Contributions of Bremsstrahlung Conversion in Trident Experiments

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Determinations of the cross section for the direct creation of electron pairs by energetic charged particles in nuclear emulsions are dependent on the experimental criteria by which the observations are accepted, phenomenologically, as tridents. This paper discusses the extent to which materialization of a bremsstrahlung photon close to the path of the parent particle becomes experimentally indistinguishable from direct pair production. A study of 200 pairs arising from the conversion of bremsstrahlung photons from electrons with energies between 0.1 and 10 Bev leads to the conclusion that about 6 percent of the pairs materialize so close to their parent that they would be interpreted as tridents. An extension of the analysis to electrons of incident energy 100 Bev demonstrates that under typical experimental conditions about 70 percent of such secondary pairs would be accepted as tridents. Recently reported experiments which indicate a cross section for direct pair production by fast electrons much larger than that predicted by theory are discussed.

I. INTRODUCTION

EXPERIMENTS on tridents—the direct production of electron pairs by charged particles of great energy—have been reported in the literature.[1–5] These phenomena, found chiefly in photographic emulsions, consist of a primary track characteristic of a relativistic singly charged particle and three thin emergent tracks. The majority of tridents occur on electronic primaries. It has been pointed out[3] that a basic difficulty in the study of the direct production process arises from the inclusion of a fraction of those pairs—called B.S. pairs—which are due to the conversion of bremsstrahlung photons close to the track of the parent electron. For this fraction the point of materialization of the B.S. pair is superposed on the parent track within the spatial resolution possible under the experimental conditions. We refer to events of this type as "pseudo-tridents." Figure 1 is a photomicrograph which illustrates the track pattern of a typical B.S. pair. The orthogonal distance from the pair origin to the parent track is called r, and for Fig. 1 the projection of r in the emulsion plane is about 3 microns. The main limitation on spatial resolution of a close B.S. pair is determined by the dimensions ($\sim$0.4 micron) of the developed grains in nuclear emulsions. In our experience it is possible to distinguish a B.S. pair if the separation of the pair origin from the line of the parent track exceeds $\sim$0.2 micron.

We develop a theory to show that the occurrence of pseudo-tridents is greater the higher the energy of the incident electron. The mean angles of both B.S. photon emission and the electron multiple scattering, which are the principal factors in the determination of the orthogonal distance r, are inversely proportional to the energy. At very great primary energies the major fraction of the B.S. photons are emitted in such a narrow forward cone that a substantial fraction materialize within the resolution distance of the parent track and are erroneously counted as tridents. In order to correct for the inclusion of these pseudo-tridents, we have undertaken a calculation of the distribution in r to be expected under various experimental conditions. In addition, the corresponding experimental distribution has been constructed from 200 events of the type shown in Fig. 1.

II. THEORETICAL TREATMENT

The calculation proceeds by computing the probability of the real and projected separations between the parent track and the origin of a B.S. pair. Figure 2 represents schematically the emission of a B.S. photon at $t=0$, which is assumed to be the point of entrance of the electron into the emulsion, and the subsequent conversion of the photon at distance t measured in the forward direction of the emitting electron. At $t=T$, the parent electron leaves the emulsion. The three-dimensional distance between parent and pair origin is r, while the projection of r in the plane of the emulsion is x. The major contribution to r is due to the multiple scattering of the electron in traversing the distance t; the contribution arising from the angles of radiative emission is small for most values of t employed and has been neglected.[6] All distances will be expressed in radiation lengths ($\sim$3.0 cm for nuclear plates).

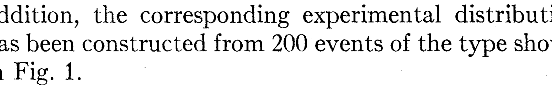

FIG. 1. A photomicrograph of a typical B.S. pair found in a G5 emulsion. The projected distance between the pair origin and the track of the parent electron has been measured for 200 observations of this type and is here about three microns.

