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Range and Charge of Energetic Nitrogen Ions in Nickel

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The range of nitrogen ions in nickel was measured for energies from 8 to 29 Mev. The rate of energy loss is nearly constant over this range and has a value of 3.7 Mev/(mg cm^{-2}). The average charge of nitrogen ions in nickel was determined as a function of velocity. At a velocity of 1.93×10^9 cm/sec (27.3 Mev), the average charge is 6.2.

A STUDY of the range-energy characteristics of particles heavier than helium has been greatly hampered by the lack of such high-energy particles in the laboratory. The ORNL 63-inch cyclotron provides 29-Mev nitrogen ions which can be used in such an investigation. For ion velocities much greater than the velocity of the particle's most tightly bound electron, the usual range relation,

$$R_Z(V)=(m/Z^2)R_p(V), \tag{1}$$

where R_p is the range of a proton at velocity V and where m and Z are the nuclear mass and charge, has been successfully applied by cosmic-ray workers and others. However, as the velocity of an ion approaches the velocity of its inner electrons its effective charge decreases as it picks up electrons from the surrounding medium. The proper Z to use in the above relation for medium- and slow-moving ions is unknown.

The range characteristics, in gases, of ions heavier than helium were investigated by Blackett,[1] Blackett and Lees,[2] and Feather,[3] who observed recoil atoms from alpha particles in cloud chambers. The velocities covered were considerably below those that will be discussed here. Knipp and Teller[4] used the early range data to determine the charge behavior of ions as a function of velocity, as will be discussed later. Bohr[5] has given a complete discussion of the information available up to 1948. Bell[6] has treated the problem of the charge of fission fragments in low-pressure gases.

A 2-microampere deflected beam of 29-Mev nitrogen ions is available at the 63-inch cyclotron. This beam was used to determine the range-energy relation of nitrogen ions in nickel, and the charge state of the ions as a function of velocity in nickel. Nickel was chosen as an absorber because it is commercially available in the form of thin, strong, uniform foils which make excellent windows for gas-target chambers. Where a nuclear physics program necessitates the use of such windows a knowledge of the energy lost by nitrogen ions passing through them becomes imperative. Nickel absorbers are also used in investigations of excitation functions.[7]

EXPERIMENTAL METHOD—RANGE-ENERGY RELATION

The energy of the nitrogen ions as a function of nickel absorber thickness was determined by allowing the ions to pass into a chamber of hydrogen and observing the energy of recoil protons at zero degrees to the incoming nitrogen beam. A foil changer, Fig. 1, was placed before the hydrogen chamber to allow nickel foils of various thickness to degrade the energy of the incoming nitrogen ions. The energy of the recoil protons was determined by measuring their range in nuclear emulsions. In order to obtain tracks of a convenient length in the emulsions, aluminum absorbers were interposed between the chamber exit

[1] P. M. S. Blackett, Proc. Roy. Soc. (London) **107**, 349 (1925).
[2] P. M. S. Blackett and D. S. Lees, Proc. Roy. Soc. (London) **134**, 658 (1932).
[3] N. Feather, Proc. Roy. Soc. (London) **141**, 194 (1933).
[4] J. Knipp and E. Teller, Phys. Rev. **59**, 659 (1941).
[5] N. Bohr, Kgl. Danske Videnskab. Selskab, Mat-fys. Medd. **18**, 8 (1948).
[6] G. I. Bell, Phys. Rev. **90**, 548 (1953).
[7] A. Zucker and H. L. Reynolds, Phys. Rev. **94**, 784 (1954); also H. L. Reynolds and A. Zucker, Meeting of the S. E. Section, Am. Phys. Soc., April, 1954.

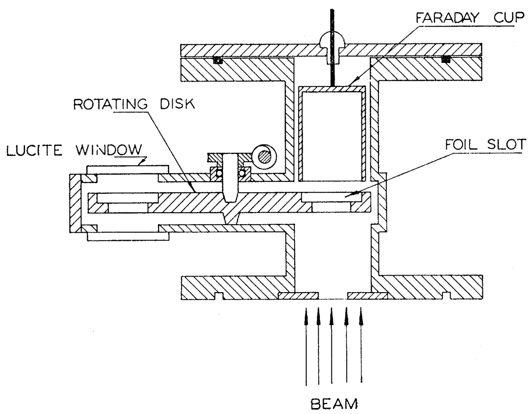

FIG. 1. Foil changer. Schematic drawing of the rotating foil and target changer, with Faraday cup in place. The disk can be readily removed from the chamber by disengaging the center shaft and removing the left end of the chamber.

