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Theory of Unstable Heavy Particles*

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In order to explain the nature of unstable heavy particles which are found in the penetrating components of cosmic rays, a possible model is presented. A mass relation for elementary particles is derived from the conditions for the elimination of divergences of the nucleon and heavy-nucleon self-energies. By choosing a suitable set of interactions, the lifetimes of these particles are calculated. Their production is evaluated, and their contribution to the magnetic moments of nucleons is also discussed. It is found that the predictions of the present theory are not qualitatively at variance with experimental results.

INTRODUCTION

THE results of several experiments[1] on the nature of particles in the penetrating component of cosmic rays indicate the presence of unstable heavy particles which have long lifetimes and which seem to be produced with comparatively large cross section. That is, they have lifetimes $\sim 10^{-9}-10^{-10}$ sec, and their abundance in nature is about 1/100 that of π mesons.

Attempts to account for these results in terms of current field theory have been made by many authors.[2,3] In particular, in his paper on the so-called V particles, Pais[3] put forward a theory in a general and purely deductive manner without referring to any particular set of interactions. Since the results obtained are promising, there appears to be general acceptance of the ideas involved. However, it was felt that because there was no satisfactory indication of the masses of the new particles, further investigations should be pursued in connection with the problem of the mass spectrum.

With regard to the last point, one of the present authors[4] has suggested a new theory of cohesive mesons which are responsible for the elimination of the nucleon self-energy divergences by π mesons, and which lead to a mass relation for some of the unstable heavy particles. Indeed, it appears that this theory would lead to values of the masses which are in qualitative agreement with those of V particles, notwithstanding the uncertainty attached to the present field theory and to the method of approximation. Since the possibility of identifying V particles with the particles anticipated by this theory is still open, it is necessary to investigate more closely their lifetimes for disintegration into lighter particles, their production rates, and other properties.

The purpose of the present paper is to find a model which leads to results in comparison with the majority of the experimental facts concerning the nature of V particles. Our survey differs slightly from those of other authors in that the coupling scheme is specified as far as possible. We shall show that by a suitable choice and by consideration of previous results, at least the neutral V particle problem may be well treated by means of a simple but definite model.

MASS RELATION

Several experiments in a number of different laboratories have shown evidence of the existence of various new particles. These facts seem to be of decisive importance for the development of quantum field theory. Up to now, however, no attempt has been made to solve the problem of mass quantization. In this connection, it was first suggested by Nambu[5] that, together with nucleons, π mesons, and μ mesons, the mass values of new particles correlate in an interesting fashion to Sommerfeld's fine structure constant α. That is, if all mass values are multiplied by α, the results will approximately be equal to the integers or half-integers which are shown in Table I. An interesting correspondence exists between these mass numbers and the statistics of elementary particles mentioned above: the boson possesses an integral value and the fermion has

TABLE I. Masses of elementary particles and magic numbers.

Particle	e	μ	π	ζ	v	τ	χ	N	V
Mass	1	210	276	$\sim$550	$\sim$800	977	$\sim$1470	1836	$\sim$2200
Mass/137 (nearest integer or half-integer)	0.008	1.5	2.0	4.0	6.0	7.0	{10.5, 11.0	13.5	{16.0, 16.5

* A preliminary presentation of this work was made by H. Enatsu, Phys. Rev. **85**, 483 (1952).

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[1] Report on Copenhagen Conference, June, 1952 (unpublished). Proceedings of the Third Annual Rochester Conference on High Energy Physics, 1952 (Interscience Publishers, Inc., New York, 1953).

[2] Nambu, Nishijima, and Yamaguchi, Progr. Theoret. Phys. Japan **6**, 615, 619 (1951); K. Aidzu and T. Kinoshita, Progr. Theoret. Phys. Japan **6**, 630 (1951); H. Miyazawa, Progr. Theoret. Phys. Japan **6**, 630 (1951); S. Oneda, Progr. Theoret. Phys. Japan **6**, 633, 1015 (1951); R. Sachs, Phys. Rev. **84**, 305 (1951); D. C. Peaslee, Phys. Rev. **86**, 127 (1951); H. P. Noyes, Phys. Rev. **87**, 344 (1952); R. J. Finkelstein, Phys. Rev. **88**, 555 (1952).

[3] A. Pais, Phys. Rev. **86**, 663 (1952).

[4] H. Enatsu, Progr. Theoret. Phys. Japan **6**, 643 (1951); Phys. Rev. **85**, 483 (1952); **89**, 1304 (1953). These papers will be referred to as I, II, and III, respectively.

[5] Y. Nambu, Progr. Theoret. Phys. Japan **7**. 595 (1952); and T. Inoue (private communication).

a half-integral value, while this rule is not applicable to the electron itself. We desire to make a modification of the above description as follows: "This scheme of mass magic numbers for elementary particles is provided when they are measured in Heisenberg's natural units ($\hbar=c=r_0=1$) where $r_0=2.8\times10^{-13}$ cm is the classical electron radius." This can easily be seen by the fact that the mass of the electron is 1/137 in Heisenberg's units. One must recognize at this point that, as stressed by Heisenberg, the classical electron radius r_0 has a universal significance. An interpretation of these magic mass numbers as fundamental numbers of field theory has not yet been found.

