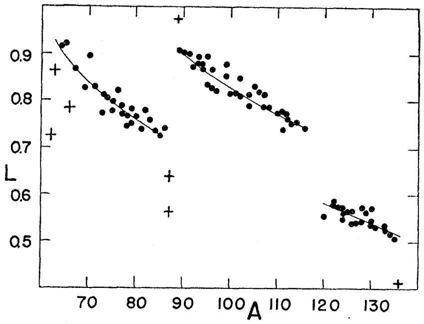

FIG. 1. Calculated values of the potential energy parameter L of the Wigner formula for isobars with $\Delta Z=2$. Crosses indicate anomalous results.

by means of semiempirical mass formulas, that the energy available for double-beta decay would be much smaller than in the lower regions. Hogg and Duckworth,[4] however, have recently found a large mass difference between Nd^{150} and Sm^{150}.

Table I contains a summary of the stable isobars which have been found with mass differences greater that about 2 mMU (1.86 Mev). The agreement between calculated and experimental values is seen to be adequate for our purpose.[5] It is believed that this table is a reliable guide in the choice of elements for experimental study of possible double-beta activity. In Table I are found ten nuclei capable of emitting two negative electrons with a combined energy of about 2 Mev or more. Among the isobars for which no experimental information is available, calculations show that only Mo^{100} and Xe^{136} are good candidates for double-beta emission. Six nuclei have been found for which double-positron emission is energetically possible, as shown by the six positive values in the table. When the necessary four electron masses are subtracted from these values, it is seen that in all cases the two positrons would share an energy less than 1 Mev. This process would therefore have an extremely long half-life.

[4] B. G. Hogg and H. E. Duckworth, Can. J. Phys. **32**, 65 (1954).

[5] *Note added in proof.*—The second, third, and fourth mass differences in the above table have been recently measured as −2.02, 2.81, and −3.18 mMU, respectively, by Collins, Johnson, and Nier in Phys. Rev. **94**, 398 (1954).

PHYSICAL REVIEW VOLUME 95, NUMBER 2 JULY 15, 1954

Interactions between Some Two-Nucleon Configurations*

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(Received April 12, 1954)

The shell model wave functions may often be mixtures of those for two or more configurations. Calculations based upon a harmonic oscillator central potential with scalar Gaussian interactions between nucleons indicate that there is substantial interaction between various two-nucleon configurations of the $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$ shell. The admixtures of the $1f_{7/2}$ subshell, which belongs to the next main level, are, however, small. A mixture of several neighboring configurations accounts for the ft value of $F^{18}(\beta^+)O^{18}$ better than any single configuration.

I. INTRODUCTION

THE j-j coupling shell model of the nucleus has had remarkable success in accounting for many qualitative regularities in nuclear structure. Moreover, it is possible to calculate quantitatively on the basis of this model numerical values of several properties of those nuclei which have both closed neutron and proton shells$\pm$one nucleon, and to do so in a rather unambiguous way. All available data except one, the magnetic moment of Bi^{209}, are in approximate agreement with such calculations.

The situation is far different for the many other nuclei. It is necessary to make additional assumptions regarding configurations, the order of filling of subshells, and many other details. It has not appeared possible to account with any set of assumptions even roughly for the quantitatively measured properties of these nuclei, which have two or more nucleons outside double closed shells. Indeed, not even the properties of nuclei within any single shell have been accounted for in a consistent way.

In view of the similarity between the atomic and nuclear shell models, it might be expected that states of many-particle nuclei are, in general, superpositions of states of two or more configurations. This effect is well known for complex atoms; for example, superpositions of states of configurations $4d^n$, $4d^{n-1}5s$, and $4d^{n-2}5s^2$ are common. Such mixing occurs when inter-configuration matrix elements of the electrostatic inter-electron potential operator $\sum_{i>j}(e^2/r_{ij})$ are not zero. The mixing may therefore be said to be due to "configuration interaction."

