Independent Particle Model and the Nuclear Photoeffect*

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(Received April 7, 1954)

The very low value calculated by Levinger and Bethe for the harmonic mean energy for the nuclear photoeffect is increased by a factor of 3 for Cu and 4 for Ta if correlations between nucleons due to the Pauli principle are included in the calculation using an independent particle model (IPM). Our calculation removes the discrepancy between the sum-rule result and Burkhardt's calculation using the IPM for Cu. If we use a smaller nuclear radius, and include the increase of mean energy due to exchange forces, we find reasonable agreement between experimental results for Cu and Ta and the harmonic mean energies calculated for an IPM in a finite square well: i.e., 18 Mev for Cu and 12 Mev for Ta.

I. INTRODUCTION

IN a previous paper[1] (here referred to as LB) one of the present authors and H. A. Bethe calculated the effects of exchange forces on the integrated cross section and on the mean energies for electric dipole transitions in the nuclear photoeffect. LB used an approximate independent particle model (IPM) in which the nucleons were treated as a degenerate Fermi gas, with no correlations among the different nucleons. The calculated integrated cross section was in reasonable agreement with experiment.[2,3] The mean energy $\bar{W}$ was not in clear disagreement with experiment; but the calculated harmonic mean energy W_H was extremely low. LB therefore argued that there must be strong correlations among the nucleons: e.g., as in an alpha-particle model.

In the past several years the shell model, or IPM, has had striking successes in many fields of nuclear physics, including some calculations concerning the nuclear photoeffect.[4] We therefore wish to re-examine the argument against the IPM advanced by LB. A more pressing reason for this reexamination is Burkhardt's IPM calculation[5] of electric dipole transitions for the neutrons in the Cu^{63} nucleus. He finds that the cross section for photon absorption has a sharp peak at about 9 Mev. This result is in marked contrast with the LB values (for ordinary forces) of 2.9 Mev for W_H and 16 Mev for $\bar{W}$, corresponding to a very broad curve of cross section *vs* photon energy. In particular Burkhardt does not find the absorption of low-energy photons corresponding to the low value of W_H given by LB. While there is some difference between the details of IPM used by Burkhardt and LB, this difference seems much too minor to explain these major discrepancies between Burkhardt's direct calculation of the absorption cross section, and LB's sum-rule calculation of mean energies for photon absorption.

We discuss below the nucleon correlations present even in the IPM due to the Pauli principle. These correlations have a large effect on sum-rule calculations of the mean energies. Including these correlations we find good agreement between Burkhardt's results and our revised sum-rule calculation. (The two calculations must agree, provided that the same nuclear model is used.[1,4])

In the next section we set up the calculation of mean energies including nucleon correlations based on the Pauli principle. In Sec. III we find the mean energies using the LB model of a degenerate Fermi gas. In the following section we find the mean energies for an IPM in a square well for Cu and Ta and compare with Burkhardt's results, and with experiment. We also present IPM calculations of the nucleon density as a function of radius.

II. PAULI PRINCIPLE CORRELATIONS

Using the notation of LB we define two mean energies: the harmonic mean energy,

$$W_H=\sum_n f_{on}/[\sum_n f_{on}/(E_n-E_o)]$$
$$=\sigma_{\rm int}\Big/\Big[\int(\sigma/W)dW\Big], \quad (1)$$

and the mean energy,

$$\bar{W}=\sum_n f_{on}(E_n-E_o)/\sum_n f_{on}=\int\sigma W dW/\sigma_{\rm int}. \quad (2)$$

Here o refers to the ground and n to the various excited states; E_n-E_o equals the photon energy W for a transition from o to n, and $\sigma_{\rm int}=\int\sigma dW$.

The summed oscillator strength $\sum_n f_{on}$ is independent of correlation among the nucleons. It has the value NZ/A for ordinary forces, and is increased by about $0.8x$, where x is the fraction of attractive exchange (Majorana) force. For the present let us restrict the discussion to ordinary forces, as was done by Burkhardt in his calculation for Cu. The denominator in Eq. (1) and the numerator in Eq. (2) are both dependent on correlations among the nucleons.

The denominator of Eq. (1) is given in LB as

$$\sum_n f_{on}/(E_n-E_o)=(2M/\hbar^2)$$
$$\times\sum_n\{[(N/A)\sum_i z_i-(Z/A)\sum_j z_j]_{on}\}^2. \quad (3)$$

* A preliminary account is given in Phys. Rev. **93**, 932 (1954).
[1] J. S. Levinger and H. A. Bethe, Phys. Rev. **78**, 115 (1950).
[2] L. W. Jones and K. M. Terwilliger, Phys. Rev. **91**, 699 (1953).
[3] R. Nathans and J. Halpern, Phys. Rev. **93**, 437 (1954).
[4] J. S. Levinger, Ann. Rev. Nuclear Sci. (to be published).
[5] J. L. Burkhardt, Phys. Rev. **91**, 420 (1953).

