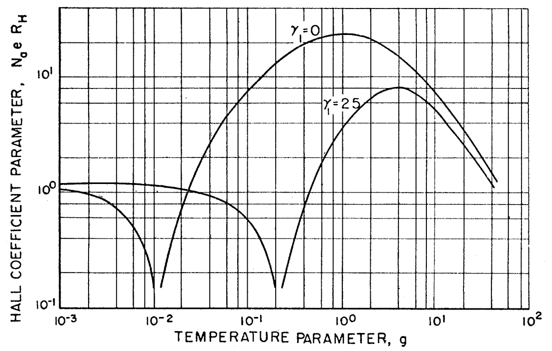

FIG. 1. Hall coefficient parameter $N_a e R_H$ as a function of the temperature parameter g for $c=85$, $\gamma_1=0$, 25.

defining $g\equiv n_1\mu_1/n_2\mu_2$, $c\equiv\mu_1/\mu_2$, one obtains

$$\frac{\Delta\rho}{\rho_H}=1-\frac{gK_1(\gamma_1)+K_1(\gamma_2)}{g+1}-\frac{\gamma_2\{gcK_2(\gamma_1)-K_2(\gamma_2)\}^2}{(g+1)\{gK_1(\gamma_1)+K_1(\gamma_2)\}}, \tag{12}$$

and

$$R_H=-3\pi^{\frac{1}{2}}/4n_2e\cdot\frac{\{gcK_2(\gamma_1)-K_2(\gamma_2)\}}{\{gK_1(\gamma_1)+K_1(\gamma_2)\}^2+\gamma_2\{gcK_2(\gamma_1)-K_2(\gamma_2)\}^2}. \tag{13}$$

For a p-type semiconductor with an acceptor density N_a, assumed completely ionized, one has

$$n_2=n_1+N_a, \tag{14}$$

$$n_1n_2\equiv n_i^2=4[2\pi(m_1m_2)^{\frac{1}{2}}kT/h^2]^3e^{-W/kT}. \tag{15}$$

Hence,

$$g=c\left\{\frac{-N_a+(N_a^2+4n_i^2)^{\frac{1}{2}}}{+N_a+(N_a^2+4n_i^2)^{\frac{1}{2}}}\right\}. \tag{16}$$

Also, Eq. (13) may be written

$$R_H=-\frac{3\pi}{8N_ae}\cdot\frac{2}{\pi^{\frac{1}{2}}}\cdot\frac{\{gcK_2(\gamma_1)-K_2(\gamma_2)\}\{1-g/c\}}{\{gK_1(\gamma_1)+K_1(\gamma_2)\}^2+\gamma_2\{gcK_2(\gamma_1)-K_2(\gamma_2)\}^2}. \tag{17}$$

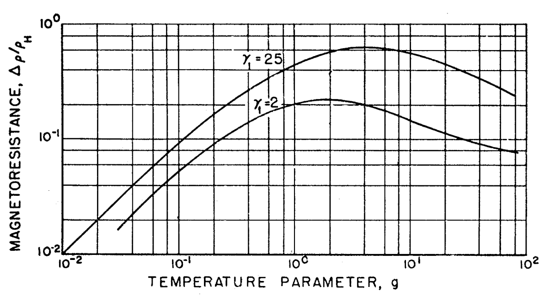

FIG. 2. Magnetoresistance $\Delta\rho/\rho_H$ as a function of the temperature parameter g for $c=85$, $\gamma_1=2$, 25.

The preceding relationships are sufficient to determine $\Delta\rho/\rho_H$ and R_H as functions of temperature, impurity concentration, and magnetic-field strength for those cases where the conditions previously enunciated are satisfied.

DISCUSSION OF THE THEORETICAL RESULTS

Assuming for a particular specimen of germanium or InSb that the lattice mobility ratio and the density of ionized impurities can be considered constant through-

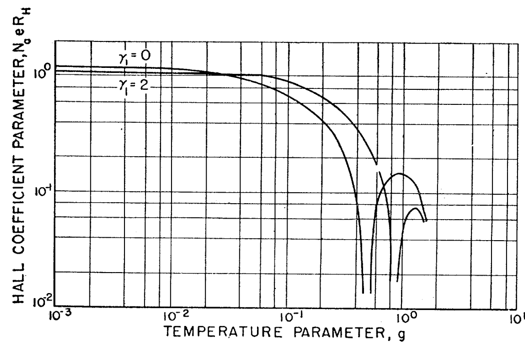

FIG. 3. Hall coefficient parameter as a function of the temperature parameter g for $c=2$, $\gamma_1=0$, 2.