It thus appears that, so far as the mass spectrum is concerned, the situation is in some respects analogous to the case of the optical spectrum in the pre-Bohr epoch, and the mass spectrum suggests a profound relation between elementary particles, which is entirely lacking in the current quantum theory of fields.

As previously noted, one of the present authors (H.E.)[4] pointed out an intimate relation between masses of elementary particles by setting up conditions to compensate divergences in the self-energies of nucleons. Since the argument presented there is incomplete, we will supplement it here with a fuller account and some minor modifications. The method of mixed fields for the nucleon, in terms of which the compensation of the diverging parts of the self-energy would be achieved, was worked out in detail by Pais.[6] At that time it seemed to be of academic interest only. In several instances, however, recent observations have confirmed the existence of numerous unstable particles. Under these circumstances, the introduction of the so-called realistic viewpoint for the self-energy problem[7] is again required. Indeed, the experimental evidence reveals that the V particles are created and annihilated as the result of a nuclear interaction in which the electromagnetic field is of minor importance. This suggests that it would be useful to try to carry out a compensation program for the self-energy of nucleons due to π mesons, although the realistic viewpoint is unfavorable for the solution of the problem involving vacuum polarization. Furthermore, the particular stability of the proton makes it attractive. Thus, applying the recent covariant formalism, an attempt along this direction was made by one of us[4] as a representative case of mixed fields.

In the first place, we tentatively assume that π mesons are of the symmetrical pseudoscalar type with pseudovector coupling (for simplicity here we take a neutral form)

$$L_1=-i\left(\frac{g}{\kappa_0}\right)\bar{\psi}_N\gamma_5\gamma_\nu\psi_N\frac{\partial\phi_\pi}{\partial x_\nu}+\text{h.c.},\quad \kappa_0=\kappa c/\hbar. \tag{1}$$

Of course, we have some doubts about the type and coupling of π mesons, especially of the neutral π meson; but as for the theoretical situation, it may be stated at the present stage that conclusions derived from the assumption that the π meson is of pseudoscalar type are not inconsistent with the experimental results. The justification for such a postulate of a symmetrical nature for π mesons will depend on further development of theory and experiment. Here we shall invoke the principle of simplicity. The calculations of nucleon self-energies have been performed by the Feynman-Dyson techniques with inclusion of the so-called translation effects, which were described fully in paper I. Unfortunately, the present field theory contains some ambiguities. Thus, the method adopted there is not completely satisfactory. A practical advantage of the present field-theoretical method is that it leads us to results which are in agreement with observation when used for the calculation of physical processes in quantum electrodynamics such as the Lamb shift and the additional spin magnetic moment of electron. One may say, therefore, that this attempt to find a mass relation of elementary particles is of heuristic character.

Next, we consider the case of cohesive mesons (c mesons, for short) for π mesons. Generally, as was shown in I, the self-energy of a nucleon arising from a coupling with the matrix γ_5 has opposite sign to that from the corresponding coupling without it. This is true exactly for the spin-zero meson field with an interaction of ordinary type. Thus, cohesive scalar mesons with vector coupling should be chosen to cancel out divergences caused by π mesons which include quadratic and logarithmic parts.

By using the following interaction, the compensation problem was solved in I,

$$L_2'=-i\left(\frac{G}{\mu_0}\right)\bar{\psi}_N\gamma_\nu\psi_N\frac{\partial\phi_c}{\partial x_\nu}+\text{h.c.},\quad \mu_0=\frac{\mu c}{\hbar}. \tag{2}$$

The convergence relations have been expressed in a simple way, namely,

$$G^2=3\left(\frac{\mu}{\kappa}\right)^2 g^2, \tag{3}$$

and

$$8m^2+75\kappa^2-15\mu^2=0,\quad (m=1836m_e). \tag{4}$$

It was found that the theory in its original form gave us an interesting mass for the c meson, i.e., $\mu=1474m_e$, which seemed to be within the limit of error of recent experimental determinations of the masses of new particles. One might try to take the point of view that, since the mass obtained does not differ too much from the observed mass and the V particle is not involved in our model, a modification in the right direction might be made by changing slightly one of the nucleon masses in the intermediate states. In an attempt to see how this may be done, we observed that, in the c meson case, its mass could be reduced by assuming still heavier masses of intermediate-state nucleons. As to the interaction (1) itself, no change will be necessary.

[6] A. Pais, Verhandel. Koninkl. Akad. Wetenschap. Amsterdam Afdeel Natuurk. **19**, 1 (1947).

[7] W. Pauli and F. Villars, Revs. Modern Phys. **21**, 434 (1949).