Here i stands for protons and j stands for neutrons; the matrix element is calculated for the complete nuclear wave function. Consider, for example, the term for pairs of protons. Omitting the constant factors, and using closure to express the sum over n in terms of the ground-state wave function alone we have

$$\sum_n (\sum_i z_i)_{on}(\sum_i z_i)_{no} = (\sum_i z_i^2)_{oo} + (\sum_{i\neq i'}\sum z_i z_{i'})_{oo} = D+B. \quad (4)$$

In the last term on the right i is unequal to i'. If there is no correlation in the ground state between pairs of protons, these off-diagonal terms give zero. The diagonal terms were calculated by LB using the model of uniform proton density in a sphere of radius R.

$$D = (\sum_i z_i^2)_{oo} = (1/5)ZR^2. \quad (5)$$

This expression, together with a similar expression for the terms in Eq. (3) involving neutrons was used in LB to give the value for the harmonic mean energy as

$$W_H = 5\hbar^2/2MR^2 = 46A^{-\frac{2}{3}} \text{ Mev}. \quad (6)$$

The numerical result is given for $r_o = 1.5$, and is proportional to r_o^{-2}. (The nuclear radius $R = r_o A^{\frac{1}{3}} \times 10^{-13}$ cm.) As stated above, the value of 2.9 Mev for the harmonic mean energy for photon absorption by Cu is in serious disagreement with Burkhardt's calculated value of 8.4 Mev, and in even worse agreement with experiment.

However, even in the IPM there is correlation among the nucleons, since two nucleons of the same charge must have antisymmetric total (spatial and spin) wave functions. In three-fourths of the cases the spin wave functions are symmetric, and therefore the spatial wave functions are antisymmetric. The negative contribution to the last term on the right of Eq. (4) from these triplet states is larger than the positive contribution from the singlet states. Then the denominator of Eq. (1) is decreased, and the harmonic energy is increased.

Consider a pair of protons in states k and l, respectively. If the spatial wave function normalized to 2 particles is antisymmetric, we have in Eq. (4):

$$(z_i z_{i'})_{oo} = \int [\psi_k(r_i)\psi_l(r_i') - \psi_k(r_{i'})\psi_l(r_i)]^2 z_i z_{i'} d^3r_i d^3r_{i'}. \quad (7)$$

Terms such as

$$\int [\psi_k(r_i)]^2 [\psi_l(r_{i'})]^2 z_i z_{i'} d^3r_i d^3r_{i'}$$

vanish because of the definite parity of the single particle wave functions. The exchange terms do not vanish. For an antisymmetric spatial wave function, Eq. (7) gives

$$(z_i z_{i'})_{oo} = -2\int \psi_k(r_i) z_i \psi_l(r_i) d^3r_i \int \psi_l(r_{i'}) \times z_{i'} \psi_k(r_{i'}) d^3r_{i'} = -2(z_{kl})^2. \quad (8)$$

For a spatially symmetric wave function occurring in $\frac{1}{4}$ of the cases we find the same result but with the sign changed. The complete result for the off-diagonal terms is then

$$B = (\sum_i \sum_{i'} z_i z_{i'})_{oo} = -\sum_k \sum_l (z_{kl})^2. \quad (9)$$

The correction term B then involves the squared electric dipole matrix element with single particle wave functions for transitions between occupied states. [Eq. (9) could also be derived by using a product of single-particle wave functions for the ground state, and taking account of the Pauli principle by subtracting transitions from one occupied level to another, which should not be counted in the $\sum_n$ of Eq. (3).]

In the approximation that $N = Z$, used in this paper, the correction factor C to the harmonic mean energy is

$$C = (1+B/D)^{-1}. \quad (10)$$

Here B, the negative contribution of the off-diagonal terms for pairs of protons, is given by Eq. (9), while the contribution D of the diagonal terms is given in Eq. (5).

