Threshold Field Properties of Some Superconductors*

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Some refined measurements of the critical field curves for tin, thallium, indium, and mercury have been completed and the results compared with the specific predictions of the Gorter-Casimir and the Koppe versions of the two-fluid model of superconductivity. Neither version is completely adequate, although each has points in its favor. The Koppe prediction of a universal critical field curve for all superconductors is not verified. The Gorter-Casimir α model has greater flexibility than the Koppe model, and although it is capable of giving a fair description of the critical field data, it is in some respects also inconsistent with the data. The isotope effect in thallium has been observed and is consistent with the half-power law.

INTRODUCTION

THE threshold field curve, which defines the phase boundary between the normal and superconducting states, is a fundamental characteristic of superconductors. From the thermodynamics of the superconducting phase transition it follows that the difference in free energies of the normal and superconducting states is proportional to the square of the critical field. Consequently one can check the applicability of thermodynamic reasoning to the problem by quantitatively comparing threshold field data with calorimetric data for mutual consistency. Such comparisons have been made in the past by other observers, and the correctness of the thermodynamic treatment has been adequately confirmed.

This paper is primarily concerned with another aspect of the threshold field curve. Since the threshold field is determined once the free energies of the normal and superconducting states are given it provides a means of testing theoretical models of superconductivity in which the free energy is explicitly given as a function of temperature. The two-fluid model of superconductivity is such a model, and in the present paper we shall use some recently obtained precise critical field measurements on tin, mercury, indium, and thallium to compare with some specific forms of the two-fluid model.

Because of the relatively high precision of the present data it becomes practicable to try to interpret some of the finer details of the critical field curve. In a recent paper[1] a very general form of the two-fluid model was described and some of the present experimental data were briefly presented in a preliminary form. In this paper we shall be more directly concerned with describing the experimental aspects of the investigation and giving a more detailed analysis of the data.

The data for tin were given in an earlier paper[2] in connection with an investigation of the isotope effect. We give here some additional parameters which have been calculated from the original data. The thallium data also stem from an investigation of the isotope effect and have not been previously published in detail. These measurements on tin and thallium revealed systematic departures of the critical field curves from parabolic form of a kind which were generally consistent with some forms of the two-fluid model. It was therefore also thought advisable to investigate the behavior of indium and mercury with high precision techniques. A limited comparison of this sort using the tin data has been given by Bender and Gorter.[3]

EXPERIMENTAL METHOD

The method of measurement was the null magnetic method described in an earlier paper.[2] The identical apparatus was used for the thallium measurements. For mercury and indium an apparatus with another pickup coil system was used which differed from the one described chiefly with respect to number of turns and coil resistance. Some typical transition curves observed in the course of these experiments are given in Figs. 1, 2, and 3 as well as in Fig. 4 of the earlier paper.[2] The critical field is taken as that field at which the specimen would become completely normal if there were no rounding-off of the upper portion of the transi-

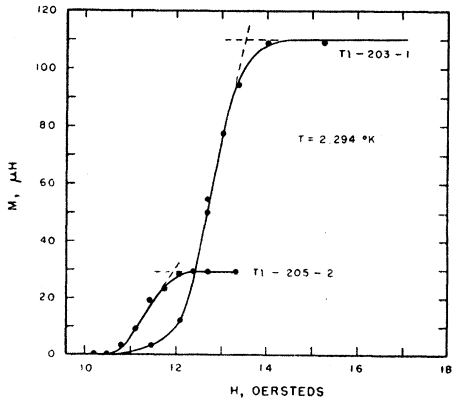

FIG. 1. Transition curves for thallium.

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[1] P. M. Marcus and E. Maxwell, Phys. Rev. **91**, 1035 (1953).
[2] E. Maxwell, Phys. Rev. **86**, 235 (1952).
[3] P. L. Bender and C. J. Gorter, Physica **18**, 597 (1952).