port and the emulsion. The energy of nitrogen ions is simply related to the energy of recoil protons at zero degrees as follows:

$$E_{\mathrm{N}}=3.99E_p. \tag{2}$$

The experimental arrangement is shown in Fig. 2. The entrance foil for the hydrogen chamber was 0.025-mil nickel (0.59 mg/cm²). The foil separating the hydrogen from the atmosphere was 0.1-mil nickel (2.38 mg/cm²). The hydrogen pressure was 7.4 mm of mercury. The energy loss of the nitrogen ions in the length of the hydrogen chamber was estimated to be less than 50 kev. Both foils were attached to the brass support plates with Zapon cement. An external collimator allowed only recoil protons making an angle of less than 5 degrees with the incoming nitrogen beam to enter the emulsion. To monitor the beam a small ring

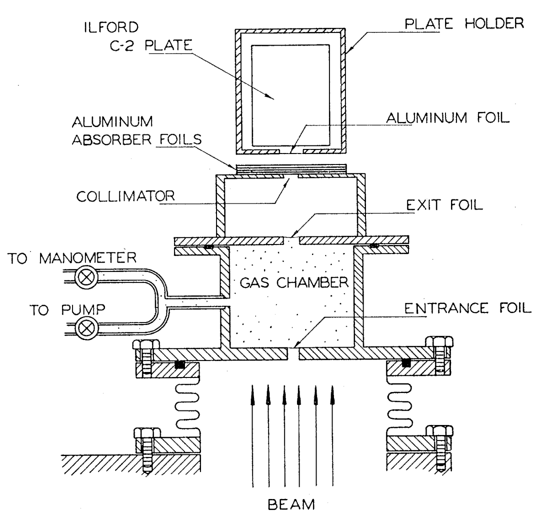

FIG. 2. Gas-scattering chamber. Protons scattered at zero degrees end their range in the emulsion. The aluminum absorbers are used to insure a convenient proton range in the emulsion. Foil changer (Fig. 1) is usually inserted between the gas chamber and bellows.

around the entrance port was painted with Aquadag so that the beam would produce nuclear reactions in the carbon. A Geiger counter was placed close to the port to count gamma rays from these reactions. The monitor served to give a qualitative indication that the beam was entering the scattering chamber.

The emulsions were C-2 Ilford, 50 microns thick. Background protons were negligible as compared with the main recoil proton peak. The exposure time with each nickel absorber was approximately 10 minutes.

EXPERIMENTAL METHOD—CHARGE-VELOCITY RELATION

When the nitrogen ions reach the deflected beam port they have a charge of three, since they have satisfied the cyclotron resonance conditions and have passed through the deflector channel. The charge-velocity experiment consisted of measuring the charge collected in a Faraday cup, Fig. 1, after the beam had passed through a thin nickel foil and comparing it with the charge collected with no nickel foil present. If the beam of nitrogen ions remained constant the charge of the attenuated beam particles could be compared directly with the triply charged particles of the unattenuated beam. The velocity of the particles upon leaving the foils was known from the previously measured range-energy relation.

A foil changer, consisting of a flat disk with twelve 1-inch holes along the periphery, was rotated at a constant velocity through the beam. Nickel foils of various thickness were placed in some of the holes while others were left empty. The nitrogen beam passed through successive holes as the disk was rotated. After passing through a hole or foil the beam current was collected in a Faraday cup 2 inches deep by $1\frac{3}{4}$ inches in diameter, which was spaced about $\frac{1}{4}$ inch from the foils. A $\frac{5}{8}$-inch diameter copper collimator was placed in the path of the beam ahead of the foil changer so that only one hole could be presented to the beam at a time. The rotation of the foil changer disk was adjusted so that the beam passed through each foil or hole for approximately 10 seconds. The current passing through the foil was measured with a vibrating reed electrometer. Comparing the maximum current reading for each foil thickness with the reading for the empty holes gave an average charge as a function of absorber thickness.

The whole apparatus was operated in the 2000-oersted fringing field of the cyclotron. This magnetic field should prevent electrons from migrating into or out of the Faraday cup. To check this possibility, however, a larger Faraday cup approximately 4 inches deep and 4 inches in diameter was placed about $3\frac{1}{2}$ inches beyond the foils. The results obtained with this cup were identical with the results obtained with the smaller cup. A run was performed without the copper collimator; collimation was then effected by just the 1-inch hole under each foil. No difference in the results could be seen, which indicated that edge effects due to collimation were not important. Calculations indicate that multiple scattering in the foils should not have influenced the results.

RESULTS

The range-energy results are presented in Fig. 3. Because of the absorbers between the hydrogen chamber and the emulsion, consisting principally of the exit foil, the minimum nitrogen energy which could be measured was 8 Mev. The total range of the full-energy (29.1 Mev) beam was found, from the extrapolated charge *versus* absorber thickness curve presented in Fig. 4, to be 7.85 mg/cm^2 of nickel. The equilibrium charge as a function of ion velocity is given in Fig. 5. It should be pointed out that for velocities below 10^9 cm/sec the curve is based on an extrapolation of the range-energy relation, Fig. 3.