The contribution of off-diagonal terms to the numerator of Eq. (2) for the mean energy can be calculated in an analogous manner, using matrix elements for p, the momentum component along the polarization direction of the photon. We find a correction factor C' less than unity given by

$$C' = 1+B'/D', \quad (11)$$

$$B' = -\sum_k \sum_l (p_{kl})^2, \quad (12)$$

$$D' = (\sum_i p_i^2)_{oo}. \quad (13)$$

LB assumed no correlations, and therefore calculated just the term D', giving a mean energy for the degenerate Fermi gas model of 4/5 the Fermi energy, or 16 Mev for $r_o = 1.5$.

III. FERMI GAS MODEL

Calculations of the correction factors C and C' to the LB values for W_H and $\bar{W}$, respectively, must be based on some specific model for the nucleon wave functions. In this section we use the model of a degenerate Fermi gas of constant density in a sphere of radius R. This model is different from Burkhardt's, so our result will not be in perfect agreement with his. This model is not self-consistent, as the nucleon density cannot be constant inside the nucleus, suddenly changing to zero at the nuclear surface. It turns out that surface terms are of importance in these

TABLE I. Correction factor for harmonic mean energy, using Fermi gas model.[a]

Mass number A	Function $g(y)$	Correction factor C_F
35	0.246	1.64
69	0.295	1.88
119	0.329	2.10
174	0.356	2.30
283	0.384	2.57

[a] The function $g(y)$ and the correction factor C_F are given in Eq. (15) of the text.

calculations, particularly for the correction factor C'; so that the Fermi gas model is not reliable for quantitative results. Still, it gives semiquantitative results applicable to all nuclei.

The matrix element z_{kl} depends on q, the magnitude of the difference of the propagation vectors for nucleons in states k and l:

$$z_{kl}=(3i\cos\theta'/q)j_2(qR). \tag{14}$$

Here θ' is the angle between the vector q and the direction of polarization of the photon, while j_2 is a spherical Bessel function. We see that the matrix element has a maximum value of order of magnitude R, corresponding to pairs of states k and l where q is small, of magnitude $1/R$. This matrix element is squared, and summed over all pairs of states k and l by integrating over the distribution function[6] for q, giving

$$B=-(ZR^2/\pi)g(y),$$
$$y=(9\pi A)^{\frac{1}{3}},$$
$$g(y)=\int_0^y [j_2(x)]^2(1-x/y)^2(2+x/y)dx, \tag{15}$$
$$C_F=(1+B/D)^{-1}=[1-(5/\pi)g(y)]^{-1}.$$

In this approximate model the correction factor is independent of the parameter r_o for the nuclear radius,

TABLE II. Proton binding energies in Mev for IPM in square well.[a]

Shell	Cu ($N=Z=34$)		Ta ($N=Z=92$)	
	$r_o=1.5$	$r_o=1.2$	$r_o=1.5$	$r_o=1.2$
$1h^{22}$	···	···	8.0	8.0
$3s^2$	···	···	8.03	8.21
$2d^{10}$	···	···	9.29	10.06
$1g^{18}$	···	···	12.72	15.21
$2p^6$	···	···	14.48	17.97
$1f^{14}$	10.0	10.0	16.92	21.71
$2s^2$	14.15	16.35	19.13	25.08
$1d^{10}$	16.7	20.2	20.55	27.28
$1p^6$	22.4	28.9	23.61	31.99
$1s^2$	27.0	35.9	26.04	35.74
Well depth V_o	31.4	42.6	28.38	39.35

[a] The binding energy of 10 Mev for $1f$ protons in Cu, and 8 Mev for $1h$ protons in Ta is estimated from experiments. This determines the well depth V_o, and the binding energies of the other protons. The Coulomb potential is neglected. The radius parameter $r_0=RA^{-\frac{1}{3}}\times10^{13}$, where R is the radius of the square well.

[6] A. Winslow, Ph.D. thesis, Cornell University, 1952 (unpublished).

since both B and D are proportional to r_o^2. The correction factor C_F for the harmonic mean energy W_H is given for various values of A in Table I. We see that C_F is about two for moderate and heavy nuclei, and increases with A. Then $W_H=C_F(W_H)_{\rm LB}$ decreases with A less rapidly than $A^{-\frac{2}{3}}$.

We have also calculated the correction factor C' for the mean energy $\bar{W}$, but this result is unreliable, since it becomes negative for large A. The surface effects, poorly treated by our Fermi gas model, are of more importance for the calculation of C' than of C, and even lead to appreciable errors in the calculation of C, as we find below by comparison with our calculation of C using a square well IPM.