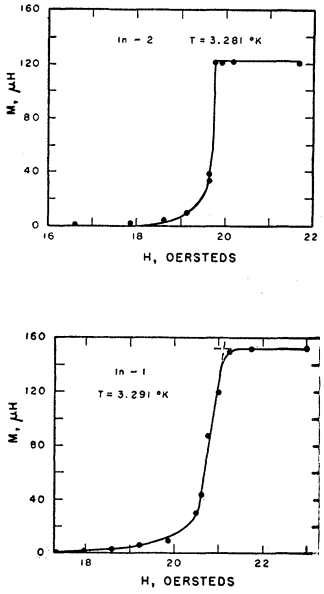

FIG. 2. Transition curves for indium. In-1 cast in Pyrex capillary; In-2 single-crystal free rod of indium.

tion curve. The steep slope of the transition curve is extrapolated to the level of M (the mutual inductance setting for balance) in the normal state, and the corresponding value of the field is taken as the critical field.

In some cases where the transitions were sharp, as for the mercury and indium single crystals, it was possible to fix the critical field to within about one tenth of an oersted. In the case of thallium, however, some of the transitions were less sharp and have a probable uncertainty of three or four tenths of an oersted. The errors due to nonuniformity of the field and to uncertainty in the calibration constant of the magnet are each less than $\frac{1}{4}$ percent.

THE SPECIMENS

Some of the physical properties of the specimens are given in Table II. The isotopic tin and thallium samples were reduced from the oxide form and the chemical purity figures refer to the original oxides. The samples were melted *in vacuo* after reduction and subsequently recrystallized in the course of which process the purity was probably somewhat enhanced. The natural metal samples were in all cases Johnson-Matthey Specpure materials.

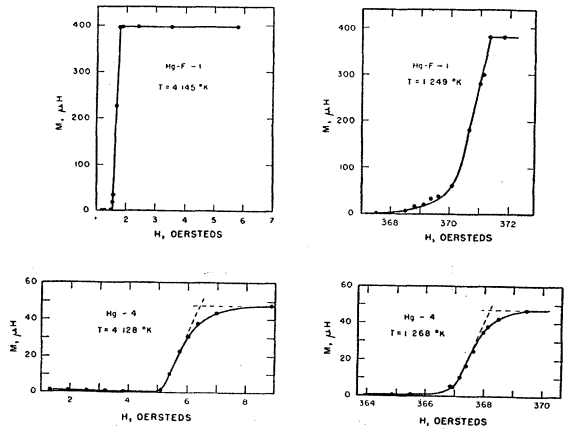

FIG. 3. Transition curves for mercury. Hg-4 sample in Pyrex tube; Hg-F-1 free rod of mercury.

The tin and thallium samples were kept in the Pyrex glass capillaries in which they were crystallized. Since tin does not stick to clean Pyrex this probably resulted in no appreciable strain when the specimens were cooled down. Thallium, however, does stick somewhat to glass so there is a possibility that strain may have been present. One of the indium samples In-1 was crystallized in a Pyrex capillary in which it was sealed off and used. Since indium also tends to stick to glass a second sample In-2 was cast and crystallized, and then the glass was etched off in HF. The crystal was kept and used in a loosely fitting glass tube.

Mercury sample Hg-4 was enclosed in Pyrex and frozen just prior to using. The second sample, Hg-F-1, was a free rod and was made by slowly freezing mercury in a glass tube lines with thin paper. The freezing mixture was a mixture of dry ice in alcohol, and the tube was slowly lowered into the bath so as to promote slow crystal growth. After freezing, the rod was slipped out of the enclosing tube, the paper lining removed and the frozen rod reinserted so that it was free of the walls. The rod was kept cold until used.

RESULTS

The collected results of the critical field measurements are given in Tables I, II, and III. The original tin data were tabulated in the earlier paper.[2] Some minor changes have been made in the values of some of the tin constants. In the earlier analysis where relative changes were more important, the computations were based heavily on difference measurements whereas in the present paper each of the isotope curves has been considered by itself.

The transitions were for the most part reasonably sharp and the frozen-in moments were no more than a few percent. The thallium data are probably the poorest, as shown both by the scatter of the points in Fig. 5 and by the breadth of the transitions in Fig. 1. This may be, in part, due to strain resulting from adhesion to the glass walls of the capillary and in part to chemical impurity.

The effect of strain may be gauged from the comparative data of the two indium specimens and the mercury specimens. In-1 was used in the original glass capillary in which it had been cast whereas the glass was removed from In-2 by etching in hydrofluoric acid. The transitions are slightly sharper for In-2, and there is a small but observable change in the critical field curve, including a small reduction in the initial slope and a lowering of the transition temperature.

TABLE I. Threshold field data for the specimens.