In this way, it seems plausible to require that the interaction scheme (2) should be replaced by the following modified expression:

$$L_2=-i\left(\frac{G}{\mu_0}\right)\bar{\psi}_V\gamma_\nu\psi_N\frac{\partial\phi_c}{\partial x_\nu}+\text{h.c.},\qquad(5)$$

of which the prototype was originally assumed by Nambu *et al.* and Oneda.[2] Here ψ_V is the wave function of the V particle. The diverging parts of the self-energy of a nucleon arising from the interactions (1) and (5) become

$$\delta m_\pi=m\left(\frac{g^2}{4\pi\hbar c}\right)\left(\frac{m}{\kappa}\right)^2\left\{\left(\frac{-3}{8\pi}\right)\frac{KK_0}{M^2}+\left(\frac{1}{120\pi}\right)\left(75\frac{\kappa^2}{m^2}+65\right)\log(K+K_0)M^{-1}\right\},\qquad(6)$$

and

$$\delta m_c=m\left(\frac{G^2}{4\pi\hbar c}\right)\left(\frac{m}{\mu}\right)^2\left\{\left(\frac{1}{8\pi}\right)(2\beta-1)\frac{KK_0'}{M^2}+\left(\frac{1}{120\pi}\right)\left(-20\beta^3+25\beta^2-22\beta-2+35\frac{\mu^2}{m^2}-40\beta\frac{\mu^2}{m^2}\right)\times\log(K+K_0')M_1^{-1}\right\},\qquad(7)$$

where

$$\begin{aligned}&M=\frac{mc}{\hbar},\quad M_1=\frac{m_1c}{\hbar},\quad \beta=\frac{m_1}{m},\\&K_0=(K^2+M^2)^{\frac{1}{2}},\quad K_0'=(K^2+M_1^2)^{\frac{1}{2}},\quad K\to\infty,\\&m_1=\text{mass of the }V\text{ particle.}\end{aligned}\qquad(8)$$

As was shown in II, the convergence relations are

$$G^2=N\left(\frac{3}{2\beta-1}\right)\left(\frac{\mu}{\kappa}\right)^2g^2,\qquad(9)$$

and

$$\frac{\mu^2}{m^2}=\frac{1}{120\beta-105}\left\{(2\beta-1)\left(75\frac{\kappa^2}{m^2}+65\right)-(60\beta^3-75\beta^2+66\beta+6)\right\}.\qquad(10)$$

The factor N has a numerical value 1 or 3 according to whether the c meson obeys the symmetrical theory or the neutral theory, because in the symmetrical theory we assume that $g^2=g_{\text{ch}}^2=2g_{\text{neut}}^2$. Although at present there is a remarkable lack of symmetry regarding the charge properties of observed particles, it seems also plausible that the limited evidence available could be reconciled with the assumption of charge symmetry. Whichever we may choose, it is evident that our theory gives the same mass relation for each of them. Taking the V masses as $2200m_e-2400m_e$, it is found that the corresponding c meson masses are $1040m_e-800m_e$.

The interaction (5) gives us a divergent self-energy of the V particle. However, as was shown in III, if we assume the coupling

$$L_3=-i\left(\frac{L}{\kappa_0}\right)\bar{\psi}_V\gamma_5\gamma_\nu\psi_V\frac{\partial\phi_\pi}{\partial x_\nu}+\text{h.c.},\qquad(11)$$

we can eliminate this difficulty. In this case, the convergence conditions are given by the following substitutions in (9) and (10), because we assume a complete symmetry between nucleon and heavy nucleon in the expressions (1), (5), and (11):

$$g\to L,\quad m\to m_1,\quad m_1\to m.$$

The numerical values for the masses of these particles were given more fully in III.

Having thus obtained some of the mass relations, we shall here discuss some problems connected with them. The first question which arises is the extent to which the ordinary perturbation method furnishes reliable answers apart from the ambiguities mentioned above. That perturbation theory is not applicable to meson theory is well known. Although the results are uncertain because of the largness of the coupling constants and most likely all of the present field-theoretical computations for events involving new particles are quantitatively unsatisfactory, many attempts are needed to find a model which leads to qualitative agreement with experiment to the second order in the coupling constant. Once we have found a suitable model, we have to ask to what extent higher-order corrections may be expected to modify the results. Considering the relatively large coupling constant, the present argument should be extended as far as possible by a fourth-order calculation, but we do not intend to do so in this paper. Therefore, higher-order corrections may seriously modify the conclusions obtained.

Apart from the higher-order problem, we shall consider some other methods of treatment that do not involve the rather problematic perturbation method. In the case of strong coupling the perturbation theory will give erroneous results which are too large by some numerical factors (>1). This fact was pointed out by Tomonaga[8] for the case of the nucleon self-energy. It is thus expected that, as far as the main term to the second order for the self-energy is concerned, we may infer the expression for it in the strong and intermediate coupling theories from our results by multiplying them by some numerical factors. Such a modification may alter the relation (9); however, the effect for the mass relation (10) may be smaller. One must bear in mind that the above qualitative reasoning might be theoretically incomplete. The main point of the foregoing

[8] S. Tomonaga, Progr. Theoret. Phys. Japan **1**, 83 (1946); also see K. M. Watson and E. W. Hart, Phys. Rev. **79**, 918 (1950).

remarks is to indicate that in our case the mass relation can be determined by the logarithmic divergences without referring directly to the coupling constants.