* This work was supported in part by the U. S. Atomic Energy Commission and the Higgins Scientific Trust Fund.
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If there are two or more electrons outside closed shells, each of them is moving in a central field determined by (1) the nucleus and closed shells, and (2) the other outer electrons. The contribution of these other electrons is in general not a central potential, and therefore the orbital angular momentum of an individual outer electron may be not even approximately a good quantum number.

Calculations of this effect for nuclei have been made on the basis of the j-j coupling shell model with a harmonic oscillator central potential and scalar Gaussian internucleon potentials. These assumptions are today often made,[1] because it seems useful to explore the consequences of any model of the nucleus which might have some validity and which is amenable to calculation.

II. FRAMEWORK OF THE CALCULATION

Calculations were made for two-particle configurations of the $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$ shell. The potential energy of one particle with coordinates (r_1,θ_1,φ_1) is

$$V(r_1)=(\hbar\nu r_1)^2/2m, \tag{1}$$

where m = the nucleon mass, and ν was determined from

$$\langle r_1^2\rangle=\int_0^\infty r_1^2R_{1d}^2(r_1)dr_1=\frac{7}{2\nu}=(1.43A^{\frac{1}{3}}\times10^{-13})^2; \tag{2}$$

i.e., the $1d_{5/2}$ particle is assumed to be just at the outer edge of the nucleus. Here $R_{1d}(r_1)/r_1$ is the radial part of the harmonic oscillator wave function $\psi_{1dm}(r_1) = [R_{1d}(r_1)/r_1]Y_2^m(\theta_1,\varphi_1)$. Calculations were made for $A=18$. This choice of ν leads to an energy difference between main harmonic oscillator levels of $\hbar\omega=\hbar^2\nu/m = 10.3$ Mev.

The interaction between two particles a distance $r=|\mathbf{r}_1-\mathbf{r}_2|$ apart is

$$V_X(r)=V_0\,[\exp(-ar^2)]P_X, \tag{3}$$

where $P_W=1$, P_M = space-exchange operator, P_B = spin-exchange operator, and $P_H=P_MP_B$. The parameters V_0 and a were taken from the data for the neutron-proton system in the triplet state.[2] These lead to $V_0 = -70.8$ Mev, and $a^{-1}=2.245\times10^{-26}$ cm^2. It is well known (see, e.g., reference 2) that the experimental data for low energies can equally well be accounted for by other potentials, for example, the Yukawa potential. The equivalent Yukawa potential for $V_W(r)$ with the parameters just given is

$$-\frac{98.1}{10^{13}r}\exp\left(-\frac{10^{13}r}{1.015}\right)\text{ Mev.} \tag{4}$$

This may be compared with the two-nucleon potential given in a recent book.[3] That potential has both central and tensor terms, each of the form (4). The denominator of the exponential of the central part is 1.176 instead of 1.015; i.e., the ranges are approximately equal, as would be expected. The depth of the central part, however, is 46.8 instead of 98.1. This depth would be expected to be smaller since there is a tensor force term which has about the same depth as the central term, and which contributes to the binding energy of the deuteron.

It should be noted that if there is only one outer nucleon, there is no possibility of interaction except with the core, since $V_X(r)$ is a two-particle operator. For two particles, however, there may be configuration interaction. To make calculations of this effect, it is necessary to evaluate matrix elements of the type

$$\langle j_1j_2JT|V_X(r)|j_3j_4JT\rangle. \tag{5}$$

Here j_k is the total angular momentum of a single nucleon (with orbital angular momentum l_k); J and T are total angular momentum and isotopic spin (with components J_z and T_ζ) of the two two-particle configurations. By means of transformation coefficients $\langle j_1j_2J|l_1l_2LS\rangle$, which were given by Racah,[4] the matrix elements (5) can be expanded in terms of similar ones for L-S coupling. These, in turn, can be expressed as series of certain radial integrals, which can readily be calculated for the present model by the methods of Talmi.[1] Details of the calculations and formulas for some radial integrals and matrix elements of type (5) may be found elsewhere.[5] The radial integrals depend upon the parameters ν and a only through $\beta=(a/\nu)+\frac{1}{2}$. $\beta=\infty$ corresponds to zero range, and $\beta=\frac{1}{2}$ to infinite range. $\beta=2.277$ for the parameters used here.