Tl-5		Tl-205-2		Tl-203-1		In-2		In-1	
T°K	H oersteds	T°K	H oersteds	T°K	H oersteds	T°K	H oersteds	T°K	H oersteds
2.370	2.95	2.358	3.70	2.384	1.35	1.219	238.5	1.287	237.7
2.350	4.87	2.351	4.40	2.381	2.08	1.230	237.7	1.698	205.8
2.330	7.86	2.344	5.32	2.375	2.73	1.648	207.7	1.967	181.8
2.294	12.33	2.334	6.76	2.374	2.74	1.862	188.5	2.295	146.7
2.171	28.2	2.330	7.46	2.372	2.92	2.023	173.7	2.353	141.3
2.001	47.9	2.310	9.80	2.370	3.45	2.300	144.4	2.900	74.0
1.805	69.8	2.297	11.42	2.358	4.98	2.624	107.3	3.136	42.5
1.598	91.2	2.294	11.93	2.351	5.96	2.875	75.0	3.291	21.1
1.524	99.2	2.170	27.8	2.344	6.44	3.110	43.7	3.364	9.90
1.257	122.4	2.001	47.3	2.334	7.95	3.237	26.1	3.392	5.87
		1.994	47.7	2.330	8.67	3.281	19.80	3.411	3.17
		1.805	69.0	2.310	10.92	3.321	13.70		
		1.745	75.6	2.297	12.76	3.379	5.30		
		1.598	90.2	2.294	13.52				
		1.464	103.2	2.170	29.0				
		1.257	122.3	2.001	48.9				
				1.994	49.4				
				1.745	77.1				
				1.464	104.4				
				1.237	123.9				

Hg-4		Hg-F-1	
T°K	H oersteds	T°K	H oersteds
1.268	368.2	1.249	371.3
1.913	323.3	1.935	326.8
2.456	267.8	2.512	260.6
2.888	212.8	2.924	208.8
3.292	154.5	3.385	137.8
3.580	107.5	3.750	78.0
3.862	57.7	3.992	31.9
3.986	34.1	4.098	10.85
4.014	28.6	4.145	1.84
4.064	19.1		
4.067	18.0		
4.098	11.6		
4.128	6.46		
4.148	2.32		

Both of these effects are usually associated with strain, i.e., the less the strain the lower are both T_c and $(dH/dT)_{T_c}$. The same is true for Hg-4 and Hg-F-1, the former of which was enclosed in a glass capillary while the latter was a free rod and possibly also a single crystal. The transitions were appreciably sharper for Hg-F-1 and the transition temperature slightly lower. There was little difference in the initial slope.

It is quite possible that the thallium data may have been influenced somewhat by strain. The tin was probably strain free because of the fact that tin does not usually adhere to clean glass.

The critical field curves are closely parabolic in the lower range of temperatures, but all except those for mercury clearly deviate from the simple parabolic form at higher temperatures. The mercury curve is however closely parabolic over the entire range. The over-all departures from the parabola, $h=1-t^2$, are exhibited in Figs. 4 to 7 where the differences between a parabola drawn through H_0 and T_c, and the observed data, are plotted in normalized form. H_0 is the critical field at absolute zero, T_c is the zero-field transition temperature, and h and t are the reduced field and temperature H/H_0 and T/T_c, respectively. The differences, although small, are quite observable. The significance of the solid and dashed curves will be explained later.

The critical field curves all have this feature in common. At low temperatures they are parabolic and at temperatures near $T=T_c$ they are all linear. We can therefore specify four observable parameters for each curve, H_0, T_c, $[dH/d(T^2)]_{T=0}$ and $(dH/dT)_{T_c}$, without reference to the detailed intermediate form of the curve. These quantities are tabulated in Table II, and were obtained by least squares adjustment of straight lines to selected portions of the data, namely $H_c(T^2)$ near $T=0$ and $H_c(T)$ near $T=T_c$. It turns out that both slopes are related to interesting physical quantities in the two-fluid description of a superconductor. If $H_c(T)$ is expressed as a polynomial in T, then the slope $[dH/d(T^2)]_{T=0}$ is the coefficient of the leading term. Other coefficients may be evaluated from T_c, $(dH/dT)_{T_c}$ and intermediate points. Some fourth degree polynomial representations found in this way

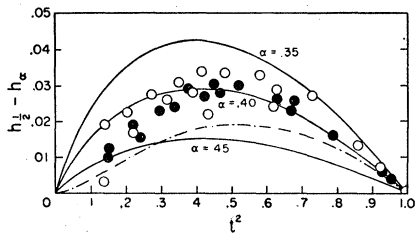

FIG. 4. Plot of the deviation of the critical field data for tin from the parabola $1-t^2$. The solid curves are the deviations calculated for the α model. The broken curve is the deviation calculated from Koppe's model. ○ Sn-112-1; ● Sn-124-1.

