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Theory of the Aurora Based on Magnetic Self-Focusing of Solar Ion Streams

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It is assumed that the sun emits a jet of an electrically neutral mixture of fast electrons and positive ions, and that the space between the sun and earth is filled with an ionized, electrically neutral gas. As the jet proceeds the fast electrons are slowed down and scattered by encounters with the space ionization, being continually replaced by slow electrons from space. The stream of positive ions continues as an electric current, but at all times the stream and surrounding gas are closely electrically neutral. Due to the magnetic field of the electric current, the fast moving positive ions are continually compressed into the stream and the fast electrons and slower ions are ejected from the stream. This is "magnetic self-focusing." The stream develops into a narrow core which if correctly aimed enters the earth's magnetic field and is diverted to auroral zones according to the calculations of Störmer. Meinel's observations indicate that the primary solar particles in the stream are mainly protons of speed greater than 3×10^8 cm sec^{-1}. A speed of order of 10^{10} cm sec^{-1} of the protons is required for correct Störmer orbits into auroral latitudes and penetration to 60- to 80-km levels in the upper atmosphere. For a stream which carries sufficient energy for an aurora, the density of space ionization between the sun and earth is calculated to be at least 10^{-5} electron ion pairs cm^{-3} to support magnetic self-focusing and not more than 10^9 pairs cm^{-3} for the fast protons to be able to reach the earth. It is suggested that the auroral rays are not the solar proton stream but are formed when the protons are stopped in the upper atmosphere. Due to the charges and absorbed energy of the protons, oxygen and nitrogen particles of the atmosphere are ejected outward to great heights along the lines of magnetic force and form the auroral rays and streamers.

INTRODUCTION

THE identity of the primary particles incident upon the earth's upper atmosphere and responsible for the aurorae has remained in question[1] until recently when Meinel reported his measurements[2] of the H_α line in the quiet arc which occurs at the beginning of an auroral display. Meinel observed a broadening but no Doppler shift in the H_α line when observing towards the magnetic horizon, but found and measured a Doppler shift when observing towards the magnetic zenith. He was able to show that protons reach the earth's atmosphere with velocities of at least 3.3×10^8 cm/sec corresponding to more than 50 000 electron volts of proton energy.

The hydrogen which is responsible for the H_α line having the observed Doppler shift would have to approach the earth as ions in order for the earth's magnetic field to be able to restrict such particles to the auroral zone of about 23° of latitude around the magnetic poles of the earth. Calculations of the kind first published by Störmer[3] show that such ions would have to have velocities of at least 10^{10} cm sec^{-1} corresponding to energies of the hydrogen ions of 65 Mev. Meinel pointed out that these much larger velocities are not precluded by his observations because an incoming hydrogen ion would first have to be slowed down by collisions in the earth's atmosphere to the order of 10^8 cm sec^{-1} before it could attach a slow atmospheric electron into the excited atomic state from which H_α could be radiated. The maximum intensity in Meinel's Doppler-shifted H_α line occurs at a velocity of 4×10^7 cm/sec and the intensity falls steadily to approximately zero at a velocity of 3.3×10^8 cm/sec. The likelihood of electron attachment is negligible until the relative velocity of ion and electron has decreased to the order of magnitude of the Bohr-orbital velocity of the electron in the atom to be formed[4] which for the excited state necessary for the H_α radiation is about 7×10^7 cm/sec. That high-velocity ions were present in the upper atmosphere in auroral latitudes was discovered by Meredith, Van Allen, and Gottlieb[5] by means of cosmic-ray equipment on high-altitude rockets. They observed ionic particles having a penetrating power greater than that of 18-Mev protons at north magnetic latitudes 64° and 74°, which are near the region of maximum auroral frequency, and did not observe such ionic particles at north magnetic latitudes 89° and 56°, which are remote from the auroral zone.