Next, we can make some further remarks concerning the numerical values of the various masses. According to recent experiments, there seem to be several heavy mesons, i.e., χ, v, and ζ, which decay into two π mesons or τ and π mesons. Accepting this result for the moment, this means that they are of the scalar or vector type. At the present stage we are not sure about the exact relationship between these heavy mesons and the c meson. As pointed out in III, it may be necessary for us to assume more c mesons. One further point may be noted. In the formula (10), the masses of V particles and c mesons depend on those of nucleons and π mesons. Since we cannot know their mechanical masses accurately, we are still at liberty to select values for the masses for the V particle and the c meson.

We also note that some other particles could play a part in the self-energy of nucleons. For instance, the τ meson might conceivably be related to the self-energy of nucleons. According to the detailed discussion of the τ meson given by Fukuda *et al.*,[9] its coupling constant is taken to be $\sim 1/1000$; thus it is inferred that the effect of the τ meson on the self-energy of a nucleon would be negligible in comparison with those of the π meson and the c meson. Furthermore, the existence of other particles (κ mesons, etc.) must be taken into account. In this respect our model will still not be free from serious modification as a result of future experimental developments.

Finally, in the present situation it is impossible to exclude ambiguities involved in the mass determinations. Nevertheless, even in the mass values and Q values for the V particles the experimental information seems to indicate an appreciable fine structure. If this is so, it would be necessary to devise some additional mechanism in order to explain it. In this connection we shall employ an alternative treatment of the proton self-energy caused by the electromagnetic field without employing the original c-meson theory developed in the previous paper.[10] As was shown already, the divergent mass correction caused by the electromagnetic field for the proton is

$$\delta m_{el.} = m\left(\frac{e^2}{4\pi\hbar c}\right)\left(\frac{3}{2\pi}\right)\log(K+K_0)M^{-1}. \quad (12)$$

TABLE II. Masses of V particles and c mesons for the proton in units of m_e. (It is assumed that $m=1836m_e$, $\kappa=276m_e$, and $g^2=10e^2$.)

V mass	1900	2000	2100	2200	2300	2400	2500
c mass	1406	1288	1175	1065	947	813	650

[9] Fukuda, Hayakawa, and Miyamoto, Progr. Theoret. Phys. Japan **5**, 283, 352 (1950); Ozaki, Oneda, and Sasaki, Progr. Theoret. Phys. Japan **5**, 25, 165 (1950).

[10] H. Enatsu and P. Y. Pac, Progr. Theoret. Phys. Japan **6**, 665 (1951).

Accordingly, in the proton case the condition (10) is modified as follows:

$$\frac{\mu^2}{m^2} = \frac{1}{120\beta-105}\left\{(2\beta-1)\left(75\frac{\kappa^2}{m^2}+65\right) + 180(2\beta-1)\left(\frac{e\kappa}{gm}\right)^2 - (60\beta^3-75\beta^2+66\beta+6)\right\}. \quad (10')$$

At the present stage nothing can be said about the relation between the coupling constants e^2 and g^2. Here the numerical values of the c-meson mass are given in Table II, with the convention that the constant g^2 is ten times as large as e^2. Comparing them with the result in II, we see that the shift of the mass values of V particle is found to be about $\sim 30m_e$ in the mass or ~ 15 Mev in the Q value. The foregoing discussion is somewhat premature in the sense that such fundamental problems as the proton-neutron mass difference and the charge independence of nuclear forces remain untouched.

We have thus far considered the mass relation of elementary particles from the point of view of the compensation to which strong fermion-boson interactions are correlated. We can, in principle, extend this method to all other types of couplings. To carry out such a program would be a profitable subject for the mass spectrum theory, but we shall not discuss here the detailed analysis of other cases.

DECAY OF HEAVY PARTICLES

We now proceed to investigate the decay of the heavy particles into lighter ones. Our treatment will not go beyond that previously obtained by many authors. First we shall be concerned with the determination of a scheme of interactions which is necessary to allow the unstable particles to disintegrate with lifetimes of the order of $\sim 10^{-9}-10^{-10}$ sec.