TABLE II. Observed threshold field curve parameters.

Sample	T_c °K	H_0 oersteds	$-\left(\frac{dH}{dT}\right)_{T_c}$ oersteds/°K	$-\left[\frac{dH}{d(T^2)}\right]_{T=0}$ oersteds/°K²	Remarks
Sn 112-1	3.8090	312.3	144.3	23.42	42 mm×0.33 mm
	(0.0089)	(0.63)	(1.3)	(0.66)	99.58% pure
Sn 118-1	3.7452	303.3	139.8	22.92	57 mm×0.39 mm Cast and used in Pyrex capillaries;
	(0.0010)	(1.52)	(0.49)	(0.13)	99.19% pure probably single crystals
Sn 124-1	3.6657	298.4	141.0	23.98	23 mm×0.31 mm
	(0.0027)	(0.54)	(1.54)	(0.18)	99.91% pure
Tl-5	2.3917	172.8	125.8	31.85	64 mm×0.5 mm
(nat. Tl)	(0.0025)	(1.4)	(5.6)	(0.71)	specpure thallium
Tl-205-2	2.3861	171.4	129.2	31.62	37 mm×0.35 mm Cast and used in Pyrex capillaries:
($M=205.02$)	(0.0010)	(1.1)	(2.0)	(0.50)	99.73% pure presumably single crystals
Tl-203-1	2.3956	170.0	130.5	30.40	52 mm×0.51 mm
($M=203.33$)	(0.0007)	(1.1)	(1.7)	(0.33)	99.74% pure
In-1	3.4314	278.9	149.6	25.16	52 mm×1 mm; single crystal cast in Pyrex capillary and kept
(nat. In)	(0.0010)	(0.78)	(1.8)	(0.21)	in glass envelope; specpure indium.
In-2	3.4178	275.0	142.2	24.68	45 mm×1 mm; single crystal cast in Pyrex capillary and glass
(nat. In)	(0.0019)	(0.46)	(1.5)	(0.14)	etched off in HF; specpure indium.
Hg-4	4.1596	404.5	196.2	22.81	62 mm×1 mm; 99.999 Hg in Pyrex capillary.
	(0.0011)	(2.7)	(2.1)	(0.49)	
Hg-*F*-1	4.1538	410.7	196.9	23.56	50 mm×2.5 mm; 99.999 Hg free rod (presumably single
	(0.0008)	(4.0)	(1.6)	(0.70)	crystal).

(Figures in parentheses are the standard deviations of the numbers above them.)

are as follows:

$$\text{Sn} \quad h=1-1.078t^2-0.103t^3+0.181t^4,$$
$$\text{In} \quad h=1-1.051t^2-0.123t^3+0.1745t^4,$$
$$\text{Tl} \quad h=1-1.053t^2-0.152t^3+0.206t^4,$$
$$\text{Hg} \quad h=1-t^2.$$

From the observable parameters we can also calculate values for γ, the molar specific heat coefficient associated with the electrons in the normal state. These values, calculated from Eq. (3) below, are as follows:

$$\text{Sn} \quad \gamma=4.45\times10^{-4} \text{ cal/deg}^2 \text{ mole},$$
$$\text{Hg} \quad \gamma=5.3\times10^{-4} \text{ cal/deg}^2 \text{ mole},$$
$$\text{In} \quad \gamma=4.0\times10^{-4} \text{ cal/deg}^2 \text{ mole},$$
$$\text{Tl} \quad \gamma=3.65\times10^{-4} \text{ cal/deg}^2 \text{ mole}.$$

ISOTOPE SHIFT IN THALLIUM

In the course of these experiments the isotope shift between the two thallium samples, of mass 205.01 and 203.33, was observed. The value of the exponent ϵ in the relation $M^{\epsilon}H_0=\text{const}$ was calculated using the approximation $\epsilon=(\Delta H/H_1)_{T=0}/(\Delta M/M_2)$, where ΔH and ΔM are the differences in critical field and mass for the two samples. $(\Delta H)_{T=0}$ was obtained by extrapolation of the critical field differences to $T=0$. It was found that $\epsilon=0.49$ with a standard deviation of ±10 percent arising from the uncertainties of extrapolation. Thus while the precision is not high the value of the exponent is consistent with the half-power relation previously observed in tin[2] and mercury.[4]

[4] Reynolds, Serin, and Nesbitt, Phys. Rev. 84, 691 (1951).