IV. IPM FOR SQUARE WELL POTENTIAL

Calculations of the corrections to the mean energies due to Pauli principle correlation have also been made using IPM wave functions in a square well potential. Calculations were made for two nuclei, called here by

TABLE III. Oscillator strengths for transitions between occupied shells for Cu.[a]

	$1s^2$	$1p^6$	$1d^{10}$	$2s^2$	$1f^{14}$
$1s$	···	−2.06	···	···	···
$1p$	2.06	···	−6.77	−1.27	···
$1d$	···	6.77	···	···	−13.67
$2s$	···	1.27	···	···	···
$1f$	···	···	13.67	···	···
Summed	2.06	5.98	6.90	−1.27	−13.67
Unused	−0.06	0.02	3.10	3.27	27.67

[a] The summed oscillator strength for a level equals the sum of oscillator strengths for transitions to occupied levels of higher energy minus the oscillator strengths to occupied levels of lower energy. The unused oscillator strength represents the strength of all transitions to unoccupied levels. (The unused oscillator strength should be non-negative; the value −0.06 for the $1s^2$ level shows lack of sufficient accuracy in the calculations.)

the approximate names of Cu ($N=Z=34$) and Ta ($N=Z=92$). These nuclei have closed sub-shells in the square-well IPM (no spin-orbit coupling) which simplifies the sums over k and l. We have calculated for two different values of the parameter r_o for the radius of the square well: $r_o=1.5$, and $r_o=1.2$. The former value was used by Burkhardt in calculations on Cu. The latter value of r_o is near that of recent Stanford[7] and Columbia[8] measurements. We find below that the correction factors C and C' are insensitive to the value chosen for r_o, though there is some dependence on r_o in contrast to the Fermi gas model treated above. Since the LB values for W_H and $\bar{W}$ are each proportional to r_o^{-2}, the mean energies calculated here keep a strong dependence on the value chosen for r_o. We shall find that use of the smaller value for r_o does give reasonable agreement between IPM calculations and experiment; but this agreement may well be fortuitous.

We shall first present our results on the IPM energy

[7] Hofstadter, Fechter, and McIntyre, Phys. Rev. 92, 978 (1953).
[8] V. L. Fitch and J. Rainwater, Phys. Rev. 92, 789 (1953).

TABLE IV. Oscillator strengths for transitions between occupied shells for Ta.[a]

	$1s^2$	$1p^6$	$1d^{10}$	$2s^2$	$1f^{14}$	$2p^6$	$1g^{18}$	$2d^{10}$	$3s^2$	$1h^{22}$
$1s$	···	−1.95	···	···	···	−0.05	···	···	···	···
$1p$	1.95	···	−6.45	−1.20	···	···	···	−0.15	−0.06	···
$1d$	···	6.45	···	···	−14.20	−2.35	···	···	···	···
$2s$	···	1.20	···	···	···	−3.11	···	···	···	···
$1f$	···	···	14.20	···	···	···	−23.08	−3.55	···	···
$2p$	0.05	···	2.35	3.11	···	···	···	−8.78	−2.41	···
$1g$	···	···	···	···	23.08	···	···	···	···	−35.91
$2d$	···	0.15	···	···	3.55	8.78	···	···	···	···
$3s$	···	0.06	···	···	···	2.41	···	···	···	···
$1h$	···	···	···	···	···	···	35.91	···	···	···
Summed	2.00	5.91	10.10	1.91	12.43	5.68	12.83	−12.48	−2.47	−35.91
Unused	0.00	0.09	−0.10	0.09	1.57	0.32	5.17	22.48	4.47	57.91

[a] For interpretation see notes on Table III.

levels, and matrix elements z_{kl} for electric dipole transitions. We then calculate the correction factor C for the harmonic mean energy by performing the sums over k and l. We shall compare this correction factor with that found above using the Fermi gas model. We then give the correction factor C' for the mean energy, and finally compare various calculations and experimental results for Cu and Ta.

The energy levels for protons in Cu and in Ta are given in Table II for two different radii for the square well. In each case we have determined the well depth by using rough experimental values for the binding energy of the least tightly bound proton or neutron. Note that we are making the approximation $N=Z$, and neglecting both the Coulomb potential and spin-orbit coupling.

The matrix elements z_{kl} needed for the calculation of the off-diagonal terms B and B' are found using the method of Courant[9] and others. In Tables III and IV we give the oscillator strengths of each shell for electric dipole transitions of protons between the various occupied shells. (Results for $r_o=1.5$ are very close to the tabulated values for $r_o=1.2$.) We have summed over the m values for the closed shells using an equation given by Bethe[10] and have also summed over spins. The oscillator strengths are presented as a square matrix with elements above the diagonal negative since the energy difference is negative. The oscillator strength summed for each shell is found by adding each column, giving the next to the bottom row of the tables. The difference between the oscillator strength summed for the shell and the number of protons in the shell gives the "unused oscillator strength" given in the bottom row of the tables. We see that the lower levels have used almost all of their oscillator strength, transferring it to the higher levels, with the result that for Ta the 32 protons in the $2d$ and $1h$ levels have about 90 percent of the total oscillator strength of 92. (Since the oscillator strength is merely transferred from one level to another, the sum of the unused oscillator strength is the total number of protons.)