III. INTERACTIONS BETWEEN MAIN HARMONIC OSCILLATOR SHELLS

Let us examine whether there is mixing between two configurations belonging to different main harmonic oscillator levels. The third main level contains the subshells $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$; the fourth, $1f_{7/2}$ and others

TABLE I. Matrix elements, $\times(-1)$, in Mev, for some two-particle configurations.

		$\langle aa,JT\|V_W\|cc,JT\rangle$			$\langle aa,JT\|V_M\|cc,JT\rangle$		
T	J	$a=c=1d_{5/2}$	$a=c=1f_{7/2}$	$a=1d_{5/2}$ $c=1f_{7/2}$	$a=c=1d_{5/2}$	$a=c=1f_{7/2}$	$a=1d_{5/2}$ $c=1f_{7/2}$
0	1	3.26	3.00	−2.72	0.94	0.43	−0.61
	3	1.77	1.49	−1.37	1.16	0.60	−0.75
	5	2.84	1.74	−1.40	2.84	1.41	−1.40
	7		2.36			2.36	
1	0	4.55	4.08	−3.78	2.90	2.08	−2.20
	2	2.08	1.98	−1.75	0.10	0.02	−0.06
	4	1.24	1.13	−0.98	−0.11	−0.10	0.08
	6		0.87			−0.19	

[1] I. Talmi, Helv. Phys. Acta **25**, 185 (1952). Earlier references are quoted there.

[2] J. M. Blatt and J. D. Jackson, Phys. Rev. **76**, 18 (1949).

[3] R. G. Sachs, *Nuclear Theory* (Addison-Wesley Press, Cambridge, 1953), p. 153.

[4] G. Racah, Physica **16**, 651 (1950).

[5] M. G. Redlich, Princeton dissertation, 1954 (unpublished).

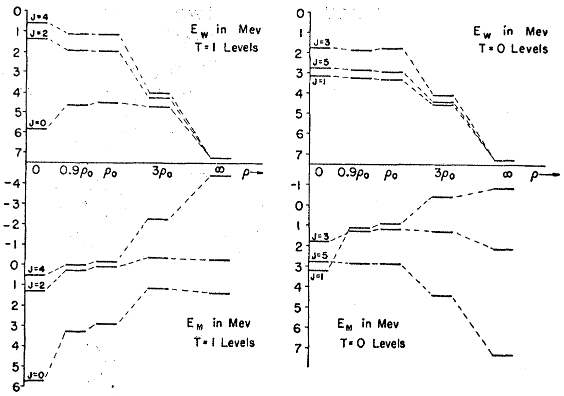

FIG. 1. The effects of changes in the range of nuclear forces on the levels of $(1d_{5/2})^2$.

which are empirically found to lie higher. Both diagonal and off-diagonal matrix elements are given for all states of $(1d_{5/2})^2$ and $(1f_{7/2})^2$ in Table I. Space-and-spin-exchange (H) and spin-exchange (B) interactions are readily obtained from this table by the formulas

$$\langle A|V_H|C\rangle=(-1)^T\langle A|V_W|C\rangle, \quad\text{and}\quad \langle A|V_B|C\rangle=(-1)^T\langle A|V_M|C\rangle.$$

Parity conservation prohibits configuration interaction between $(1d_{5/2})^2$ and $1d_{5/2}1f_{7/2}$. It is evident from Table I that mixing between $(1d_{5/2})^2$ and $(1f_{7/2})^2$ is largest for $T=1$, $J=0$ and ordinary (W) forces. From first-order perturbation theory, the amplitude of admixture of $(1f_{7/2})^2$ is

$$\alpha(1f_{7/2}{}^2)=\frac{3.78}{-2\times 10.3}=-0.183. \tag{6}$$

This amplitude is small because the denominator, which is the difference between the energies of the main harmonic oscillator levels times the number of particles, is large. The interconfiguration matrix elements are actually about equal to the diagonal ones. Even for this state, α^2 equals 0.034, so that mixing is small.