out the temperature interval investigated, then the Hall coefficient and the magnetoresistance are functions of only two independent variables, namely, g and γ_1. Since g is a monotonic function of temperature, plots of $\Delta\rho/\rho_H$ or R_H as functions of g for various values of γ_1 reveal the essential temperature-dependent characteristics of the magnetoresistance and Hall effects. A representation of this type gives a good portrayal of

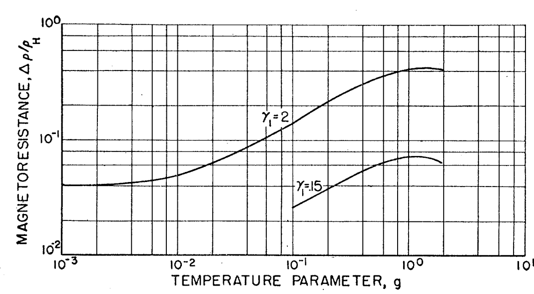

FIG. 4. Magnetoresistance $\Delta\rho/\rho_H$ as a function of the temperature parameter g for $c=2$, $\gamma_1=0.15$, 2.

the temperature dependence of the effects since g varies much more rapidly over the temperature interval of interest than does γ_1.

Such a plot for the Hall effect is shown in Fig. 1. A mobility ratio value 85, representative of InSb,[5] was used. It is readily seen that (1) the Hall coefficient maxima shift to larger values of g (thus to higher temperatures) with increasing magnetic-field strength, (2) the magnitudes of the Hall coefficient maxima decrease

[5] M. Tannenbaum and J. P. Maita, Phys. Rev. **91**, 1009 (1953).

with increasing field, (3) the temperature at which the Hall coefficient crossover occurs increases with increasing field, (4) the variation in Hall coefficient with field in the extrinsic and intrinsic regions are small. A similar presentation for magnetoresistance is shown in Fig. 2. The following points are obvious: (1) the magnetoresistance maxima shift to higher temperature with increasing field, (2) the magnetoresistance maxima occur at approximately the same temperature as the Hall coefficient maxima, (3) at a given temperature, the magnetoresistance increases with magnetic field, (4) as $g \rightarrow 0$, corresponding to the extrinsic region, all carrier-density parameters vanish, leaving Eq. (12) a function only of μ_2. This suggests the occurrence of a minimum at low temperatures in those specimens where lattice scattering still predominates in the extrinsic region so that μ_2 rises with decreasing temperature.

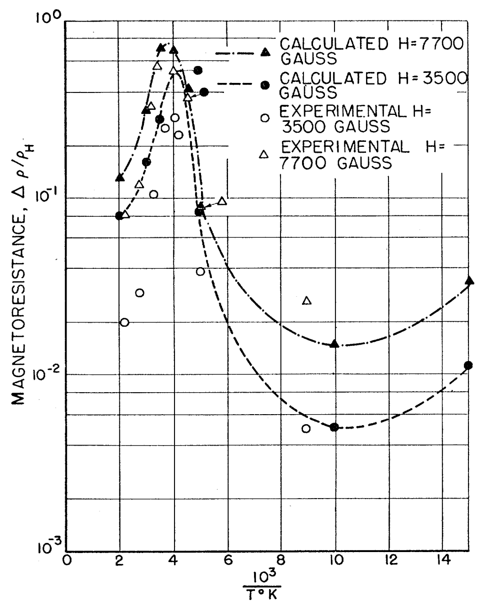

FIG. 5. Calculated and experimental Hall coefficient of a p-type InSb specimen as a function of inverse absolute temperature.

Figures 3 and 4 present similar curves computed for a mobility ratio characteristic of germanium. There is little difference in shape between these curves and the InSb curves. Consequently, this treatment indicates the variation of resistivity and Hall coefficient with magnetic field to be qualitatively the same for the two materials.

COMPARISON OF THEORY AND EXPERIMENT

The magnetoresistance[6] and Hall coefficient[7] of an InSb specimen with 2×10^{16} ionized impurities has been measured as a function of temperature for two magnetic-field strengths. In Fig. 5 and Fig. 6, these experi-

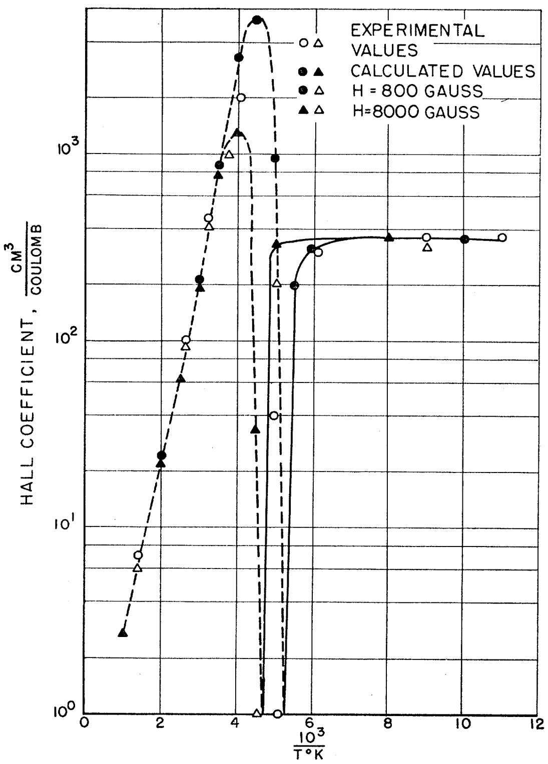

FIG. 6. Calculated and experimental magnetoresistance of a p-type InSb specimen as a function of inverse absolute temperature.