We now find the correlation correction to the harmonic mean energy by calculating B, given by Eq. (9), for the sum of the squared dipole matrix elements between occupied levels, using the oscillator strengths tabulated in Tables III and IV. While the summed oscillator strength for transitions between bound levels cancels to zero, this cancellation clearly does not occur for the sum of $(z_{kl})^2$, or $(p_{kl}{}^2)$. The numerical results for B are given in Table V for the two nuclei and two values of r_o treated in this paper. The diagonal term was found by numerical integration of the proton density distribution to find its moment of inertia. In all four cases the value found for D/R^2 is quite close to the value $Z/5$ given by a uniform charge distribution in a sphere of radius R. The correction factor C for the harmonic mean energy is then found by Eq. (10). The last column gives C_F, using the Fermi gas model of the previous section. We see that the correction factor C found for the IPM in a square well is much larger than the more approximate correction C_F. The difference in the two correction factors is due to the difference in the matrix elements z_{kl}, and remains large even for nuclei with very many nucleons. The IPM square well correction factor C is very insensitive to the value of r_o, and increases with A about as

TABLE V. Correction factor for W_H using IPM in square well.[a]

		B/R^2	D/R^2	Correction factor C	Previous section C_F
Cu	$r_o=1.5$	−4.26	6.63	2.8	1.9
	$r_o=1.2$	−4.58	7.0	2.9	
Ta	$r_o=1.5$	−13.7	18.0	4.2	2.3
	$r_o=1.2$	−13.9	18.5	4.0	

[a] The off-diagonal terms B are given by Eq. (9) using the oscillator strengths of Tables III and IV. The diagonal terms D are about equal to $Z/5$ corresponding to a sphere of radius R and uniform charge density. The correction factor $C=(1+B/D)^{-1}$ is compared with the correction factor C_F using a Fermi gas model, given in Table I.

[9] E. Courant, Phys. Rev. 82, 703 (1951).

[10] H. A. Bethe, *Handbuch der Physik* (J. Springer, Berlin, 1933), Vol. 24/1, Eq. (39.14).

TABLE VI. Mean energies for photon absorption by Cu.[a]

	LB $r_o=1.5$	Burkhardt $r_o=1.5$	This paper $r_o=1.5$	This paper $r_o=1.2$	Exchange $r_o=1.2$	Experiment
Harmonic mean W_H	2.9 Mev	8.4	8.1	12.7	17.8	20.
Mean $\bar{W}$	16. Mev	10.5	9.0	13.	32.	25.
$\int(\sigma/W)dW$	0.34 barns	0.11	0.12	0.08	0.08	0.08

[a] For LB see reference 1; for Burkhardt see reference 5. [We have included the protons in Burkhardt's value for $\int(\sigma/W)dW$.] The calculations of this paper are based on Table V for W_H, Eq. (11) for $\bar{W}$, and LB, Eq. (47) for the integrated cross section. The values for exchange forces ($x=\frac{1}{2}$) are based on LB. Experimental values are estimated from Jones and Terwilliger, reference 2.

$A^{0.4}$ thus decreasing greatly the $A^{-\frac{2}{3}}$ dependence of W_H found by LB.

The correction factor C' for the mean energy $\bar{W}$ is calculated in a similar manner. The off-diagonal terms B' [Eq. (12)] are calculated using the oscillator strengths for transitions between occupied levels given in Tables III and IV. The diagonal terms D' for the expectation value of the kinetic energy are fairly close to the value given by LB for a degenerate Fermi gas. They can be found more accurately using the kinetic energy for a nucleon of binding energy E as $b(V_o-E)-(1-b)E$, where b is the fraction of the time that the nucleon spends in the square well potential of depth V_o. The value of b is about 0.9 as calculated by numerical integrations. The correction factor $C'=1+B'/D'$ has the value of about 0.5 for Cu and 0.4 for Ta. C' is insensitive to r_o and to A.