It is easy to see that this result has qualitatively more general validity. Calculations of reference 5 indicate that for short ranges the matrix elements for two-particle configurations decrease with an increasing number n of nodes, and further, that for a given set of n, l, j, they decrease with increasing atomic number A as $\beta^{-3/2}$, i.e., roughly as $\nu^{3/2}\sim A^{-1}$. It is seen from Sec. IV (Fig. 1) that the results for the range of nuclear forces chosen here do not in general differ much from those for zero range. The difference $\hbar\omega$ between main levels is proportional to $\nu\sim A^{-2/3}$. Thus α varies very roughly as $A^{-1/3}$. On the basis of the present model, admixtures of two-particle configurations from a higher main shell are therefore in general even smaller than (6) for $A>18$.

Is the actual energy difference between $1d_{5/2}$ and $1f_{7/2}$ levels about equal to the calculated 10.3 Mev? Recent experiments[6] on O^{17}, which has one neutron outside double closed shells, revealed the existence of a $7/2-$ state at 3.85 Mev. The ground state has $5/2+$, corresponding to $d_{5/2}$, and its magnetic moment is in excellent agreement with that assignment. There is, however, a $\frac{1}{2}-$ state at 3.06 Mev. It seems likely that this is due to a breakup of the closed shells; for instance, the configuration might be $(1p_{1/2})^{-1}(1d_{5/2})^2$. It is possible that the $7/2-$ state also belongs to this configuration, since $(1d_{5/2})^2$ can give states with J up to 5. Even if the $7/2-$ state in O^{17} does belong to the next main shell, the $1f_{7/2}-1d_{5/2}$ energy difference may be much larger for other nuclei in this region. The fact that the $1d_{3/2}$ shell is filled before the $1f_{7/2}$ shell for ground states would be consistent with such a change. The O^{17} data indicate a $\frac{3}{2}+$ state, presumably $1d_{3/2}$, at 5.08 Mev; i.e., above the $7/2-$ state.

IV. THE EFFECTS OF CHANGES IN THE PARAMETERS

Let us examine next whether the choice of parameters used here is critical. This question is of some importance, since the results of the present calculations have at best qualitative significance. If rather small changes in the parameters, which would still be entirely consistent with experimental results, change the matrix elements substantially, one should doubt the validity of the general results.

To change the range of nuclear forces, the potential will be taken as

$$V_X(r)=V_0(\rho)\,[\exp(-r^2/\rho^2)]P_X.$$

While ρ is varied from the value $\rho_0=1/\sqrt{a}$ used in the previous calculations, the expectation value of $V_W(r)$ for two nucleons, both in a $1s$ state, is kept constant:

$$\langle 1s^2|V_W(r)|1s^2\rangle=V_0(\rho)/\beta(\rho)^{3/2}2^{3/2}=\text{constant}.$$

The effect of variation of ρ from 0 (δ-function potential) to ∞ (forces of ∞ range) upon the levels of $(1d_{5/2})^2$ is shown for W and M forces in Fig. 1. There is only slight variation in energies and no change in order of the levels in the neighborhood of ρ_0. Similar results

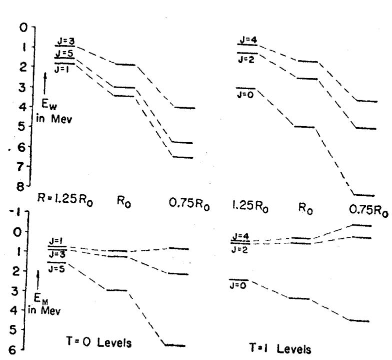

FIG. 2. The effects of changes in the nuclear radius on the levels of $(1d_{5/2})^2$.