mental data are shown, along with the calculated[8] values. It is obvious that the theory describes the essential behavior of $\Delta\rho/\rho_H$ and R_H as functions of temperature. As to actual magnitudes, however, discrepancies do exist. At the lower temperatures, this might, in part, be accounted for by impurity scattering. At the higher temperatures, there are indications that degeneracy is

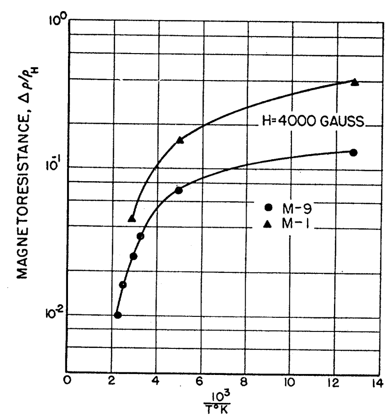

FIG. 7. Magnetoresistance of two p-type germanium specimens, $M-1$ ($\rho_{300°K}=55$ ohm-cm) and $M-9$ ($\rho_{300°K}=2.2$ ohm-cm), as a function of inverse absolute temperature.

[6] Harman, Willardson, and Beer, Phys. Rev. **93**, 912 (1954).
[7] Willardson, Beer, and Middleton, Phys. Rev. **93**, 912 (1954).

[8] The values of n_i^2 and of lattice mobilities used in the calculations are given in reference 5.

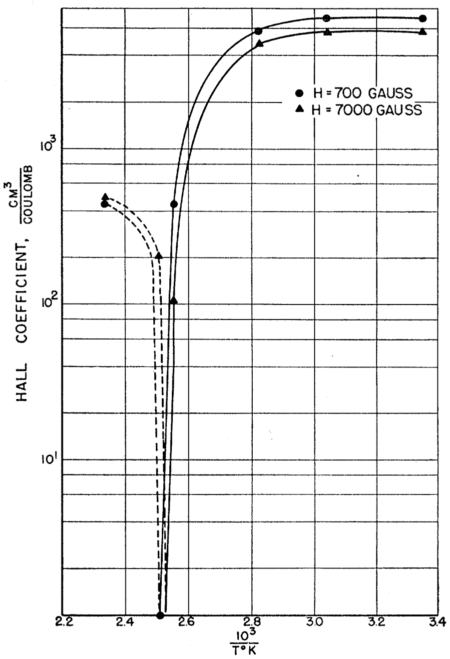

FIG. 8. Hall coefficient of p-type germanium specimen $M-9$ as a function of inverse absolute temperature for two magnetic-field strengths.

significant[9] even in p-type InSb. Finally, there is also the possibility that the mobility ratio might vary over the relatively large temperature over which the data were taken.

The results of measurements of the transverse magnetoresistance as a function of temperature in two germanium specimens are shown in Fig. 7. Unlike InSb and contrary to the theoretical predications, no magnetoresistance maxima were observed in the vicinity of the Hall coefficient maxima. A second salient feature is shown in Fig. 8, where it is seen that the variation of the Hall coefficient with magnetic field in the temperature region of the crossover is in the opposite direction to that predicted by the theory and to that observed in InSb. This radical disagreement with the theory for p-type germanium is basic and does not appear explicable by considerations of such obvious effects as impurity scattering, degeneracy, temperature dependence of mobility ratio, etc. It appears that a more fundamental modification of the theory is necessary.

Our recent investigations have shown that this anomalous behavior in p-type germanium can effectively be accounted for by the introduction of a second p-type charge carrier of very low effective mass and large mobility.[10] Such a hypothesis is consistent with recent cyclotron resonance experiments[11] which yielded two effective mass values for the holes in p-type germanium specimens.

ACKNOWLEDGMENTS

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[9] Beer, Willardson, and Middleton, Phys. Rev. **93**, 912 (1954).

[10] Willardson, Harman, and Beer, Bull. Am. Phys. Soc. **29**, 4, 9 (1954).

[11] Dresselhaus, Kip, and Kittel, Phys. Rev. **92**, 827 (1953).