Using the correction factors C and C' for the harmonic mean energy W_H and the mean energy $\bar{W}$, respectively, we obtain the results given in Tables VI and VII for the mean energies for photon absorption by Cu and Ta. In Table VI for Cu ($N=Z=34$) the column labelled LB is calculated neglecting the Pauli principle correlation. We observe the large discrepancy between LB's results and Burkhardt's mentioned in our first section. The results of this paper, for $r_o=1.5$, are in reasonable agreement with Burkhardt's values. (We should note that our models are quite similar, but not identical, as Burkhardt includes spin-orbit coupling, which we have neglected. Incidentally, we believe that Burkhardt treats spin-orbit coupling in an inconsistent manner. Instead of treating each level as split into two different levels for different relative orientations of L and S, Burkhardt treats each level as a single level, but fills them in the order of the Mayer-Jensen spin-orbit coupling model.) The next column gives W_H and $\bar{W}$ increased by a factor of $(1.5/1.2)^2=1.56$, using the smaller nuclear radius. Finally, the next to the last column gives still larger calculated mean energies, using the LB corrections for half exchange force. [This calculation, based on reference 1, Eqs. (43) and (46), uses $r_o=1.37$. W_H is insensitive to r_0, but $\bar{W}$ is quite sensitive to r_o. Also, we have here modified the LB equations to include the Pauli principle correlation.] The calculated mean energies are compared with experimental values, estimated from measurements of neutron yield made by Jones and Terwilliger.[2] [These mean energies are found by making an approximate correction for neutron multiplicity, and integrating to 70 Mev. See reference 4 for a discussion of the appropriate upper energy in the integrals of Eqs. (1) and (2). The value for W_H should be fairly good, but that for $\bar{W}$ is unreliable.]

We see that the three successive increases in the LB value of W_H, due to Pauli principle correlations, use of a smaller radius, and exchange forces, gives a calculated value of W_H of 17.8 Mev for Cu in reasonable agreement with the experimental value of about 20 Mev. The calculated value of $\bar{W}$ is 32 Mev if exchange effects are included, or rather higher than the experimental value of about 25 Mev. Here neither the calculated or experimental values can be considered reliable. (Our main purpose in calculation of $\bar{W}$ is to compare with Burkhardt for ordinary forces, and to show that for pure ordinary forces the photon absorption curve exhibits quite a sharp peak, since $\bar{W}$ is close to W_H.)

We also compare calculated and experimental values of the integrated cross section $\int(\sigma/W)dW$, as here the calculated value does not depend on exchange forces, but only on the nuclear wave function. Again we see the discrepancy between the LB and Burkhardt values. (We have added the proton contribution to Burkhardt's calculation of the neutron contribution to this integral.) The results of this paper for $r_o=1.5$ are in good agreement with Burkhardt. (This check is redundant, as this paper and Burkhardt agree on the value of σ_{int} and W_H, and therefore must also agree on the value of $\int(\sigma/W)dW$.) The calculated value using $r_o=1.2$ is in good agreement with the Jones and Terwilliger measurements, where we have included the effects of proton emission.

In Table VII we compare calculations and experiment for Ta($N=Z=92$). The LB value for W_H is only 1.4 Mev, but it is increased by the Pauli principle correlation, by use of a smaller nuclear radius and by half exchange forces to give 12.2 Mev, in reasonable agreement with an experimental value of about 15 Mev. The calculated mean energy $\bar{W}$ is smaller than experiment, if exchange forces are omitted, and larger than experiment if half exchange forces are included; but as discussed above, neither the calculations nor measurements of $\bar{W}$ are reliable. The integrated cross section $\int(\sigma/W)\,dW$ is in good agreement with experiment if we use the smaller radius $r_o=1.2$.

TABLE VII. Mean energies for photon absorption by Ta.[a]

	LB $r_o=1.5$	This paper $r_o=1.5$	This paper $r_o=1.2$	Exchange $r_o=1.2$	Experiment
Harmonic mean W_H	1.4 Mev	6.0	8.7	12.2	15.
Mean energy $\bar{W}$	16. Mev	6.5	10.4	30.	20.
$\int(\sigma/W)dW$	1.9 barns	0.45	0.31	0.31	0.32

[a] For interpretation see notes on Table VI.

We note in passing that for pure ordinary forces Ta, like Cu, shows a sharply peaked photon absorption curve, since $\bar{W}$ is not much larger than W_H. [For $r_o=1.5$ this corresponds to several strong transitions to bound levels: 61 percent of the total oscillator strength is used up by the $1h$–$1i$ transition (5.2 Mev), 20 percent of the oscillator strength is used in the $2d$–$2f$ transition (5.6 Mev), 17 percent by transitions to other bound levels, and only 2 per cent by transitions to the continuum.]