A 65-Mev proton has a penetrating power of approximately 26 meters of air at normal pressure and temperature. If these protons were to enter the earth's atmosphere vertically they would penetrate to an altitude of 40 kilometers whereas auroral arcs are usually seen at higher altitudes than this. Störmer calculations of the paths of such protons in the earth's magnetic field show that the paths are turned away from the vertical and that the protons spiral or curve around the magnetic lines of force while approaching the earth, reaching the end of their allotted 26 meters

[1] A recent review, "The Aurorae," V. C. A. Ferraro in Advances in Physics, a supplement of the Phil. Mag. 2, (7), 265–320 (1953).
[2] A. B. Meinel, Astrophys. J. **113**, 50 (1951), and Mém. soc. roy. sci. Liège **12**, 203 (1952).
[3] C. Störmer, Kristiania Ske. Vid. Selsk **1**, 3 (1904) and Arch. sci. phys. et nat. **32**, (1911–12). See also Thomas H. Johnson, Revs. Modern Phys. **10**, 193 (1938) and R. A. Alpher, J. Geophys. Research **55**, 437 (1950).
[4] See N. F. Mott and H. S. W. Massey, *The Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), second edition, Chap. XII, Sec. 2.
[5] Meredith, Van Allen, and Gottlieb kindly gave us advance information of their results.

of air (at N.T.P.) at altitudes above 40 kilometers. Meinel found that H_α comes from lower altitudes than the oxygen and nitrogen auroral emissions.

Although the 10^{10} cm sec^{-1} protons must be slowed down to the order of 10^8 cm sec^{-1} before attachment of electrons to radiate H_α is likely, protons moving at an intermediate velocity of say 10^9 cm sec^{-1} are moving at the same velocity as 250-ev electrons, and this is the right order of magnitude of velocity for the ionizing particles to excite optically the atmospheric molecules and produce the main auroral displays.

Many of the features of auroral phenomena were explained in the early work of Birkeland[6] and Störmer[3] on the supposition that streams of particles of one polarity of charge moved from the sun to the earth. The theory was developed to explain the restriction of the incoming ions to auroral zones around the magnetic poles by the earth's magnetic field, the delay between the passage of a sun spot past the line of centers between the sun and earth and the incidence of the auroral disturbance at the earth, the correlation between auroras and magnetic storms, and the location of auroras at 80 to 140 kilometers in altitude.

Explanations of the auroras involving streams of charged particles were rejected by Schuster[7] and Lindemann[8] (now Lord Cherwell) and later by Chapman and Ferraro[9] for the reason that such streams would disperse due to mutual electrostatic repulsion of the particles in the stream. To overcome this objection, various theories[9–11] have been advanced based on electrically neutral streams of particles of both signs of charge, but the theories have not been carried completely or satisfactorily to the point of explaining how the particles are diverted to auroral latitudes. In this paper a theory is presented which is based on an effect not heretofore considered in auroral stream theories, namely the magnetic self-focusing action of ionized streams first presented in 1934 and developed in a more general form recently.[12]

SELF-FOCUSING STREAM THEORY

It is generally supposed that interplanetary space is filled with an electrically neutral mixture of positive ions and electrons but the various estimates of the density of this matter differ by several orders of magnitude. In the theory of magnetically self-focusing streams it is shown[12] that a stream of charged particles of only one sign entering a region containing an electrically neutral mixture of charges of both signs with random velocities small compared with the velocity of the particles in the entering stream can form a magnetically self-focusing stream. The condition that must be met in order for the stream to be self-focusing is that the total electric current i in the stream of fast particles must be greater than

$$i \geqq 2c^2(\theta_1+\theta_2)/eu, \quad (1)$$

where θ_1 is the average kinetic energy of the particles in the entering stream due to components of velocity transverse to the axis of the stream, θ_2 is the average transverse energy of the residual slow particles of opposite sign, u is the average velocity in the direction of the stream of the particles in the entering stream, e is the charge of each such particle, and c is the velocity of light.