[6] R. K. Adair, Phys. Rev. 92, 1491 (1953).

would hold for the integrals $\langle(1d_{5/2})^2J|V_X|(1f_{7/2})^2J\rangle$, and hence for the admixture $\alpha(1f_{7/2}{}^2)$ of formula (6).

Only one crossover occurs, for M forces: The state with $J=1$, $T=0$ lies lowest for $\rho=0$, but highest for $\rho=0.9\rho_0$ and all larger ranges. The explanation is readily seen. Let $\langle(1d_{5/2})^2J|V_M|(1d_{5/2})^2J\rangle=[J]$ and, for L-S coupling, $\langle 1d^2\,{}^{2S+1}L|V_W|1d^2\,{}^{2S+1}L\rangle=({}^{2S+1}L)$. Then

$$\begin{aligned}[1]&=0.28({}^3S)-0.56({}^1P)+0.16({}^3D);\\ [3]&=0.62({}^3D)-0.36({}^1F)+0.02({}^3G);\\ [5]&=({}^3G).\end{aligned}\tag{7}$$

For a δ-function potential $({}^1L)=0$ if L is an odd number. This is so because the wave function is space antisymmetric for odd L and is therefore 0 when the two particles are at coincidence. The δ-function potential is 0 except at coincidence. The matrix elements $({}^1L)$ with odd L rapidly increase with increasing ρ, until at ∞ range they equal those with even L. It is plain from formulas (7) that the effect of this increase will be largest for $[1]$, and hence the crossover between the $J=1$ level and the $J=3$ and 5 levels occurs.

For the $(1d_{5/2})^2$ configuration, the state with $J=1$ is the lowest of the $T=0$ states for W forces and all ranges $<\infty$, but only for very short range M forces. The ground state of F^{18}, which is expected to have this configuration, probably has $J=1$ and $T=0$. A state with these (T,J) may be the lowest even for $\frac{1}{2}(V_W+V_M)$ if admixtures of neighboring configurations are taken into account (see Sec. V).

The effect of a change in the nuclear radius R for constant range ρ_0 is shown in Fig. 2. The value $R=R_0$ was used in the calculations for Table I. A change of 25 percent from R_0 in either direction changes the magnitude of the energies of $(1d_{5/2})^2$ levels substantially but leaves their order and generally their relative positions about the same. For these three radii the interconfiguration integrals of V_W for $T=1$, $J=0$ and corresponding values of $\alpha(1f_{7/2}{}^2)$ are

R	$1.25\,R_0$	R_0	$0.75\,R_0$
$\langle(1d_{5/2})^2, J=0\|V_W\|(1f_{7/2})^2, J=0\rangle$	2.39	3.78	6.22 Mev
$\alpha(1f_{7/2}{}^2)$	−0.181	−0.183	−0.170.

The change in mixing is thus very small, even though the interconfiguration integral changes substantially. This is so because

$$\alpha(1f_{7/2}{}^2)=\langle(1d_{5/2})^2,\ J=0|V_W|(1f_{7/2})^2,\ J=0\rangle/(-2\hbar\omega),$$

and ω increases with decreasing R as R^{-2}. Similar results can be expected for other (T,J).