Thus we find in Tables VI and VII that the IPM in a square well, with $r_o=1.2$ and half exchange forces, gives reasonable agreement between calculations and experiment for the harmonic mean energy of photon absorption by Cu and Ta. Our conclusion is in disagreement with Burkhardt's, who argued that his calculations showed that the IPM was unsuccessful for calculations of the nuclear photoeffect in Cu. We have increased his mean energy by about a factor of two, by using a smaller nuclear radius and by including half exchange forces, thus removing almost all the discrepancy between his calculations and experiment.

Of course the IPM is an extreme idealization. In the high-energy photoeffect, for example, one of us[11] found good evidence for strong nucleon-nucleon correlations, as in a quasi-deuteron. We have only tried to show that the IPM might well be a reasonable starting point for less approximate calculations of the nuclear photoeffect. We should point out that since we have used sum-rules, we have only used the IPM for the nuclear ground state, and do not concern ourselves here as to whether the IPM applies to the excited states reached by photon absorption.

In Table VIII we present our calculations of the nuclear density $\rho(r)$ *vs* the radial distance in units of the radius of the square well potential (i.e., r/R). The density is normalized to $\int_0^\infty \rho(r) r^2 dr = 1$. This calculated density might be of interest for comparison with recent Stanford measurements[7] of the nuclear form factor for electron scattering. Our work is in reasonable agreement with Malenka's[12] exact calculations for an infinite square well, and approximate calculations for a potential intermediate between a square well and an oscillator potential. In Fig. 1 we plot the nuclear density for Ta with $r_o=1.2$.

Touschek[13] estimated the position of the lowest level of a nucleus that could make electric dipole transitions to the ground state. It is clear that there is at least one dipole level below the excitation energy W_H. However, W_H depends on the possible correlation of nucleus into subunits, such as alpha particles. Touschek estimates the position of lowest dipole level by summing up to an excitation energy which should include transitions for the nucleons not in alpha particles, but sufficiently low so that the alpha particles are not dissociated. He finds that the position of the lowest dipole level is insensitive to the correlation of nucleons in alpha particles, and has approximately the very low LB value, given by Eq. (6) of this paper. Our present work shows that one must include the Pauli principle correlations among nucleons, greatly increasing the value of W_H above that given in Eq. (6). This increase by a factor of three to four gives an estimate of the lowest dipole level which is no longer in disagreement with experiment. For example,

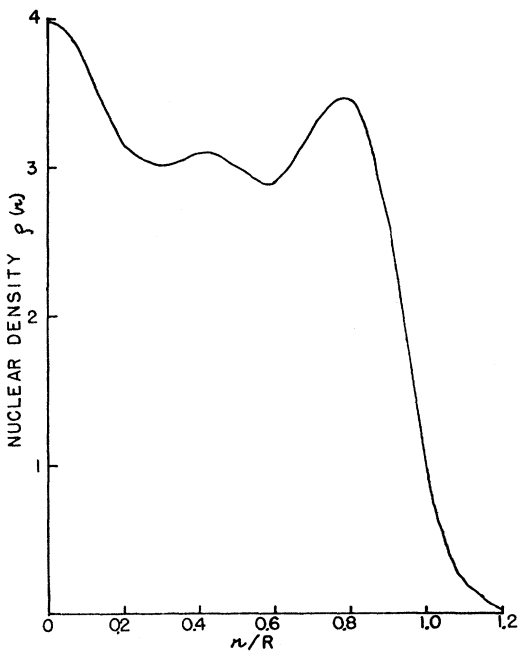

FIG. 1. Nuclear density $\rho(r)$ *vs* radial distance r/R for Ta. The nuclear density $\rho(r)$ is calculated using the IPM for Ta($N=Z=92$) for a square well potential with radius parameter $r_0=1.2$. Numerical values are given in Table VIII.

TABLE VIII. Nuclear density for Cu and Ta, for IPM in square well.[a]

	Nuclear density $\rho(r)$			
	Cu($N=Z=34$)		Ta($N=Z=92$)	
r/R	$r_o=1.5$	$r_o=1.2$	$r_o=1.5$	$r_o=1.2$
0	3.68	3.50	4.15	3.99
0.1	3.47	3.32	3.77	3.65
0.2	2.97	2.86	3.24	3.13
0.3	2.54	2.47	3.10	3.01
0.4	2.55	2.45	3.18	3.10
0.5	3.00	2.87	3.08	2.99
0.6	3.65	3.47	2.99	2.91
0.7	3.88	3.73	3.36	3.26
0.8	3.35	3.29	3.49	3.47
0.9	2.14	2.18	2.59	2.60
1.0	0.847	0.947	0.899	0.983
1.1	0.226	0.315	0.158	0.217
1.2	0.067	0.106	0.039	0.053

[a] The well-depths and the IPM energy levels are given in Table I. The nuclear density $\rho(r)$ is normalized for radial integration: $\int_0^\infty \rho(r) r^2 dr = 1$. With this normalization, the numerical value would be 3 for a nucleus of constant density.