Regarding the interpretation of the decay processes

$$\text{(i)}\ V^{\pm}\to V^0+\pi^{\pm},\quad V^0\to N+\pi, \quad (13)$$

and

$$\text{(ii)}\ v^{\pm}\to v^0+\pi^{\pm},\quad v^0\to 2\pi. \quad (14)$$

Nambu *et al.* and Oneda[2] have pointed out that among many possible couplings the following ones would be allowed or should at least be taken into account:

$$\left.\begin{array}{l} G_1(VN\pi),\quad G_2(VNv),\quad G_3(VV\pi),\\ \qquad G_4(VVv),\quad g_1(NN\pi),\quad g_2(NNv)\\ G_1^2\sim 10^{-11}-10^{-13},\quad G_2^2\sim 10^{-2}-10^{-3},\\ \qquad g_2^2\sim 10^{-7}-10^{-9},\\ G_3^2\gtrsim g_1^2,\quad G_4^2\leq g_2^2,\quad g_1^2\sim 1-10^{-1} \end{array}\right\}, \quad (15)$$

where $G(VNv)$ denotes an interaction which relates the V particle, nucleon, and v meson and which has a coupling constant given symbolically by G; the estimation of coupling constants might be accompanied by uncertainties of the order of 100 owing to the method of calculations.

Let us inspect the interactions from the standpoint of the self-energy. Calling our attention to the order of magnitude of coupling constants, we find that G_1, G_4, and g_2 are extremely small compared with the others, so that we discard them entirely. Thus, we are left with

$$G_2(VNv),\quad G_3(VV\pi),\quad \text{and}\quad g_1(NN\pi). \tag{16}$$

In other words, if one analyzes the processes (13) and (14) in which the typical decay modes

$$V\rightarrow N+\pi \tag{17}$$

and

$$v\rightarrow \pi+\pi \tag{18}$$

are involved, one sees that the only couplings which may have a possibility of contributing seriously to the self-energy of nucleons and V particles are those given in (16). This result is very similar to that drawn from our reasoning in the preceding section. The agreement is seen not only for the type of coupling, but also for the order of the coupling constant. Our analysis is in substance identical with that given by Pais.[3]

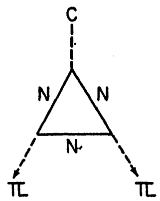

FIG. 1. Decay of a neutral c meson into two charged π mesons.

So far we have restricted our considerations to an analysis of the work of preceding authors. For later calculation the results may be summarized as follows.

(1) To compensate for the self-energy of nucleons and V particles caused by π mesons whose couplings are given by (1) and (11), it is necessary to introduce c mesons having the coupling (5).

(2) The c mesons, which may be identified with the χ, v, and ζ particles obey the symmetrical (or neutral) theory.

(3) For the order of magnitude of the coupling constants one estimates

$$\frac{g^2}{4\pi\hbar c}\sim\frac{L^2}{4\pi\hbar c}\sim 1-10^{-1},\quad \frac{G^2}{4\pi\hbar c}\sim 10-1. \tag{19}$$

(4) In order to allow V particles and c mesons to decay with lifetimes $\sim 10^{-9}-10^{-10}$ sec, one needs only assume the following interactions:

$$f(nnc),\quad \frac{f^2}{4\pi\hbar c}\sim 10^{-7}-10^{-9},\quad (n=N,\,V). \tag{20}$$

In order to find the exact form for the coupling (20), we shall first confine ourselves to the c-meson decay (Fig. 1), in which a selection rule will be effective.

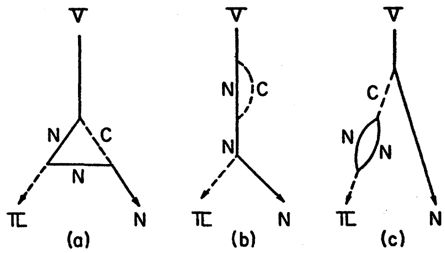

FIG. 2. Decay of a V particle into a proton and a π meson.

Then, from the work of Fukuda *et al.* and others,[9] it is readily seen that there remains only one possible case, that is, the neutral scalar type with scalar coupling

$$L_4=-f\bar{\psi}_n\psi_n\phi_c+\text{h.c.} \tag{21}$$

The evaluation is carried out by the standard Feynman-Dyson method using regulators. For simplicity, we discard the so-called translation effect,[11] and the couplings (11) and (21) for $n=V$ in the present and following sections. With $m=1836m_e$ and $\mu=800m_e$, we find

$$\tau_1^{-1}\approx 1.2\times 10^{24}\left(\frac{f^2}{4\pi\hbar c}\right)\left(\frac{g^2}{4\pi\hbar c}\right)^2 \text{sec}^{-1}, \tag{22}$$

for the 2π decay of the c meson. The competing processes are

$$c\rightarrow\pi+\gamma \tag{23}$$

and

$$c\rightarrow\gamma+\gamma. \tag{24}$$

We have ascertained that the former is forbidden and the latter is a relatively rare event compared with the 2π decay.

