

The crossover transition was too weak to permit an accurate estimate of its electron intensities. Comparing its  $K$  and  $L$  electrons to the  $M$  electrons of the "stop over" 127-keV transition, we find for the intensity of the crossover transition  $0.8 \pm 0.5$  percent, in good agreement with the expectations for a one-particle transition  $h_{11/2} \rightarrow d_{3/2}$ . The  $K/L$  ratio was found to be  $2 \pm 1.3$ , which is more consistent with the empirical  $K/L$  ratio<sup>9</sup> for an  $M4$  than for an  $E4$  transition.

The most plausible shell model configurations for the odd proton and odd neutron of  ${}_{55}\text{Cs}^{134,79}$  are shown in Fig. 3. Mixed states are expected,<sup>10</sup> since  $d_{5/2}$  and  $g_{7/2}$  proton levels in Cs nearly coincide in energy. These configurations indicate that, in contrast to the  $M4$  and  $M1$  transitions, the  $E3$  transition of  $\text{Cs}^{134m}$  should be "forbidden" as a one-particle transition, both because the particle making the transition is a neutron, and because in strict  $j$ - $j$  coupling the transition is forbidden; it is indeed one of the slowest known  $E3$  transitions,<sup>9</sup> with an  $|M|^2 \approx 10^{-6}$ .

It is interesting to note that the addition of a proton to  $\text{Xe}^{133}$  leads to a lowering of the energy difference between metastable and ground state, i.e., the energy of the  $M4$  transition is reduced from 232 keV to 137 keV in  $\text{Cs}^{134}$ , whereas the addition of two protons to  $\text{Xe}^{133}$  increases the difference between the corresponding states to 275 keV<sup>5</sup> in  $\text{Ba}^{135}$ .

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## Radiative Capture of Orbital Electrons

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THE continuous spectra of  $\gamma$  rays accompanying orbital electron capture have recently been measured for a number of elements.<sup>1-3</sup> These have thus far been compared with a theoretical spectrum of the

form  $x(1-x)^2$  (where  $x = E/E_{\text{max}}$ ), obtained under highly simplified assumptions by Morrison and Schiff.<sup>4</sup> For low-energy  $\gamma$  rays, the observed intensities are far higher than those predicted. In particular, the spectra  $\text{Fe}^{55}$ ,  $\text{Cs}^{131}$ , and  $\text{Ge}^{71}$  have been determined down to energies bordering on the characteristic x-ray region; in each, there is an unexplained and precipitous increase of intensity with decreasing photon energy. Deviations from the form  $x(1-x)^2$  are, in fact, to be expected. The assumptions of Morrison and Schiff, that Coulomb effects may be neglected and that capture occurs only from the  $K$ -shell, are quantitatively valid only for capture processes far more energetic than those mentioned.

We wish to indicate that the above approximations may be avoided; the internal bremsstrahlung spectra may be found almost exactly without difficulty. In particular, the effects of the Coulomb field can be treated precisely. When this is done, the spectrum of magnetic dipole radiation accompanying  $K$  capture is found indeed to have the form  $x(1-x)^2$ , for all energies of interest. Further considerations show the remaining and dominant part of the observed low-energy radiation to be electric dipole in character and to arise from the capture of electrons from  $P$  states.

We assume that the electron capture process is an allowed one, with a matrix element of the most general form,

$$H_c = \sum_{\lambda} c^{(\lambda)} \langle T_{\mu}^{(\lambda)} \rangle (\bar{\chi}(0) T_{\mu}^{(\lambda)} \psi(0)), \quad (1)$$

in which  $\psi(0)$  and  $\chi(0)$  are the electron and neutrino wave functions evaluated at the nucleus, the  $c^{(\lambda)}$  are linear combination coefficients for the various  $\beta$ -coupling operators  $T_{\mu}^{(\lambda)}$ , and  $\langle T_{\mu}^{(\lambda)} \rangle$  is a nuclear matrix element. The radiative transition of an electron from the state  $\psi_i(\mathbf{r})$  to the state  $\psi_f(\mathbf{r})$  has the familiar matrix element<sup>5</sup>

$$(H_R)_{fi} = -e(2\pi/k)^{\frac{1}{2}} \int \bar{\psi}_f(\mathbf{r}) e_{\mu} \gamma_{\mu} e^{-i\mathbf{k} \cdot \mathbf{r}} \psi_i(\mathbf{r}) d\tau, \quad (2)$$

where  $\mathbf{k}$  and  $\mathbf{e}$  are the propagation and polarization vectors of the emitted photon. The processes of interest combine both radiation and capture; an electron initially in an  $S$  or a  $P$  state will ordinarily emit a photon during a virtual transition to an  $S$  state, from which it is subsequently captured by the nucleus. The virtual transitions to  $S$  states already occupied in the atom are, however, forbidden by the exclusion principle. The absence of these terms in the summations over intermediate states, it may be shown, is precisely compensated for by the occurrence of transitions in which the capture of an electron from the occupied state precedes the radiative transition.