[11] J. S. Levinger, Phys. Rev. **84**, 43 (1951).
[12] B. J. Malenka, Phys. Rev. **86**, 68 (1952).
[13] B. F. Touschek, Phil. Mag. **41**, 849 (1950).

for Pb^{208}, Touschek (using $r_o=1.5$) would estimate that there should be a dipole level below about 1.4 Mev, in contradiction to the experiments of Elliott *et al.*[14] who find no dipole levels up to an excitation energy of 3.7 Mev. The Pauli principle correlation increases the value of W_H, and estimate of the position of the lowest dipole level, to a value of about 6 Mev, which is not in disagreement with present experiments. (As discussed above, the value of W_H is raised still further by use of a smaller radius, and by including exchange forces.)

We believe that we have shown that the IPM is not in clear disagreement with experiments on the nuclear photoeffect. The IPM should provide a good starting point for making less approximate calculations.

ACKNOWLEDGMENT

We wish to thank J. L. Burkhardt for communication of his results, and for helpful discussions on these problems.

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PHYSICAL REVIEW VOLUME 95, NUMBER 2 JULY 15, 1954

Analysis of Proton-Proton Scattering at 9.7 Mev*

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(Received April 6, 1954)

The Berkeley data on the 9.7-Mev differential scattering cross section have been subjected to a phase shift analysis. An S phase shift in agreement with earlier work has been found, but with somewhat better precision. This value agrees with that expected from a Yukawa potential.

This experiment shows some suggestion of a repulsive P-state interaction, while the Los Alamos data show a similar but attractive P-state interaction. It is observed that the apparent discrepancy in the small-angle differential scattering cross section is large enough to warrant further measurements in the small-angle region.

THE scattering of protons by protons near 10 Mev has been measured by several experimenters.[1–3] This report summarizes the analysis of the most recent experiment.

The differential scattering cross section obtained by Cork and Hartsough[3] has been fitted in a least-squares sense to a two-parameter phenomenological angular distribution by simultaneous adjustment of the S and P phase shifts.[4] Two systems of weighting were used, Method I weighting observations at a given angle by the reciprocal square of the fractional absolute error, and Method II using the reciprocal square of the fractional statistical error. Method I gave nearly equal weights, the ratio of maximum to minimum weight being 2.6. Method II assigned markedly unequal weights, the corresponding ratio being 31. The phase shifts resulting from the two assignments were in good agreement, as shown by Table I.

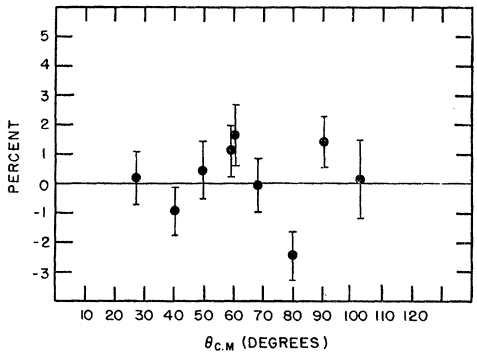

FIG. 1. The percentage difference between the experimental cross section and the least-squares fitted theoretical cross section involving only S and P scattering, as a function of angle.

Inasmuch as the differences are negligible, further comments will be restricted to the first set of phase shifts for simplicity.

The fitting of nine observations by a two-parameter function is over-determinate, so the character of the residuals, $(\sigma_{\text{exp}}-\sigma_{\text{th}})/\sigma_{\text{th}}$, is a gauge of the goodness of fit achieved. Figure 1 exhibits these residual deviations together with the absolute uncertainties, both expressed in percent.

There is no apparent suggestion that the residuals have the angular distribution characteristic of D scattering, since this would require opposite signs above and below 55° for a small D phase shift.

Figure 2 presents the matter in a somewhat more revealing light. Here the isolated points represent the

* This work was performed under the auspices of the U. S. Atomic Energy Commission.
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[4] H. H. Hall and J. L. Powell, Phys. Rev. **90**, 912 (1953).