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[1] C. F. Powell, Nuovo cimento Suppl. **6**, 379 (1949).
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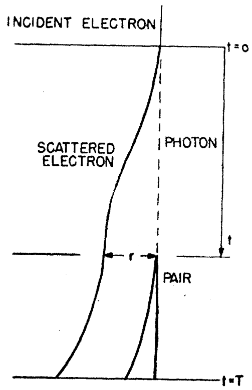

FIG. 2. A schematic diagram to represent the formation of a B.S. pair. The parent electron enters the emulsion at $t=0$, at which point the B.S. photon is assumed to originate. The effect of the angles of emission in the radiative collision has been neglected. At t the photon forms a pair and the parent electron has been scattered away a distance r, leaving the emulsion at $t=T$.

For our purpose it is necessary to calculate the probability that an electron has been scattered a distance x away from the path of the photon at the point of pair conversion. Let $H(t,x)dx$ be the probability that an electron traversing a distance t is scattered a projected distance between x and $x+dx$. Multiple scattering theory[6] indicates that $H(t,x)$ is Gaussian, and thus we can put

$$H(t,x)=\frac{1}{(2\pi)^{\frac{1}{2}}\alpha t^{3/2}}\exp\left(-\frac{x^2}{2\alpha^2 t^3}\right). \tag{1}$$

In order to evaluate the parameter α, we note from (1) that

$$\langle x^2\rangle_{\text{Av}}=\int_{-\infty}^{\infty}x^2H(t,x)dx=\alpha^2t^3.$$

It can be shown[6] that $\langle x^2\rangle_{\text{Av}}=t^2\langle\theta_p{}^2\rangle_{\text{Av}}/3$, where $\langle\theta_p{}^2\rangle_{\text{Av}}$ is the mean square of the projected angle of multiple scattering. $\langle\theta_p{}^2\rangle_{\text{Av}}$ may be expressed by the experimentally determined[7] relation for $G5$ emulsion, $\langle\theta_p{}^2\rangle_{\text{Av}}{}^{\frac{1}{2}}=25t^{\frac{1}{2}}/E$, where E is the electron energy in Mev, t is in units of 100 microns, and θ_p is in degrees. Expressed in more convenient units, θ_p in radians, t in radiation lengths (cm), and E in Mev, we obtain $\langle\theta_p{}^2\rangle_{\text{Av}}{}^{\frac{1}{2}}=7.57t^{\frac{1}{2}}/E$. Thus $\alpha=4.35/E$.

Further, let $Q(t)dt$ be the probability that a photon traversing a distance t materializes between t and $t+dt$. Since we find only those B.S. photons which form pairs in T, we normalize so that $\int_0^TQ(t)dt$ is unity. Then

$$Q(t)=\frac{7/9\exp(-7t/9)}{1-\exp(-7T/9)},\quad 0\leqslant t\leqslant T, \tag{2}$$

where the conversion length was taken to be 9/7 radiation lengths. Since T is usually small in the conditions of the experiment, we will approximate (2) by

$$Q(t)dt=dt/T,\quad t\leqslant T\ll 1. \tag{3}$$

We are now able to evaluate $N(x')$, the fraction of the B.S. pairs for which $-x'\leqslant x\leqslant x'$. Since (1) and (3) are independent, the simultaneous probability is given by their product. Thus

$$N(x')=\int_{-x'}^{x'}\int_0^T H(t,x)Q(t)dxdt.$$

After transformation to the dimensionless variables

$$\eta=x'/\alpha t^{3/2},\quad \eta_{x'}=x'/\alpha T^{3/2},$$

$$N(x')=(4/3)\eta_{x'}{}^{2/3}I(\eta_{x'}) \tag{4}$$

where

$$I(\eta_{x'})=\int_{\eta_{x'}}^{\infty}E(\eta)\eta^{-5/3}d\eta,$$

and

$$E(\eta)=\int_0^{\eta}\frac{\exp(-z^2/2)}{(2\pi)^{\frac{1}{2}}}dz.$$

In addition, we consider the quantity $N(r')$, the fraction of the B.S. pairs within a true distance r' from the parent. Expressed in radial distance, (1) is replaced by

$$G(t,r)=\frac{r}{\alpha^2t^3}\exp(-r^2/2\alpha^2t^3), \tag{1a}$$

where $G(t,r)dr$ is the probability that an electron travelling through t scatters between r and $r+dr$. From (1a) and (3), proceeding as before but now employing the dimensionless variables

$$\eta=r'/\alpha t^{3/2}\quad\text{and}\quad \eta_{r'}=r'/\alpha T^{3/2},$$

we find

$$N(r')=\tfrac{2}{3}\eta_{r'}J(\eta_{r'}), \tag{4a}$$

where

$$J(\eta_{r'})=\int_{\eta_{r'}}^{\infty}\frac{1-\exp(-\eta^2/2)}{\eta^{5/3}}d\eta.$$

Functions (4) and (4a) were numerically evaluated and are plotted in terms of the quantities $\eta_{x'}$ and $\eta_{r'}$ in Fig. 3.