When a current of high-speed protons, large enough to be magnetically self-focusing, emerges from the sun and proceeds into the electrically neutral mixture in interplanetary space, the protons introduce an excess positive charge which repels the residual slow positive ions from the stream. The residual slow ions have negligibly small magnetic forces acting on them because their velocities are small, while the high-speed ions are attracted towards each other magnetically. The residual slow ions ejected from the stream produce electric fields in the electrically conducting medium permeating the interplanetary space and the charges in that medium shift under the influence of those electric fields to relieve those electric fields. The residual slow electrons are held in the stream by the excess positive charge in the stream. Any of these electrons with which high-speed protons collide and which recoil in the direction of motion of the protons will be magnetically repelled by the preponderant magnetic field of the high-speed protons, and so the stream continues to repel slow ions and fast electrons keeping only the fast protons and slow electrons. Streams which are established in a gas at low pressure are essentially stable as discussed in reference 12.

In order to estimate the magnitude of the minimum electric current which a stream of 10^{10} cm sec^{-1} protons must have in order to be self-focusing, it is necessary to know the value of the average kinetic energy of the protons due to components of velocity transverse to the axis of the stream. These transverse components arise both by virtue of the thermal spread in velocity of the protons around the mean velocity of the protons in the stream and also because of the divergence of the stream away from the axis of the stream as the stream starts away from the sun. Table I gives some representative values of minimum currents in amperes of 10^{10} cm sec^{-1} protons required for self-focusing, for various temperatures of the protons and various maximum slopes of the direction of motion of protons from the axis in a diverging cone of protons. From Table I it is seen that the divergence of the stream of protons as it emerges from the sun is probably the principal

[6] K. Birkeland, Arch. sci. phys. et nat. **4**, 497 (1896).
[7] A. Schuster, Proc. Roy. Soc. (London) **85**, 44–50 (1911).
[8] F. A. Lindemann, Phil. Mag. **38**, 669–684 (1919).
[9] S. Chapman and V. C. A. Ferraro, Terrestial Magnetism and Atm. Elec. **36**, 77–97; **46**, 1–6 (1941) and S. Chapman, Ann. geophys. **6**, 205–224 (1952).
[10] D. F. Martyn, Nature **167**, 92 (1951).
[11] H. Alfvén, Kgl. Svenska Vetenskapsakad. Handl., 3rd Ser. **18**, No. 3, 1–39 (1939) and No. 9, 1–39 (1940).
[12] W. H. Bennett, Phys. Rev. **45**, 890–897 (1934); Phys. Rev. (to be published).

factor determining the minimum current of protons which the stream must contain in order to become self-focusing and so to reach the earth in an intense enough bundle to produce an aurora.

ORIGIN OF A STREAM

It is assumed that the jet of material which first emerges from the sun consists of an electrically almost neutral mixture of fast protons and fast electrons moving along together at about the same velocity, so that the electric current in the jet is zero; such a jet would not be magnetically self-focusing. The jet would diverge increasingly with increasing altitude above the sun's surface both because of the random thermal velocities of the particles and also under the effects of the gradients of radiation density—the same kind of effect which caused the material in the jet to rise out of the sun and attain such a high velocity in the first place. The jet moves out through the sun's atmosphere which may be supposed to consist mostly of an electrically neutral mixture of slow electrons and slow protons which decreases in density with altitude.

The effects of collisions of the rapidly moving electrons and protons in the outermost parts of the jet with the slow electrons and protons in the sun's atmosphere are calculated using the methods published by Thomas.[13] By using Thomas' equation (4.31) for the time rate at which a charged particle of mass m_1 and large velocity v_1 loses momentum when moving through an atmosphere of slow charged particles of mass m_2 and particle density N_2 per unit volume, the mean rate of deceleration of the particle of kind 1 may be written

$$dv_1/dt = 4\pi e^4 A\{\log(2B/\log B) - \gamma - 1\} N_2/v_1^2 m_1, \qquad (2)$$

where $A = (m_1 + m_2)/m_1 m_2$, $B = 3v_1^6/4\pi A^3 e^6 N_2$, and $\gamma = 0.5772$ is Euler's constant.