V. INTERACTIONS WITHIN THE 1d, 2s SHELL

There are six two-particle configurations in the $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$ shell. They lead to the states listed in Table II. The energy differences between the single-particle levels are assumed to be due to spin-orbit splitting and to equal in magnitude the ones observed in O^{17} between $\frac{3}{2}+$ and $5/2+$ states (5.08 Mev) and $\frac{1}{2}+$ and $5/2+$ states (0.875 Mev). It is assumed, contrary to evidence from O^{17} discussed in Sec. III, that the $1f_{7/2}$ levels lie much higher. Let us define an operator $\Delta(j_1)$ as follows:

$$\langle j_1j_2JT|\Delta(j_1)|j_1'j_2'JT\rangle=a(j_1)\delta(j_1,j_1')\delta(j_2,j_2');$$

$$a(j_1)=\begin{cases}0 & \text{if } j_1=5/2\\ 0.875\text{ Mev} & \text{if } j_1=1/2,\\ 5.08\text{ Mev} & \text{if } j_1=3/2.\end{cases}$$

TABLE II. States of the two-particle configurations in the 1d, 2s shell. Expectation values of $\frac{1}{2}(V_W+V_M)+\Delta(j_1)+\Delta(j_2)$ are listed in Mev, $\times(-1)$, for all states permitted by the Pauli principle for space, spin, and isotopic spin.

Configuration	$T=0$ $J=1$	0 2	0 3	0 4	0 5	1 0	1 1	1 2	1 3	1 4
$1d_{5/2}{}^2$	2.10		1.47		2.84	3.72		1.09		0.57
$1d_{3/2}{}^2$	−8.39		−7.78			−7.68		−9.52		
$2s_{1/2}{}^2$	1.46					1.46				
$1d_{5/2}1d_{3/2}$	−0.47	−2.81	−3.80	−2.23			−5.08	−4.53	−5.08	−2.80
$1d_{5/2}2s_{1/2}$		0.17	1.74					0.69	−0.87	
$1d_{3/2}2s_{1/2}$	−3.35	−4.39					−5.95	−4.91		

Calculations have been made for the $(T,J)=(1,0)$ and $(0,1)$ states. These are expected to be the quantum numbers of the ground states of F^{18} and O^{18}. The interaction was taken as $\frac{1}{2}(V_W+V_M)$. The matrices of $\frac{1}{2}(V_W+V_M)+\Delta(j_1)+\Delta(j_2)$ are given in Table III. The predominant configurations for $(T,J)=(0,1)$ are $1d_{5/2}{}^2$, $1d_{5/2}1d_{3/2}$, and $2s_{1/2}{}^2$. Diagonalization of the part of the submatrix for these configurations leads to the maximum eigenvalue $\lambda_{01}=4.03$ Mev. The wave function $\Psi(T,J)$ is easily calculated. Admixtures of the other two configurations are small and can be calculated by first-order perturbation theory. Then

$$\Psi(0,1)=0.732\psi(d_{5/2}{}^2)+0.477\psi(d_{5/2}d_{3/2})+0.464\psi(s_{1/2}{}^2)-0.131\psi(d_{3/2}{}^2)-0.009\psi(d_{3/2}s_{1/2}).\tag{8}$$

Diagonalization of the (1,0) matrix yields $\lambda_{10}=5.05$ Mev. The wave function is

$$\Psi(1,0)=0.895\varphi(d_{5/2}{}^2)+0.370\varphi(s_{1/2}{}^2)+0.243\varphi(d_{3/2}{}^2).\tag{9}$$

Mixing is high for some states of larger J also. For instance, the matrix of $\frac{1}{2}(V_W+V_M)+\Delta(j_1)+\Delta(j_2)$ for (0,3) is given in Table IV. This time the predominant configuration is $d_{5/2}s_{1/2}$ with a very large admixture of $d_{5/2}{}^2$, and small admixtures of the other two configurations.

The expectation values in Table II for the $T=0$ states would indicate, at least for the equal mixture of

TABLE III. Matrices of $\frac{1}{2}(V_W+V_M)+\Delta(j_1)+\Delta(j_2)$, in Mev, $\times(-1)$.

