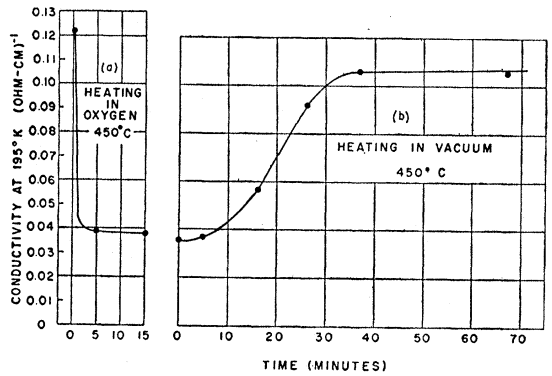

FIG. 2. (a) Conductivity after heating in oxygen; (b) conductivity after heating in vacuum.

perature. The reduction in conductivity with reaction between germanium and oxygen suggests that oxygen produces electron traps on the germanium surface, and if we assume bulk mobilities[6] the observed reductions in conductivity correspond to a trapping of about 5×10^{11} electrons/cm^2 of surface.

The above suggests that if oxygen is allowed to react with germanium having a fairly wide p-type or near-intrinsic surface region, the conductivity will increase rather than decrease, with the electrons entering the oxygen levels from the valence band leaving mobile holes behind; we have observed this in earlier work.[4]

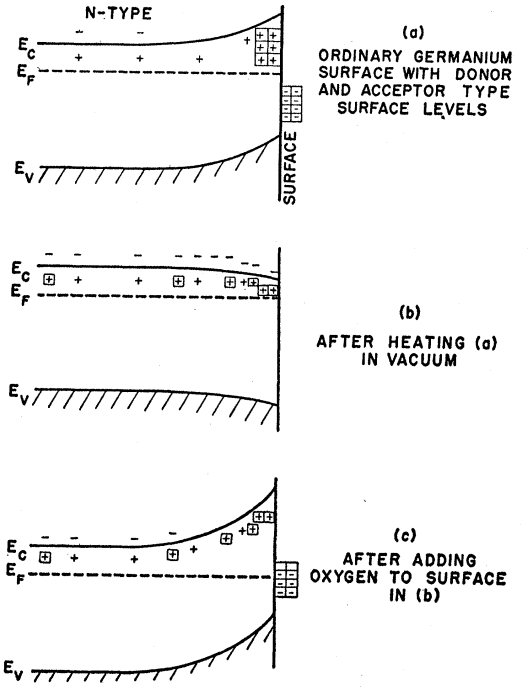

FIG. 3. Proposed model of the etched germanium surface after heating in vacuum and after admitting oxygen to the surface. E_C is the bottom of the conduction band, E_V the top of the valence band, and E_F the Fermi level.

It is found that the excess conductivity results from impurities, some of which have diffused deeply into the germanium. With removal of the germanium from the vacuum system, the conductivity decreases corresponding to oxygen reacting with the germanium. However, there is still much excess conductivity remaining. Rectification measurements show that this is n type. Slow etching of the surface in conjunction with conductivity measurements show that about 70 percent of the excess conductivity is limited to a surface region several microns in thickness. With further etching, the conductivity levels off at a value much larger than that of the original crystal indicating that some of the donors have diffused deeply into the germanium.

Figure 3 shows a model of the etched germanium surface as it is affected by heating in vacuum, and by reaction with oxygen.

The nature of the donor centers is as yet unknown. However, there exists the interesting possibility that oxygen might also be responsible for the donor levels. In this case, the oxygen might be that which has entered the germanium lattice rather than sitting on the surface where it produces electron traps.

Whether or not oxygen can account for most of the surface levels on the etched germanium surface remains to be seen. Our work, thus far, suggests that oxygen can account for at least 10^{12} to 10^{13} surface levels per cm^2.

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Improved Calculation of the P-Wave Pion-Nucleon Scattering Phase Shifts in the Cut-Off Theory*

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IN earlier papers[1,2] the author has proposed a perturbation method for evaluating the cut-off form of the Yukawa theory. This method was applied in lowest order to the problem of pion-nucleon scattering and results presented[3] for a renormalized $f^2=0.2$ and a cut-off energy, $\omega_{max}=3.2\mu$. However, these values of the fundamental parameters do not lead to an actual resonance in the state of isotopic spin 3/2 and total angular momentum 3/2, a resonance which is now strongly indicated experimentally.[4,5] A larger cut-off energy is required to give genuine resonance and with such a larger cutoff the variational approximation used in reference 3 is not quantitatively reliable.[6] Also, higher-order effects, neglected previously, become more

important. The purpose of this note is to report a calculation of the P-wave phase shifts which is improved with respect to these earlier deficiencies.