The distribution $P(x)dx$, the probability for B.S. pairs to lie in the projected interval between $|x|$ and $|x+dx|$, is immediately obtainable by differentiation of the integral distribution $N(x')$ of Fig. 3, and is

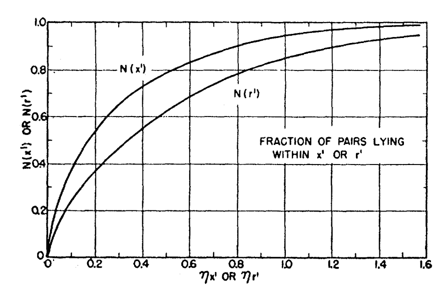

FIG. 3. The fraction of B.S. pairs formed within a projected distance x' or radial distance r' from the track of the parent electron as a function of the dimensionless variables $\eta_{x'}$, $\eta_{r'}$, respectively.

[7] Gottstein, Menon, Mulvey, O'Ceallaigh, and Rochat, Phil. Mag. **42**, 708 (1951).

given by

$$P(x)=\left[\frac{dN(x')}{dx'}\right]_{x'=x}=\left[\frac{dN(x')}{d\eta_{x'}}\cdot\frac{d\eta_{x'}}{dx'}\right]_{x'=x}$$

$$=\frac{1}{\alpha T^{3/2}}\left[\frac{dN(x')}{d\eta_{x'}}\right]_{x'=x}. \quad (5)$$

In the application of the theory to experiment three effects have not been considered. Firstly, it has been assumed that the photon is emitted at the point of entry of the electron into the emulsion. A more exact theory would take into account the distance traversed by the electron before photon emission. In practice this distance can be shown to be small; if it is neglected in relation to T, an underestimate of $N(x')$ or $N(r')$ results. Secondly, if two or more B.S. pairs are formed alongside the same parent the above effect is more pronounced. This, however, is relatively rare in comparison with single photon conversion. Thirdly, if the path length $T \gtrsim 1$, so that (3) is no longer valid, the calculated $N(x')$, $N(r')$ are too small since materialization is more probable at smaller distances. These effects are generally rather small and their neglect leads to a considerable simplification of the analysis. As a result, our calculation of the number of unresolved B.S. pairs is a minimum estimate.

In order to examine the general character of these distributions as functions of track length and energy, it is instructive to consider $\langle r^2\rangle_{\mathrm{Av}}$, the mean square radial separation, which is given from (1a) and (2) by

$$\langle r^2\rangle_{\mathrm{Av}}=\int_0^{\infty}\int_0^{T}\frac{r^3}{\alpha^2t^3}\exp\left(-\frac{r^2}{2\alpha^2t^3}\right)\frac{7/9\exp(-7t/9)}{1-\exp(-7T/9)}dtdr.$$

Within the conditions of present experiments where $T\ll 1$, $\langle r^2\rangle_{\mathrm{Av}}^{\frac{1}{2}}=\alpha T^{3/2}/\sqrt{2}$, a result which corresponds to the distribution developed from (4a). As experimental techniques improve, it will be possible to examine electromagnetic cascades of great path length, $T\rightarrow\infty$. It is noteworthy that under such circumstances $\langle r^2\rangle_{\mathrm{Av}}^{\frac{1}{2}}\rightarrow 5.05\alpha$, and the distribution width becomes independent of path length. For small T, a more exact expansion of $\langle r^2\rangle_{\mathrm{Av}}$ in T leads to

$$\langle r^2\rangle_{\mathrm{Av}}^{\frac{1}{2}}=(\alpha T^{3/2}/\sqrt{2})(1-3T/20).$$

In our experimental comparison the largest value of T employed is 0.5 and for this figure we see that the error introduced by neglect of the expansion in T is an overestimate in $\langle r^2\rangle_{\mathrm{Av}}^{\frac{1}{2}}$ of only $\sim$7 percent.