$T=0$, $J=1$

	$d_{5/2}{}^2$	$d_{5/2}d_{3/2}$	$s_{1/2}{}^2$	$d_{3/2}{}^2$	$d_{3/2}s_{1/2}$
$d_{5/2}{}^2$	2.10	2.15	0.82	−1.61	−0.60
$d_{5/2}d_{3/2}$	2.15	−0.47	1.24	−0.51	0.79
$s_{1/2}{}^2$	0.82	1.24	1.46	−0.44	0
$d_{3/2}{}^2$	−1.61	−0.51	−0.44	−8.39	1.12
$d_{3/2}s_{1/2}$	−0.60	0.79	0	1.12	−3.35

$T=1$, $J=0$

	$d_{5/2}{}^2$	$d_{3/2}{}^2$	$s_{1/2}{}^2$
$d_{5/2}{}^2$	3.72	3.04	1.21
$d_{3/2}{}^2$	3.04	−7.68	0.98
$s_{1/2}{}^2$	1.21	0.98	1.46

forces used here, that for a pure $d_{5/2}{}^2$ configuration a state with $J=5$ is the ground state. This is in contradiction to the allowed ft value of the $F^{18}(\beta^+)O^{18}$ transition. It is readily seen from Fig. 1 that a shorter range would, in fact, give the $J=1$ state as the lowest even for space-exchange (M) forces. However, the preceding calculation, which yielded $-\lambda_{01}=-4.03$ Mev, points the way to a more plausible explanation. The $J=5$ state belongs only to $(d_{5/2})^2$, and so its energy remains -2.84 Mev when configuration interaction is taken into account. Similarly, the (0,4) state belongs only to $d_{5/2}d_{3/2}$ and has energy 2.23 Mev. A glance at the (0,3) matrix shows that its largest eigenvalue will be less than 4.03 Mev. The same can be expected for (0,2). Thus, the present model accounts directly for the probable spin of F^{18}. It should be noted that the (0,1) state is the lowest, even though its predominant configuration is $(d_{5/2})^2$. The $(d_{5/2})^2$ probability, $|\alpha(d_{5/2}{}^2)|^2=54$ percent, while $s_{1/2}{}^2$ and $d_{5/2}d_{3/2}$ admixtures are only of the order of 20 percent.

VI. THE TRANSITION $F^{18}(\beta^+)O^{18}$

The wave functions (8) and (9) may be used to calculate the ft value for the following transition:[7]

F^{18}	$\beta^+\rightarrow$	O^{18}	$ft=4170\pm330$ (10)
$N, Z=9, 9$		$10, 8$	
$J_i=1$ (assumed)		$J_f=0$ (measured)	
$T_i=0$ (from experiment)		$T_f=1$ (from assumption that there are just two neutrons outside double closed shells).	

The reaction $Ne^{20}(d,\alpha)F^{18}$ has been observed; therefore the ground state of F^{18} must have $T=0$. Its J could be only 0 or 1, since the ft value for the transition to O^{18}, which has $J=0$, lies in the allowed favored range. A state with $T=0$ and $J=0$ does not occur for any two-particle configuration.

The Fermi matrix element is 0 and, using constants obtained from single-particle and single-hole transitions,[8] one obtains

$$ft=\frac{5300}{\frac{1}{2J_i+1}\sum_{M_i,M_f}\sum_q|\langle\alpha_iJ_iM_i|A_q{}^{(1)}|\alpha_fJ_fM_f\rangle|^2}. \quad (11)$$

TABLE IV. Matrix of $\frac{1}{2}(V_W+V_M)+\Delta(j_1)+\Delta(j_2)$, in Mev, $\times(-1)$, for $T=0$, $J=3$.

	$d_{5/2}{}^2$	$d_{5/2}d_{3/2}$	$d_{3/2}{}^2$	$d_{5/2}s_{1/2}$
$d_{5/2}{}^2$	1.47	0.91	−0.63	1.18
$d_{5/2}d_{3/2}$	0.91	−3.80	0.84	−0.91
$d_{3/2}{}^2$	−0.63	0.84	−7.87	0.20
$d_{5/2}s_{1/2}$	1.18	−0.91	0.20	1.74

[7] Data are taken from F. Ajzenberg and T. Lauritsen, Revs. Modern Phys. **24**, 321 (1952).