In Table II are given some values, calculated from (2), of the deceleration of 10^{10} cm sec^{-1} electrons and of 10^{10} cm sec^{-1} protons in densities N of slow electrons and N of slow protons. Given also are the total deceleration of the electrons and protons in mixtures of the two polarities of slow particles and the ratios of the decelerations in various densities of slow particles. It is seen that the electrons are slowed down at more

TABLE I. Minimum current of 10^{10} cm sec^{-1} protons for magnetic self-focusing for various proton temperatures and various cone angles. (ξ = half-angle of the cone.)

Proton temperature	tan ξ 0.001	0.01	0.1
10 000°K	5.9 amp	570 amp	57 000 amp
100 000	7.3	570	57 000
1 000 000	21	590	57 000

[13] L. H. Thomas, Proc. Roy. Soc. (London) **121**, 464 (1928).

TABLE II. Deceleration in cm sec^{-2} of 10^{10} cm/sec electrons and protons in electrically neutral mixtures of slow electrons and protons of density N.

N	1 cm^{-3}	10^6 cm^{-3}	10^{12} cm^{-3}	10^{18} cm^{-3}
Fast electrons due to slow electrons	1.2	9.6×10^5	7.3×10^{11}	5.1×10^{17}
Fast electrons due to slow protons	0.6	4.9×10^5	3.8×10^{11}	2.8×10^{17}
Fast protons due to slow electrons	3.3×10^{-4}	2.7×10^2	2.1×10^8	1.5×10^{14}
Fast protons due to slow protons	4.5×10^{-7}	0.4	3.2×10^5	2.6×10^{11}
Total deceleration of electrons	1.8	1.5×10^6	11.1×10^{11}	7.9×10^{17}
Total deceleration of protons	3.3×10^{-4}	2.7×10^2	2.1×10^8	1.5×10^{14}
$\frac{\text{Decel. of electrons}}{\text{Decel. of protons}}$	5400	5400	5400	5300

than 5000 times the rate at which the protons are slowed down at all densities of the atmosphere. As the jet of material rises out of the sun, the electrons in the jet are retarded and scattered out of the jet more than 5000 times as rapidly as the protons are, whenever the jet of material has decreased in density enough to permit the atmosphere to penetrate into the jet. In such outermost parts of the jet, the excess of fast protons over fast electrons which accompanies such selective scattering and slowing of electrons, produces a positive electric current outward from the sun. This selective scattering and slowing of the electrons continues with a continuing increase in positive electric current outward from the sun in the jet until the current has risen to exceed the critical current for magnetic self-focusing given by Eq. (1). At this point, the magnetic field of the magnetically self-focusing stream exerts a repellent force on the remaining fast electrons in the jet strong enough to repel them all from the stream, keeping only the slow electrons which were already there before the stream arrived or which drift into the stream from the outside of the stream assisted by the electric attraction of the excess positive charge in the magnetically self-focusing stream. From this point on, the stream is self-focusing and does not spread out to a greater mean square radial distribution than the stream had where self-focusing first set in. Once self-focusing has set in, the magnetic field produced from there on also reacts backward along the on-coming material to eject the fast electrons and assists to some extent in propagating the self-focusing conditions backwards towards the sun. As the self-focusing conditions propagate backwards towards the sun, the divergence of the material in the jet is decreased, and the current which must be exceeded in order to attain self-focusing conditions is correspondingly reduced, as illustrated in Table I.

In addition to the effect of the sun's atmosphere in scattering electrons out of the mixed jet rising out of the sun, the atmosphere also exercises another selective effect which can be deduced from Eq. (2). If there are some ions of other elements than hydrogen in the jet of material, these will not be slowed down and scattered out as much as the protons and so the stream of positive ions which eventually emerges from the sun's atmosphere will be richer in ions of the heavier elements than is the matter in the sun. Such heavier ions are held in the magnetically self-focusing stream at least as strongly as the protons, and if they are multiply charged, as they will doubtless be, they will be held in magnetically even more strongly. Such heavy ions would probably be separated from the proton stream when the stream enters the magnetic field of the earth, but whether they actually exist or where they would go is speculative.