DISCUSSION

Before discussing the relation of these results to the two-fluid model we shall briefly summarize some of the features of this model from the standpoint of the treatment referred to earlier.[1] From this point of view superconductivity is associated with changes in the energies of the electrons near the surface of Fermi distribution, which may be described by saying that a fraction of the electrons enter a condensed state, losing their thermal excitation and entropy. The degree of condensation is described by a parameter ω which by definition takes on values between 0 and 1 as the transition temperature goes from T_c to 0. The excited and condensed fractions are treated like two phases in thermal equilibrium. The distribution between the two phases varies so that first, at $T=T_c$, all electrons are in the excited phase and then gradually condense as the temperature is lowered, until at $T=0$ all are in the condensed state. Excluding lattice terms the free energy of the superconducting state is

$$F_s=-\beta\omega-\tfrac{1}{2}\gamma T^2K(\omega), \tag{1}$$

where the first term is the condensation energy and the second is the thermal excitation term. β is given by $H_0^2V_m/8\pi$, in which V_m is the molar volume. $K(\omega)$, a function of ω, is the effective normal fraction, and takes on values from 1 to 0 as T goes from T_c to 0. Minimizing F_s with respect to ω yields the equilibrium condition

$$K'(\omega_e)=K'(0)(T_c/T)^2, \tag{2}$$

in which ω_e is the equilibrium value of ω and also a

generalized form of the Kok relation

$$K'(\omega_e)=-\frac{H_0^2 V_m}{T_c^2 4\pi K'(0)}. \tag{3}$$

Some explicit forms for the critical field curve and some related derivatives may be readily obtained. These are

$$h^2=\omega_e+t^2[1-K(\omega_e)]/K'(0), \tag{4}$$

$$dh/dt=h/t-\omega_e/ht, \tag{5}$$

$$K(\omega_e)=1-K'(0)\left[\frac{d(h^2)}{d(t^2)}\right]=1-K'(0)\left[2h\frac{dh}{d(t^2)}\right], \tag{6}$$

in which h and t are the normalized critical field and transition temperature, H/H_0 and T/T_c. It follows from these that

$$K'(0)=\left[\left(\frac{d(h^2)}{d(t^2)}\right)^{-1}\right]_{t=0}=\left[\left(2h\frac{dh}{d(t^2)}\right)^{-1}\right]_{t=0}, \tag{7}$$

$$\left(\frac{dh}{dt}\right)_{t=1}=-\left[-\left(\frac{d\omega_e}{dt}\right)_{t=1}\right]^{\frac{1}{2}}. \tag{8}$$

It will be apparent that $h(t)$ is determined once $\omega_e(t)$ and $K(\omega_e)$ are known. Conversely if the detailed form of $h(t)$ is known, both $\omega_e(t)$ and $K(\omega_e)$ may be determined. Moreover the quantities $K'(0)$ and $(d\omega_e/dt)_{t=1}$ may be evaluated if H_0, T_c, and the slopes $[dH/d(T^2)]_{T=0}$ and $(dH/dT)_{T_c}$ are known.

We shall compare the observed form of $h(t)$ and the quantities $K'(0)$ and $(d\omega_e/dt)_{t=1}$ with those calculated on the basis of two special forms of the two-fluid model, the α model of Gorter and Casimir,[5] and the Koppe[6] model.

The α model is a purely *ad hoc* description, and while the Koppe model is based on ideas proceeding from Heisenberg's theory of superconductivity, it too may be considered on an *ad hoc* basis. However it makes a definite assumption about the statistics of the electron distribution and consequently goes back a little further than the α model.