[8] G. L. Trigg, Phys. Rev. **86**, 506 (1952); A. Winther and O. Kofoed-Hansen, Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **27**, No. 14 (1953).

TABLE V. The double-bar matrix element
$\mathfrak{A}=(j_ij_i', J_i=1, T_i=0\,\|\,A^{(1)}\,\|\,j_fj_f', J_f=0, T_f=1)$
for several transitions.

$j_ij_i'\rightarrow j_fj_f'$	$d_{5/2}{}^2\rightarrow d_{5/2}{}^2$	$s_{1/2}{}^2\rightarrow s_{1/2}{}^2$	$d_{3/2}{}^2\rightarrow d_{3/2}{}^2$	$d_{5/2}d_{3/2}\rightarrow d_{5/2}{}^2$	$d_{5/2}d_{3/2}\rightarrow d_{3/2}{}^2$
$\mathfrak{A}$	$-\sqrt{\frac{14}{5}}$	$-\sqrt{6}$	$\sqrt{\frac{6}{5}}$	$-\sqrt{\frac{8}{5}}$	$-\sqrt{\frac{12}{5}}$

Here α_i and α_f stand for the remaining quantum numbers (including isotopic spin) of the initial and final states. $M_i=J_{iz}$. $A_q{}^{(1)}$ is q component of the Gamow-Teller irreducible tensor operator. Using a well-known relation, one obtains, in the notation of Racah,[9]

$$\langle\alpha_iJ_iM_i|A_q{}^{(1)}|\alpha_fJ_fM_f\rangle=(-1)^{J_i+M_i}(\alpha_iJ_i\,\|\,A^{(1)}\,\|\,\alpha_fJ_f)\times V(J_iJ_f1;-M_iM_fq). \quad (12)$$

Then the denominator of (11) becomes

$$\frac{1}{2J_i+1}|(\alpha_iJ_i\,\|\,A^{(1)}\,\|\,a_fJ_f)|^2. \quad (13)$$

It is evident that if the initial and final states are superpositions of states for different configurations, it is permissible in calculating the ft value to add the double-bar matrix elements (with proper phases and coefficients) instead of the $\langle\alpha_iJ_iM_i|A_q{}^{(1)}|\alpha_fJ_fM_f\rangle$. The double-bar matrix elements, $\mathfrak{A}$, are readily calculated by the methods of reference 9 and given in Table V for transitions between the several configurations involved in the transition (10). Since $A_q{}^{(1)}$ operates only on the spins and isotopic spins of single particles, $\mathfrak{A}=0$ for all transitions occurring between states of (8) and (9) except those listed in Table V.

The theoretical ft value for $d_{5/2}{}^2\rightarrow d_{5/2}{}^2$ is thus 5680; for $s_{1/2}{}^2\rightarrow s_{1/2}{}^2$ it is 2650. The observed value 4170 lies between them. For the transition (8)→(9), $ft=3190$. This is close to the $s_{1/2}{}^2\rightarrow s_{1/2}{}^2$ value, even though the $s_{1/2}{}^2$ admixtures are small, because the cross terms (like $d_{5/2}d_{3/2}\rightarrow d_{5/2}{}^2$) in the total matrix element are large and have the same sign as the direct terms. It is significant that configuration interaction does increase the matrix elements over those of $d_{5/2}{}^2\rightarrow d_{5/2}{}^2$. That the increase is too great is less important; with a change in the interaction, or in the distances between the single-particle levels, none of which is accurately known, it is surely possible to obtain just the observed ft value.

It is with pleasure that I thank Professor Eugene P. Wigner for suggesting this investigation and for his continued advice and interest. I am grateful to Dr. Igal Talmi for his advice and many valuable discussions.

[9] G. Racah, Phys. Rev. **62**, 438 (1942).