For a radiative capture process which leaves vacant an initially occupied state  $\psi_n(\mathbf{r})$  (with  $E_n < 0$ ), the expressions (1) and (2) may be shown, after some re-

arrangement, to lead to the total matrix element

$$M = (ie/m)(2\pi/k)^{\frac{1}{2}} \sum c^{(\lambda)} \langle T_{\mu}^{(\lambda)} \rangle \\ \times \int (\bar{\chi}(0) T_{\mu}^{(\lambda)} \mathcal{G}_{E_n-k}(0, \mathbf{r}) \\ \times e^{-i\mathbf{k}\cdot\mathbf{r}} \mathbf{e}_{\eta} (\partial_{\eta} - \frac{1}{2}\sigma_{\eta\rho} k_{\rho}) \psi_n(\mathbf{r})) d\tau, \quad (3)$$

in which  $\sigma_{\eta\rho} = (i/2)[\gamma_{\eta}, \gamma_{\rho}]$  and  $k_{\rho} = (\mathbf{k}, k)$ . The function  $\mathcal{G}_{E_n-k}(0, \mathbf{r})$ , which is a Green's function for the second order Dirac equation, implicitly embodies the summation over intermediate states. It is quite adequately approximated in the present context, by the simpler nonrelativistic Green's function for the propagation of an electron with energy  $E_n - k$  in a Coulomb field. The latter function is defined in terms of Coulomb wave functions  $\varphi_j(\mathbf{r})$  as

$$G_{E_n-k}(0, \mathbf{r}) = \sum_i \frac{\varphi_j(0) \varphi_j^*(\mathbf{r})}{E_j - E_n + k}. \quad (4)$$

Since this expression has spherical symmetry about the origin, it is easily evaluated by solving the appropriate radial Schrödinger equation. The occurrence of a particularly tractable form for the Coulomb Green's function stems from the fact that electron capture necessarily takes place at the center of force. The solution is found to contain a Whittaker function and has integral representation,

$$G_{E_n-k}(0, \mathbf{r}) = (m\beta/\pi) e^{-\beta r} \int_0^{\infty} e^{-2\beta r u} \left( \frac{1+u}{u} \right)^{Z/\beta a_0} du, \quad (5)$$

for  $k - E_n > Z^2$  Rydbergs, where  $\beta = [2m(k - E_n)]^{\frac{1}{2}}$ , and  $a_0$  is the Bohr radius.

With the expression (5), the integrations required to find the matrix element (3) may be carried out analytically. The resulting  $\gamma$ -ray spectra for capture from the various significant electron shells have been calculated for  $\text{Fe}^{55}$  which has an energy release<sup>1</sup> of 220 kev. They are shown in Fig. 1. The spectra for capture from  $S$  states, aside from their slightly different maximum

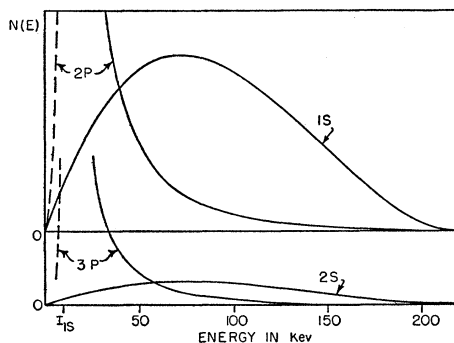


FIG. 1. Gamma-ray spectra for radiative capture from various electron shells of  $\text{Fe}^{55}$ . The characteristic x-ray region lies below  $I_{1s}$ , the  $K$ -shell ionization energy.

energies, have the general shape  $x(1-x)^2$ . The  $P$ -state spectra are by comparison quite weak at high energies and extremely intense near the characteristic x-ray lines. The processes responsible for the intensity peaks are ones in which capture of an  $S$  electron is followed by a radiative transition from a higher  $P$  state. They differ from the normal and highly probable course of electron capture and subsequent emission of characteristic x-rays only by relaxation of the requirement of energy conservation in the intermediate state. The  $P$  state spectra of Fig. 1 may, in fact, be thought of as representing the extreme wings of the characteristic x-ray lines.

For a given energy release, the intensities of the  $P$ -state spectra increase relative to those of the  $S$  states roughly as the square of the nuclear charge. Hence for  $\text{Ge}^{71}$  and  $\text{Cs}^{131}$  the  $P$ -state contributions should dominate all save the upper ends of the spectra. The shapes observed<sup>2</sup> corroborate this. Screening of the Coulomb field will act to reduce somewhat the intensities of spectra from the  $n=2$  and 3 shells and will be taken into account in seeking quantitative agreement with experiment.

The analysis described may be applied equally well to forbidden transitions which will be characterized in general by differing spectrum shapes. The capture in  $\text{Fe}^{55}$  for which  $\log ft$  equals 6.1, is evidently allowed, but quite unfavored, a fact which might be anticipated from the shell model since  $\Delta l=2$ . A detailed account of the techniques employed including an examination of the effects of screening is in preparation. We wish to thank Dr. T. Berlin for calling this problem to our attention.

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<sup>5</sup> We employ units in which  $\hbar=1$ ,  $c=1$ .

## Proton-Neutron Coincidences in the High-Energy Photodisintegration of Lithium\*

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WHEN photons with energies of several hundred Mev interact with complex nuclei, they eject high-energy protons and neutrons in greater numbers than compound nucleus formation can explain. The cross sections and angular distributions of such reactions indicate a direct interaction between the photons and individual nucleons. Of the various models proposed to explain these results, perhaps the most appealing is the "pseudodeuteron" model discussed by Levinger.<sup>1</sup> Several experiments have tended to confirm