We now consider the case of V-particle decay with the emission of a proton and a π meson. As was already suggested by Oneda,[2] because of the mass relation,

$$m_1<m+\mu, \tag{25}$$

the decay of the V particle would take place according to the scheme shown in Fig. 2. It should be noted that in our case these schemes are completely determined by the couplings mentioned above. Taking the masses of the V particle and the c meson to be $2200m_e$ and $1000m_e$, presumably, and performing the momentum integral numerically, one obtains

$$\tau_2^{-1}\approx 0.4\left(\frac{1}{2\pi}\right)^4\left(\frac{G^2}{4\pi}\right)\left(\frac{g^2}{4\pi}\right)\left(\frac{f^2}{4\pi}\right) \times\frac{mX^3}{\left(-\tfrac{1}{2}X^2+X-\dfrac{\kappa^2}{m_1^2}\right)}, \tag{26}$$

[11] According to Fukuda *et al.*[9] it often happens that the matrix element for the 2π decay of neutral heavy mesons tends to zero when the equality $\kappa^+=\kappa^-$ holds for the masses of the π mesons which are created. We have confirmed that this fact would not be changed by taking account of the translation effect. Therefore we discarded the possibility of taking a symmetrical scalar field with vector coupling for the interaction Lagrangian density L_4.

where $X=1-1/\beta^2$, and $\hbar=c=1$. In view of the internal dependence of coupling constants which is revealed in the relation (9), the result (26) becomes

$$\tau_2^{-1}\approx 2.5\times 10^{21}\left(\frac{N}{2}\right)\left(\frac{f^2}{4\pi\hbar c}\right)\left(\frac{g^2}{4\pi\hbar c}\right)^2 \sec^{-1}, \quad (27)$$

where N is either 1 or 3. Considering the lifetimes of V particles and taking $g^2/4\pi\hbar c\sim 10^{-1}$, one can estimate the order of magnitude of the coupling constant to be

$$\frac{f^2}{4\pi\hbar c}\sim 10^{-9}-10^{-12}, \quad (28)$$

which is of the order anticipated by several authors.[2,3]

It might be remarked that this estimate can hardly be trusted because of the neglect of translation effects and the adoption of the regulator method. However, so long as we limit ourselves to the neutral heavy particles, this result seems to show that the present model is reasonable. As for the decay of charged unstable particles, at the present time we know too little in the way of experimental facts; hence, we shall content ourselves with referring to the detailed discussion made by Pais.[3]

PRODUCTION OF HEAVY PARTICLES

Heavy particles are observed[1] in a cloud chamber which is operated by the passage of the penetrating particles produced in the condensed materials above the chamber, and it is confirmed that the heavy particles are generated in the nuclear reaction which gives rise to bundles of these penetrating showers. The number of heavy particles in the penetrating showers is about one hundredth that of penetrating particles. In view of the experimental information, we see that the heavy particles are produced mainly in nucleon-nucleon collisions. We shall now check to what extent the production of heavy particles may be explained by means of the set of interactions which we have introduced in former sections. The lowest-order Feynman diagrams for this production are shown in Fig. 3. In our model the production is mainly due to the processes (a) and (b) in Fig. 3. The process (c) can be disregarded because of the smallness of f^2. In the center-of-mass system the threshold energies for V production are about $(11/8)mc^2$ for process (a) and $(5/4)mc^2$ for process (b). Total cross sections of the processes are calculated numerically for incident nucleon energies of $E_0=2mc^2$ and $E_0=5mc^2$ in the cm system. The former is sufficient to produce a penetrating shower in lead. Averaging over the charges of the incident nucleons, we obtain

$$\sigma_a=\left(\frac{g^2}{4\pi\hbar c}\right)^2\left(\frac{G^2}{4\pi\hbar c}\right)\left(\frac{M^4}{\mu_0^2\kappa_0^4}\right)(\alpha_1A_1+\alpha_2A_2), \quad (29)$$

and

$$\sigma_b=\frac{\pi}{4}\left(\frac{G^2}{4\pi\hbar c}\right)^2\left(\frac{M_1-M}{\mu_0}\right)^4\left(\frac{1}{M^2}\right)(\alpha_1'A_1'+\alpha_2'A_2'), \quad (30)$$

where α_1, α_2, α_1', and α_2' are constants given in Table III, and A_1, A_2, A_1', and A_2' are calculated by numerical integration and given in Table IV. The terms α_2A_2 and $\alpha_2'A_2'$ come from the exchange effect, being much smaller than the ordinary terms α_1A_1 and $\alpha_1'A_1'$.