In the α model $K(\omega)$ is given by

$$K(\omega)=(1-\omega)^\alpha, \tag{9}$$

where α is a parameter of the superconductor. When $\alpha=\frac{1}{2}$ the critical field curve reduces to the well-known parabolic case, and both the electronic entropy and specific heat follow a T^3 law. Most previous treatments and applications of the α model have assumed that $\alpha=\frac{1}{2}$. However by taking α as a general parameter we get an extra degree of freedom which is useful in describing the departures from parabolic form which are actually observed. A more detailed discussion of the α model has been given in the paper referred to earlier.[1] We need only remark here that for the α model Eqs. (4), (7), and (8) become

$$h^2=1-\frac{t^2}{\alpha}+\left(\frac{x}{\alpha}-1\right)t^{2/(1-\alpha)}, \tag{10}$$

$$K'(0)=-\alpha, \tag{11}$$

$$(d\omega_e/dt)_{t=1}=-2(1-\alpha). \tag{12}$$

Koppe's model postulates a definite form for $K(\omega)$ based on the assumption that the density of electron states has a discontinuity at the Fermi level. Koppe's expression for $K(\omega)$ cannot be given in closed form but has been evaluated numerically[7] and is a function of only reduced variables. Consequently the same $K(\omega_e)$ and $h(t)$ apply to all superconductors according to Koppe. This model gives a critical field curve which is not quite parabolic but which has the same curvature as the parabola at $t=0$, namely $[dh/d(t^2)]_{t=0}=-1$.

Both the α model and the Koppe model are compared with the data in the difference plots Figs. 4 to 7. The solid curves are the difference between a parabola passing through H_0 and T_c and the α-model curves for various values of α. The broken-line curve is that predicted by the Koppe model.

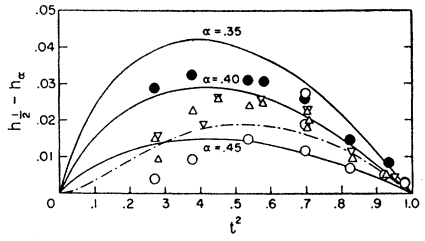

FIG. 5. Plot of the deviation of the critical field data for thallium from the parabola $1-t^2$. The solid curves are the deviations according to the α model and the broken line that according to the Koppe model. The open points represent data normalized with a value of H_0 obtained by passing a parabola through the points near $t=0$. The solid points represent data normalized with a value of H_0 found by passing a cubic through all the points. ○, ● Tl-203-1; ▽, Tl-5; △, Tl-205-2.

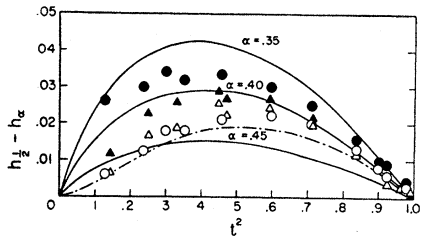

FIG. 6. Plot of the deviation of the critical field data for indium from the parabola $1-t^2$. The solid curves are the deviations according to the α model and the broken line that according to the Koppe model. The open points represent data normalized with a value of H_0 obtained by passing a parabola through the points near $t=0$. The solid points represent data normalized with a value of H_0 found by passing a cubic through all the points. △, ▲ In-1; ○, ● In-2.

[5] C. T. Gorter and H. B. G. Casimir, Physik. Z. **35**, 963 (1934).
[6] H. Koppe, Ann. Physik **1**, 405 (1947).
[7] See references 1 and 5, also B. B. Goodman, Cambridge thesis (unpublished).

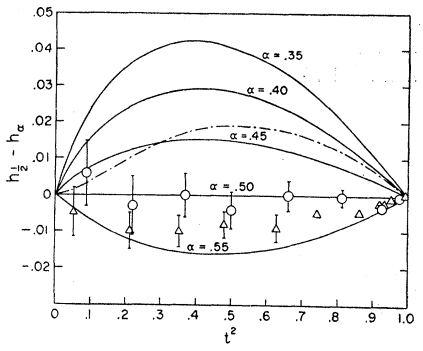

FIG. 7. Plot of the deviation of the critical field data for mercury from the parabola $1-t^2$. △ Hg-4, ○ Hg-*F*-1.

The Koppe model appears to be a satisfactory representation for the indium data and a fair representation for the thallium, for which the precision is somewhat poorer. However it is not consistent with the mercury data, which is closely parabolic, or with the tin data which has somewhat greater curvature.