The nickel foils used in these experiments were not tested for uniformity. The majority of the absorbers were sandwiches made up of several foils which should average out to some extent the effects of nonuniformity. Replacing foils by others of the same measured surface density did not appear to influence the results.[8] The error in measuring the foil surface density is estimated to be less than one percent. No attempt was made to eliminate occluded gases from the surface of the foils.

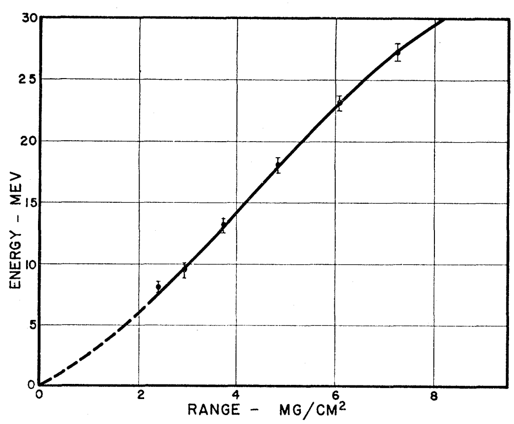

FIG. 3. Energy *vs* range of nitrogen ions in nickel.

The energy spread of the initial beam with the collimating system used for the hydrogen chamber is estimated to be approximately 600 kev, full width at half-maximum. This figure was obtained from the spread in proton energies after correcting for proton straggling and acceptance angles for the protons. The errors shown on the range-energy plot are estimated probable errors.

The errors shown on the charge plot indicate statistical probable errors due to beam intensity fluctuations while the foil changer moved from one hole to the next. Corrections due to electrometer nonlinearity and noncritical damping of the meter have been applied.

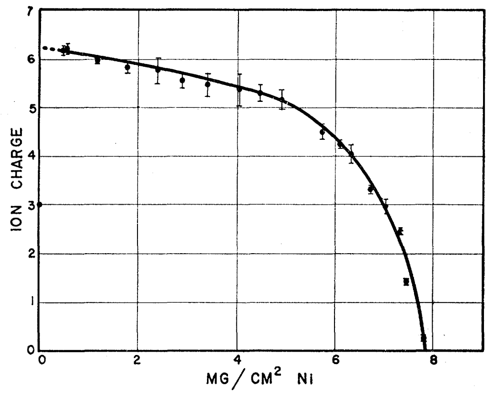

FIG. 4. Average charge of nitrogen ions *vs* nickel absorber thickness.

DISCUSSION

It has been shown[5] that the range of fission fragments is proportional to their velocity V whereas the range of protons or alpha particles is proportional to V^3. It may be seen from Fig. 2 that the range of nitrogen ions in the velocity region below 2×10^9 cm/sec varies approximately as V^2, less rapidly than alpha particles but more rapidly than fission fragments. Over most of the range of a 29-Mev nitrogen ion the pickup and loss process involves only the K electrons. The K electrons are widely spaced in velocity, and consequently one may expect the effective charge of the nitrogen ion to vary more slowly with velocity than is the case with fission fragments where only outer electrons are involved. This would lead to a larger range-velocity dependence for nitrogen than for fission fragments.

According to Bohr the ratio of electron-loss cross section to capture cross section is proportional to V^5.

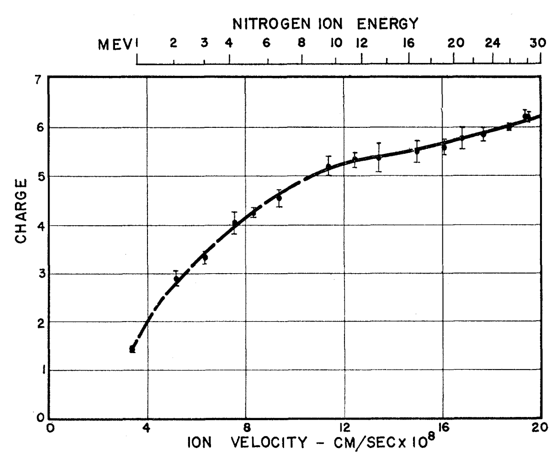

FIG. 5. Average charge of nitrogen ions *vs* energy and velocity in nickel. Region below 6 Mev is based on an extrapolation of range-energy curve, Fig. 3.

[8] Recent measurements [Chilton, Cooper, and Harris, Phys. Rev. **93**, 413 (1954)] on foils purchased from the same vendor (Chromium Corporation of America) indicate the foils are uniform to better than ±1.5 percent.