For the process (a), the cross section for $E_0=5mc^2$ is much larger than that for $E_0=2mc^2$. This suggests that our calculation is incomplete for this energy, and the effect of damping must be considered. However, for the energy $E_0=2mc^2$, which is just above the threshold, the damping effect seems to be small; hence at this energy we shall compare the cross section for the process (a) with that for the process (b). Now the ratio of the cross sections is

$$\frac{\sigma_a}{\sigma_b}\approx\left(\frac{4}{\pi}\right)\left(\frac{2\beta-1}{3N(\beta-1)^4}\right)\left(\frac{M}{\kappa_0}\right)^2\left(\frac{\alpha_1A_1}{\alpha_1'A_1'}\right). \quad (31)$$

Since $M^2/\kappa_0^2(\beta-1)^4$ is over 2×10^4, the above ratio is about 2×10^4. Thus, the heavy particles are created through the process (a) even in this energy region. By taking into account the relation (9), the total cross section for the heavy particle production becomes

$$\sigma_a\approx\left(\frac{g^2}{4\pi\hbar c}\right)^3\left(\frac{18}{2\beta-1}\right)\left(\frac{A_1}{\kappa_0^2}\right)\left(\frac{104}{5}\right)\begin{Bmatrix}3\\2\end{Bmatrix}, \quad \begin{Bmatrix}\text{symmetrical}\\ \text{neutral}\end{Bmatrix}. \quad (32)$$

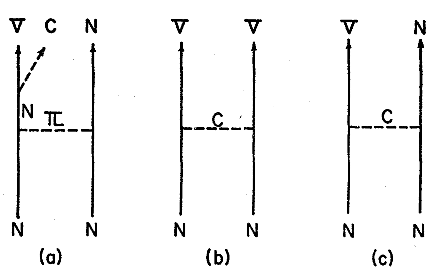

FIG. 3. Production of V particles and c mesons by nucleon-nucleon collisions.

TABLE III. Numerical values for the parameters α_1, α_2, α_1', and α_2' of Eqs. (29) and 30. Case (A): total cross section. Case (B): neutral V production only.

Type of c meson	Symmetrical theory				Neutral theory			
α	α_1	α_2	α_1'	α_2'	α_1	α_2	α_1'	α_2'
(A)	18	9/2	7/2	1/2	6	3/2	1	−1/2
(B)	9	9/4	3	3/4	3	3/4	1/2	−1/4

Alternatively this can also be written as

$$\sigma_a \approx \left(\frac{g^2}{4\pi\hbar c}\right)^3 \left(\frac{A_1}{\kappa_0^2}\right) \begin{Bmatrix} 3 \\ 2 \end{Bmatrix} 2.4\times 10^4. \tag{33}$$

If we assume that the value $g^2/4\pi\hbar c \sim 10^{-1}$, this is nearly of the order of the nucleon geometrical cross section.

At first sight, this result may be interpreted to mean that the heavy particles are produced with a cross section fairly large compared with that of the penetrating particles.[12] Consequently, we are tempted to conclude that the model is inconsistent with the experimental data. However, for comparison with experiment there are two points to be noted. The first is that the penetrating particles, which mainly consist of π mesons, are generated in multiple and their number is much larger than the number of π mesons which would be obtained by assuming single production. Accordingly, granting that the cross section for single production of π mesons is of the same order as that of the heavy particle corresponding to process (a) in Fig. 3, the number of observed π mesons would be much larger than that of the heavy particles. Second, it must be taken into account that, because of their short lifetimes, some of the heavy particles produced may decay before they enter the region of illumination of the cloud chamber, while the π meson would be able to survive for a longer time.

From these considerations we may be allowed to conclude that the predictions of the present theory regarding the production of heavy particles accompanied by penetrating showers are not unreasonable.

MAGNETIC MOMENTS OF NUCLEONS

In this section we will briefly survey the possible contribution of the c mesons to the magnetic moments of nucleons, by a method more or less similar to that of Case.[13]

As is well known, the effective interaction due to the additional magnetic moments of nucleons which are to be ascribed to the presence of mesons has a form

$$H_{\text{eff}} = K\left(\frac{F^2}{4\pi\hbar c}\right)\left(\frac{e\hbar}{2mc}\right)(-\bar{\psi}\sigma_{\mu\nu}F_{\mu\nu}\psi), \tag{34}$$

where K is a numerical constant.

Turning to our model, we have first to estimate the relative order of magnitude of the coupling constant F^2 in (34). Since we assume that π mesons and c mesons are of the types $PS(pv)$ and $S(v)$, respectively, this is done by applying the Dyson theorem concerning derivative couplings. In this way, we find

$$F_\pi^2 = \left(\frac{2m}{\kappa}\right)^2 g^2 \text{ for } \pi \text{ mesons,} \tag{35}$$

and

$$F_c^2 = [(m_1 - m)/\mu]^2 G^2, \text{ for } c \text{ mesons.} \tag{36}$$

The latter expression will be modified by means of the relation (9) as follows:

$$F_c^2 = N\left(\frac{3}{4}\right)\left(\frac{2m}{\kappa}\right)^2 g^2 \frac{(\beta-1)^2}{(2\beta-1)}. \tag{37}$$

Hence, with $\beta = 1.2$ (i.e., $m_1 = 2200 m_e$), one obtains

$$\frac{F_\pi^2}{F_c^2} = \frac{4(2\beta-1)}{3N(\beta-1)^2} = \frac{46.6}{N} \gg 1. \tag{38}$$