Fourth-order corrections to the scattering, in the sense of reference 2, have been analyzed and the only important ones found to be those resulting from nucleon

TABLE I. Theoretical P-wave phase shifts according to the Fredholm approximation.

ω_0/μ	E_{lab}(Mev)	δ_{33}	$\delta_{31}=\delta_{13}$	δ_{11}
1.2	38	+3.7°		
1.3	57	8.0°	−1.0°	−2.3°
1.4	78	14.7°		
1.5	99	24.9°	−2.0°	−4.3°
1.6	122	39.4°		
1.7	144	56.9°	−3.3°	−6.4°
1.8	167	73.2°		
1.9	190	85.8°	−4.8°	−8.4°
2.0	215	94.6°		
2.1	240	99.5°	−6.3°	−10.3°

propagation function modification. Thus Eq. (8) of reference 2 for the reaction matrix is to be altered by modifying the nucleon propagation "between" virtual scatterings; i.e., we now have

$$(k_f|K_{0\alpha}|k_0)=(k_f|U_{0\alpha}|k_0)+\int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3} \times(k_f|U_{0\alpha}|k)\frac{1}{(\omega_0-\omega_k)[1+f^2\Delta(\omega_0-\omega_k)]} \times(k|K_{0\alpha}|k_0), \quad (1)$$

where the function Δ is given by Eq. (4) of reference 2 and all symbols have the same meaning as in that reference. At the same time, the "potential" $U_{0\alpha}$ is changed to the following:

$$(k_n|U_{0\alpha}|k_m)=c_\alpha 2\pi\frac{f^2}{\mu^2}\frac{k_n k_m}{(\omega_n\omega_m)^{\frac{1}{2}}} \times\frac{v(k_n)v(k_m)}{(\omega_0-\omega_n-\omega_m)[1+f^2\Delta(\omega_0-\omega_n-\omega_m)]}. \quad (2)$$

That is, nucleon propagation "within" a fundamental virtual scattering is also modified. The two modifications are of the same order. For the problem in question the "average" value of $f^2\Delta$ is roughly -0.3 either "inside" or "outside" a virtual scattering. This may seem large for an allegedly weak coupling situation, but a preliminary investigation indicates that further important corrections to the propagation function are unlikely. For this reason no modification has been made of formula (14) from reference (2), which gives most of the scattering in the (1/2, 1/2) state.

In the following letter, a machine numerical integration of Eq. (1) above for the (3/2, 3/2) state will be described, with $f^2=0.058$ and $\omega_{\max}=5.6\mu$. These values of the fundamental parameters were based on a preliminary solution of Eq. (1) by a Fredholm approximation recommended by Gammel.[6] The values were chosen so as to give $\delta_{33}=90°$ near 200 Mev[4,5] while at the same time $\delta_{33}=9°$ at 65 Mev, as reported by the Columbia group.[7]

The detailed results of the Fredholm approximation are given in Table I. The values for δ_{11} were obtained by adding formula (14) of reference 3 to the appropriate Fredholm solution of Eq. (1) above. Since the numerical solution of the equation agrees reasonably well with the Fredholm approximation in the (3/2, 3/2) case, it has been assumed that the Fredholm results for the other P states, where the interaction is weaker, are also accurate.

The author is greatly indebted to J. Gammel for advance communication of his results on the same general problem.

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Numerical Calculation of 3/2−3/2 Pion-Nucleon Reaction Matrix and Phase Shifts*

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THE pion-nucleon reaction matrix $(k'|K_{0\alpha}|k_0)$ given by Eq. (1) of the previous letter, is a solution of the Fredholm integral equation,

$$K_{0\alpha}(k')=U_{0\alpha}(k')+\int_0^\infty dk Y_{0\alpha}(k',k)K_{0\alpha}(k), \quad (1)$$

where the matrix notation has been dropped for convenience, and $Y_{0\alpha}(k',k)$, the kernel, is equal to the coefficient of $K_{0\alpha}(k)$ in the integral as given above. Numerical solutions of (1) were obtained for $\alpha=(3/2, 3/2)$, $f^2=0.058$ and $\omega_{\max}=5.6\mu$ by the usual reduction of the integral equation to a system of simultaneous linear algebraic equations given by[1]

$$0=U_0(k_i)+\sum_{j=0}^{N}[a_j Y_0(k_i,k_j)-\delta_{ij}]K_0(k_j), \quad 0\leqslant i\leqslant N; \quad (2)$$