With such a high-velocity dependence it would not be expected that the charge of an ion would fluctuate more than one charge unit from its mean charge when it is in the velocity region of K-electron pickup, since the K electrons are widely spaced in velocity. If such is the case, it is meaningful to compare our measured charge-velocity curve with the calculation of Knipp and Teller[4] which gives the ratio of ionic to nuclear charge as a function of the velocity (V_e) of the most loosely bound electron. The usual assumption is that the capture and loss cross sections are equal when $V_e=\gamma V$. We find the value of γ for nitrogen ions to be approximately 0.65 at 1.8×10^9 cm/sec. Knipp and Teller obtained a γ value of 1.1 for nitrogen in air by analyzing the range data of Blackett and Lees.[2] However, the data fall in the velocity region below 5×10^8 cm/sec. The value of γ appears to decrease with increasing velocity.

By using relation 1 and the experimentally determined dE/dx relations for protons given by Allison and Warshaw[9] in combination with our charge-energy curve, one can calculate the range-energy relation for nitrogen in nickel. We have done this by normalizing the calculation to the experimental curve at 8 Mev to avoid the necessity of calculating nuclear stopping effects. In so doing, we find that the calculated curve falls about 2.4 Mev below the experimental curve for particles with a range of 7.5 mg/cm². The increased energy loss for nitrogen over that of equivalent protons can be attributed to several effects. The charge exchange process itself leads to an energy loss since energy is required to remove electrons from the moving ion. The magnitude of this energy loss depends, of course, upon the magnitudes of the capture and loss cross sections. Also, the charge of the ion is not necessarily the charge that contributes to energy loss. For close collisions the screening of the nucleus is not complete. If the whole 2.4-Mev discrepancy is attributed to the latter effect, the effective charge which contributes to energy loss would be about 6 percent larger than that shown in Fig. 5.

We wish to thank Mrs. W. R. Bowelle for assisting in the measurement of the proton ranges.

[9] S. K. Allison and S. D. Warshaw, Revs. Modern Phys. **25**, 784 (1953).

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Decomposition of the Scalar Product of Two Symmetric Tensors*

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A simple method is given for the decomposition of the scalar product of two symmetric tensors into products of their irreducible components, which leads also to an explicit recipe for a set of irreducible components of a symmetric tensor. Applications to dipole-dipole interaction and electrostatic multipole interaction are discussed.

In many problems of physics one begins with a scalar product of two Cartesian tensors and one is faced with the task of expressing that product in terms of the irreducible components[1] of the two tensors, i.e., of decomposing the scalar product. A discussion of the irreducible components of tensors of arbitrary symmetry has been given by Racah;[2] the scalar product of two tensors has not been dealt with explicitly, however, even for the symmetric case. It is therefore the purpose of this note to prove a decomposition theorem for the scalar product of two symmetric tensors and to give an explicit recipe for their irreducible components.

Since tensor relations are valid independent of the particular tensors through which they are exhibited, we shall work with the simple symmetric tensors,

$$X^n{}_{\alpha\beta\ldots}=x_\alpha x_\beta\cdots \tag{1}$$

and $Y^n{}_{\alpha\beta\ldots}$, formed by the n-fold direct product of the vector $\mathbf{x}$, or $\mathbf{y}$, with itself. Their scalar product is

$$S=X^n{}_{\alpha\beta\ldots}Y^n{}_{\alpha\beta\ldots}=x^ny^n(\cos\theta)^n, \tag{2}$$

where θ is the angle between $\mathbf{x}$ and $\mathbf{y}$ and where x and y are the magnitudes of $\mathbf{x}$ and $\mathbf{y}$. Expand $(\cos\theta)^n$ in Legendre polynomials[3]

$$(\cos\theta)^n=\sum_{l=0}^{n} a_{nl}P_l{}^0(\cos\theta), \tag{3}$$

where

$$a_{nl}=\frac{(2l+1)2^l n!(\frac{1}{2}n+\frac{1}{2}l)!}{(\frac{1}{2}n-\frac{1}{2}l)!(n+l+1)!};\quad n-l\geqslant0 \text{ and even};$$
$$=0;\quad n-l\leqslant0,\quad \text{or } n-l\geqslant0 \text{ and odd}. \tag{4}$$

* This material was included in a thesis submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy at the University of California.

[1] See, for example, E. Wigner, *Gruppentheorie und ihre anwendung auf die quantenmechanik der atomspektren* (Edwards Brothers, Inc., Ann Arbor, 1944), pp. 263, 264; G. Racah, Phys. Rev. **62**, 438 (1942).

[2] G. Racah, Revs. Modern Phys. **21**, 494 (1949).

[3] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis* (The University Press, Cambridge, 1952), fourth edition, pp. 310, 311.