Consequently, if the constant K for the c meson were of the same order of magnitude as that for the π meson, the contribution given by the former would be negligible in comparison with that given by the latter. To see how this can be determined, we have carried out the calculation for the case of the c meson. The result can be written in the following form. The constant K may be split into a sum of two terms which correspond to the nucleon and meson currents, respectively,

$$K_1 = K_1^N + K_1^M \text{ for the proton,} \tag{39}$$

and

$$K_2 = K_2^N + K_2^M \text{ for the neutron,} \tag{40}$$

where

$$K_1^N = -(I_1 - I_3)/4\pi, \tag{41}$$

$$K_1^M = (I_2 - I_4)/2\pi, \tag{42}$$

$$K_2^N = -(I_1 - I_3)/2\pi, \tag{43}$$

$$K_2^M = -(I_2 - I_4)/2\pi. \tag{44}$$

The explicit forms for the I's are

$$I_1 = 2\delta + 1 + (\delta^2 - \lambda)P + 2\delta(\delta^2 - 3\lambda)Q, \tag{45}$$

$$I_2 = -2\delta + 1 + (-\delta^2 + \delta + \lambda)P + 2(-\delta^3 + \delta^2 + 3\lambda\delta - 2\lambda)Q, \tag{46}$$

$$I_3 = (\beta+1)[2 + \delta P + 2(\delta^2 - 2\lambda)Q], \tag{47}$$

$$I_4 = (\beta+1)[-2 + (-\delta+1)P + 2(-\delta^2 + \delta + 2\lambda)Q], \tag{48}$$

TABLE IV. Numerical values for the parameters A_1, A_2, A_1', and A_2' of Eqs. (29) and (30).

E_0 [a]	A_1	A_2	A_1'	A_2'
2 mc^2	1.11	0.07	2.57	0.90
5 mc^2	35.50	−2.90	12.1×10^{-12}	-0.1×10^{-2}

[a] Energy of the incident nucleon in the center-of-mass system.

[12] In the same approximation, we have estimated roughly the cross section for π meson production by nucleon-nucleon collisions

$$\sigma_\pi \approx \left(\frac{g^2}{4\pi\hbar c}\right)^3 \left(\frac{B}{\kappa_0^2}\right) \times 0.2\times 10^4, \text{ for } E_0 = 2mc^2,$$

where B is a numerical constant of the order of 1. This may be compared with σ_a.

[13] K. M. Case, Phys. Rev. **76**, 1 (1949).

TABLE V. Numerical values for I_1-I_3 and I_2-I_4.

β [a]	1.09	1.14	1.20	1.25	1.31
λ [b]	0.47	0.39	0.32	0.25	0.18
I_1-I_3	−5.49	−2.45	−1.23	+0.19	+1.09
I_2-I_4	−2.86	−4.48	−6.20	−8.25	−10.62

[a] $\beta = m_1/m$.
[b] $\lambda = (\mu/m)^2$.

where

$$P = \log\left(\frac{\beta^2}{\lambda}\right), \tag{49}$$

$$Q = \frac{1}{(4\lambda-\delta^2)^{\frac{1}{2}}}\cos^{-1}\left(\frac{2\lambda-\delta}{2\beta\lambda^{\frac{1}{2}}}\right), \tag{50}$$

$$\delta = \lambda - \beta^2 + 1, \tag{51}$$

and

$$\lambda = \left(\frac{\mu}{m}\right)^2. \tag{52}$$

Numerical values of these quantities are given in Table V.

Comparing these results with those of Case, one sees that any difference in the constant K between the π meson and the c meson is not so large as to compensate for the weight owing to the relation (38). Therefore, we can conclude that in our model the effect of the c meson on the magnetic moments of the nucleon is slight. This conclusion would probably not be altered seriously in the higher order approximation because of the condition (38).

CONCLUDING REMARKS

The principal aim of this investigation has been to present an additional model which throws light on the nature of unstable heavy particles from the point of view of the self-energy. In spite of the qualitative agreement with experimental results, the model proposed here may require revision in the future, because one can not say anything about particles which may still be undiscovered. Such a possibility is likely enough in view of recent experiments. Moreover, it is uncertain at present whether or not divergences are eliminated in the higher order approximation. In other words, while it may be admitted that as a first approximation the predictions of our theory are qualitatively in conformity with experimental results, it is still a moot question whether the higher order effects may require serious modifications.

Further, even if the elimination of divergences for the self-energy could be achieved, the difficulties would not all be solved. In fact, the divergences which appear in the decay processes make the compensation problem still more serious. In this respect, the present treatment is certainly inconsistent. Noyes[2] has suggested an interesting method to avoid this difficulty. In our case the introduction of a new interaction which is the same type as that of Noyes may be considered in the case of the meson self-energy.

In conclusion, although the method shares the well-known defects of current field theory and the results obtained are preliminary, the theory seems to furnish a reasonable field-theoretical approach to the problem of explaining, by a simple model, a wide range of experimental facts (at least for the neutral heavy particles).

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