INTERPLANETARY MATTER

Intense auroral displays produce an intensity of visible light at the ground of the order of magnitude of moonlight which is approximately 1 erg cm^{-2} sec^{-1}. If one supposes that an average display reaches one percent of this intensity at its maximum, and that the order of one percent of the energy of the incoming protons produces visible light, a stream of 10^{10} cm sec^{-1} protons spread over an area 1000 km in diameter would contain a current of about 15 amperes. This is probably less than the actual auroral currents.

In order for a stream to form, it must be able to acquire slow electrons from the interplanetary matter in approximately the same density as the density of the fast protons in the stream. In Table III are given values of this density for streams of 10^{10} cm sec^{-1} protons for various currents and various equivalent stream radii, that is the radius which the stream would have if the stream were to be redistributed into a cylinder of uniform density equal to the density at its most dense part. The density of interplanetary matter does not necessarily have to be as large as the values given in Table III, because the stream of fast protons can attract slow electrons to some extent from the surrounding space.

It might be supposed that the rapidly moving protons will collide with the slow electrons in the stream, giving them increasing velocities in the direction of the stream so that the magnetic field of the stream would magnetically repel these speeded up electrons and throw them out, or that the collisions would increase the kinetic energy of the electrons due to transverse components of velocity and so increase the value of θ_2 in Eq. (1) that the current in the stream would become less than the critical current causing the stream to disperse. The following calculations indicate that neither of these possibilities is important. Consider a stream of fast protons of uniform density out to a radius r and zero beyond travelling through interplanetary matter consisting of a density n slow electrons cm^{-3} originally mixed with an equal density of slow ions. The slow electrons may be supposed to have reached thermal equilibrium characterized by a temperature T which is probably between 200°K and 10 000°K. The gas kinetic rate at which slow electrons will enter unit length of the stream is $n(kT/2\pi m)^{\frac{1}{2}}2\pi r$. The number of slow electrons in unit length of the stream is approximately $n\pi r^2$. The average time τ spent by a slow electron in the stream is the quotient of these two expressions, or $\tau=(\pi m/2kT)^{\frac{1}{2}}r$. For $r=1000$ km, τ is 23 and 3.2 sec for $T=200°$ and 10 000°K, respectively. Assume an exaggeratedly large density, 0.2 fast proton cm^{-3}, in the stream. Then from Eq. (2) the average rate at which a slow electron acquires a velocity in the direction of the stream is 0.12 cm sec^{-2}, and the average changes in velocity are 2.8 and 0.4 cm sec^{-1}, respectively, which are negligibly small compared to the velocity 10^{10} cm sec^{-1}.

From Thomas' Eq. (4.33), can be calculated the average rate at which a slow electron can acquire an increase in kinetic energy due to transverse components of velocity, θ_2 in Eq. (1). The average gain in θ_2 while an electron is in the stream is of the order of 0.003 ev which is negligible compared with the values of θ_1 to be expected from the divergence of the stream while forming.

From Eq. (2) one may estimate the density of interplanetary ionization which would slow down 10^{10} cm sec^{-1} protons by 10 percent and 100 percent in travelling from the sun to the earth, a distance of 1.5×10^{13} cm. It is found that the densities are of order 10^9 and 10^{10} slow electrons cm^{-3}, respectively, which are improbably large values. It is concluded that the present theory of self-focused streams requires a density of interplanetary matter between the rather wide limits of 10^{-5} and 10^9 ion electron pairs cm^{-3}.

TABLE III. Minimum density of ions and electrons per cm^3 in interplanetary space necessary to support a self-focused stream.