The α model, with its extra parameter, is somewhat more flexible than the Koppe model, but it too is inconsistent with some features of the data. It should be noted that the experimental data in the plots of Figs. 4–7 define curves whose curvature is sensitive to small changes in the value of H_0 used for normalization. The data represented by the open points were normalized using a value of H_0 found by passing a parabola through the data in the region of $T=0$. The solid points were obtained using a value of H_0 found by passing a cubic through all of the points. The first technique is probably better and the parameters of Tables II and III were determined by this method. The tin plots of Fig. 4 were also normalized in this way. The mercury data were normalized by passing a parabola through all the points. There was no significant difference between this parabola and that fitted locally to the points near $T=0$.

We can apply some internal consistency checks to see how well the α and Koppe models succeed in describing the data. In the α-model description α is related to the slopes at $t=0$ and $t=1$ by the following relations:

$$K'(0)=\{2[dh/d(t^2)]_{t=0}\}^{-1}=-\alpha, \tag{13}$$

$$(dh/dt)_{t=1}=-(2/1-\alpha)^{\frac{1}{2}}. \tag{14}$$

If the α model is a good representation the α determined by (13) and (14) should be the same as that giving the best fit in Figs. 4–7. In Table III we have tabulated the quantities $-K'(0)$ and $1-2[(dh/dt)_{t=1}]^{-2}$ for each of the specimens. For the mercury data, which is parabolic and which would check in any case, there is of course agreement. For the other three metals however $-K'(0)$ runs from 0.46 to 0.49 whereas the α computed from $(dh/dt)_{t=1}$ is considerably lower, of the order of 0.3–0.4. In fact if one now examines the deviation plots of Fig. 4 there is a trend in the data in the direction of increasing α, as t^2 goes from 1 to 0. Stated in another way, for $t^2<0.4$ the data is closely linear in t^2 whereas the α-model curves exhibit greater curvature. It appears therefore that the α model, with constant α, represents the data in an approximate fashion but is not an accurate description.

TABLE III. Derived quantities pertinent to α and Koppe models.

Sample	$-K'(0)$	$1-2\times[(dh/dt)_{t=1}]^{-2}$	$[(dh/dt)_{t=1}]^2$	$-K'(0)\times[(dh/dt)_{t=1}]^2$
Sn-112-1	0.46	0.35	3.1	1.42
Sn-118-1	0.47	0.33	3.0	1.41
Sn-124-1	0.46	0.33	3.0	1.39
Tl-5	0.47	0.34	3.0	1.44
Tl-205-2	0.48	0.38	3.2	1.54
Tl-203-1	0.49	0.41	3.4	1.65
In-1	0.47	0.41	3.4	1.60
In-2	0.48	0.36	3.1	1.49
Hg-4	0.50	0.50	4.0	2.0
Hg-*F*-1	0.50	0.50	4.0	2.0

It is noteworthy that Koppe's model predicts a critical field curve which has the same curvature as the simple parabola at $t=0$. In Koppe's model $K(\omega_e)$ decreases exponentially as $t\to 0$. In this region the critical field curve is an insensitive tool with which to explore $K(\omega_e)$. Measurements of specific heat and thermal conductivity below 1°K would be revealing and some recent determinations[8] of the latter have indeed indicated exponential behavior. Goodman[8] has suggested that this behavior is essentially a characteristic of a system with a finite gap between the ground state and the lowest excited state.

In the treatment of the general two-fluid model[1] the specific heat discontinuity is given by

$$(C_s-C_n)/\gamma T_c=-K'(0)[(dh/dt)_{t=1}]^2. \tag{15}$$

This quantity is tabulated for all of the specimens in Table III. Koppe has predicted that this quantity is numerically equal to $\pi^2/(12\ln^2 2)=1.7118$ for all superconductors, which is apparently not borne out.

In conclusion it appears that neither the α model nor the Koppe model gives a completely adequate description of a superconductor. The Koppe model is not quite flexible enough and its prediction of a unique reduced critical field curve for all superconductors is not verified. In particular it is inconsistent with the parabolic form observed with mercury. The α model has enough flexibility to give an approximate characterization of critical field curves, but it appears that the model with constant α is an oversimplified description.

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[8] B. B. Goodman, Proc. Phys. Soc. (London) **A66**, 217 (1953); J. G. Daunt, Proc. Schenectady Cryogenics Conference, 1952 (unpublished).