Current amperes	Equivalent stream radius 10 km	100 km	1000 km
10	0.0018	0.000018	0.00000018
100	0.018	0.00018	0.0000018
1000	0.18	0.0018	0.000018

DISCUSSION

Although many of the foregoing calculations are based on protons of speed 10^{10} cm sec^{-1}, it is hardly necessary to remark that the present theory is not limited to protons. As far as the theory is concerned the incoming ions could be of other kinds than protons and of other speeds than 10^{10} cm sec^{-1} within limits. Protons of speeds 10^{10} cm sec^{-1} spiral to the earth along the lines of magnetic force, the radii of the spiral being about 20 and 30 km at altitudes 100 and 1000 km, respectively. But the auroral rays, which lie approximately along the magnetic lines, emit mainly the radiations of oxygen and nitrogen and do not emit the hydrogen spectrum with observable intensity. The rays are observed up to 800 km, and it is not believed that the upper atmosphere normally extends to such a high altitude. It is suggested that the auroral rays arise from a secondary action. The charge carried by the ionizing stream of protons which are stopped in the earth's atmosphere at about 70-km altitude must be neutralized by motion of charges in the earth's ionized upper atmosphere—else impossibly large electrical fields would rapidly be produced around the place where fast protons are being stopped. At altitudes below about 90 km, the charge motion in the earth's atmosphere will consist mostly of positive ions moving upwards and negative ions moving downwards. There will be few free electrons here because the atmospheric oxygen is mostly molecular and dissociative attachment with free electrons to form negative atomic oxygen ions rapidly removes free electrons from the air in such conditions as these, as has recently been shown by Loeb.[14] The mobilities of negative ions and positive ions are approximately equal, and so the negative ion current downwards is about equal to the positive ion current upwards. The positive ions moving upwards continue to move upwards carrying material in the form of ions to as high altitudes as the electron field will sustain them and this field could well reach to 800-km altitude. The downward current of negative ions below altitudes of 90 km must be sustained by an equal downwards current of negative charges from higher altitudes, although above about 120 km most of the atmospheric oxygen is atomic instead of molecular and negative ion formation is unlikely. The much greater mobility of free electrons at these higher altitudes enables the electrons to move more easily and to neutralize the electric field more nearly completely at these higher altitudes, but the downward current of these free electrons cannot exceed the downward current of negative ions at the lower altitudes unless there is recombination of electrons with positive ions at the intermediate altitudes and such recombination is negligible at these low densities. Both the incoming electrons and the outgoing positive ions must spiral around the earth's magnetic lines of force. These streams are the auroral rays. This view appears to offer a reasonable explanation of the appearance of the lines of ionized nitrogen and oxygen observed in the spectrum of auroral rays up to 800 km. It further follows that groups of auroral rays, such as those which often develop out of a quiet auroral arc above 80 km are not filamented structures caused by irregularities of the solar emission but rather by irregularities of the terrestrial atmosphere.

[14] See L. B. Loeb, Phys. Rev. **86**, 256 (1952).

A solar proton speed of 10^{10} cm sec^{-1} is greater than the value 10^8 cm sec^{-1} of certain older estimates,[15] which were based on the statistical observation that 15 to 30 hours, or of order 10^5 sec, elasped between the time that a sun spot passed the meridian and the onset of an associated auroral display or magnetic storm. Dividing 1.5×10^{13} cm by 10^5 sec gives a particle speed of order 10^8 cm sec^{-1}. Such a calculation assumes that the particles travel in straight lines from the sun to the earth and is incorrect if they do not do so. Paths of protons near the earth are not straight but are bent by the terrestrial magnetic field in a direction contrary to the motion of solar rotation (an oversimplified statement, the exact path is very complicated); hence the long delay of the protons in reaching the earth is only apparent. However, to calculate the delay quantitatively requires knowledge of the geometrical shape of proton orbits in the earth's magnetic field which is not as yet available.

The present view throws doubt on the particle theory of magnetic storms of Chapman and Ferarro[9] which assumed an ionized, electrically neutral stream of particles which spread out from the sun uniformly in a vast cone and envelop the earth. Unless the particle density of interplanetary space is very low, less than about 10^{-5} cm^{-3}, such a stream would become self-focusing, and would not progress as a vast cone and would not produce the magnetic effects which they have calculated.

[15] See L. Harang, *The Aurorae* (Chapman and Hall, Ltd., London, 1951), p. 143.