III. EXPERIMENTAL COMPARISON

We have compared an experimental study of 200 B.S. pairs with the preceding analysis. The measurements were made in Ilford G5 emulsions 400 microns thick, 4×7 cm, which had been exposed to the cosmic radiation at altitudes exceeding 95 000 ft, at geomagnetic latitude 55°N, for about six hours. Data were collected from 26 plates which had been exposed under different amounts of absorbers. The search was confined to thin tracks inclined to the emulsion plane at angles less than 5°. Two hundred pairs were found which were associated with a thin and approximately parallel track; we have assumed all events of this nature to be B.S. pairs, since the occurrence of other processes of this phenomenological appearance is negligible. The projected distance between pair origin and the nearest thin parallel track was measured in each case; the fractional number of B.S. pairs per micron interval in x are given as a histogram in Fig. 4.

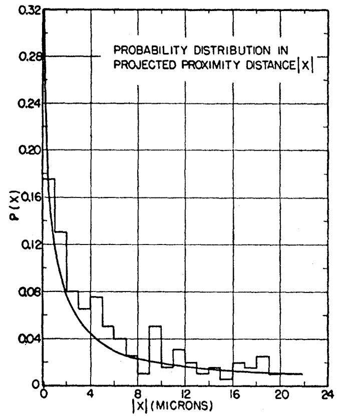

FIG. 4. Comparison of the experimental normalized histogram with the theoretical curve of the probability for a B.S. pair to form within the projected interval between $|x|$ and $|x+dx|$ from the parent track. The energies of the parent particles fall between 0.1 and 10 Bev, and experimental values of $|x|$ exceeding 20 microns have not been included.

The distribution in the emulsion path lengths of the parent tracks was obtained, and we have assumed a differential energy distribution of the form $E^{-1.6}dE$, deduced from the π^0-meson energy spectrum.[8] The probability distribution for the parameter $\alpha T^{3/2}$, which is required for the analysis, was obtained by numerical means from the above distributions in length and energy. The appropriate terms in (5) were weighted in accordance with the experimental variations in length and energy, and an averaged value of $P(x)$ obtained. This is compared with the experiment in Fig. 4. The agreement appears to be satisfactory, particularly in view of the fact that B.S. pairs originating at distances exceeding about 50 microns tend to escape observation. This leads to an error in normalization which causes a small distortion of the histogram.

IV. CONCLUSIONS

An important application of this analysis is the estimation of the fraction of B.S. pairs which are pseudotridents. In nuclear plates it is usually possible to resolve distances of 0.2 micron in projection and 0.4 micron in depth. For purposes of calculation, an elliptical resolution function with these dimensions as semiaxes has been assumed. For our experiment, the appropriately weighted values of $N(r')$ were obtained and integrated over the above elliptical resolution characteristic. We deduce that 6 percent of the total number of B.S. pairs produced under our experimental

[8] Carlson, Hooper, and King, Phil. Mag. **41**, 701 (1950).

conditions would lie within our minimum resolution criteria and are thus pseudo-tridents. A factor of two either way in the minimum projected and depth resolution distances produces a variation in this result of about 30 percent. In a subsequent paper this correction will be applied in an evaluation of the cross section for direct pair creation by fast electrons in the energy interval 0.1–10 Bev.

A recent report[5] on tridents with primary energies in the neighborhood of 100 Bev indicates a production rate notably larger than that indicated by theory.[9,10] Fifteen tridents were found in 37 cm of track. We have made a conservative estimate from electromagnetic theory which indicates that about 80 B.S. photons of quantum energy exceeding 100 Mev would be emitted and that, of these, about 14 percent would materialize as B.S. pairs under the conditions of the experiment. Thus an average of about 11 B.S. pairs would be expected. Cascading effects, which would tend to increase this number, have been neglected. An application of our analysis to the experiment has been made. If we assume a mean primary energy of 100 Bev and a mean path length of 0.47 radiation length, we find a value of 1.4×10^{-5} radiation length (cm) for the parameter $\alpha T^{3/2}$. If we employ the same elliptical resolution variation as before, the expected proportion of B.S. pairs which are pseudo-tridents is 67 percent. Therefore, it is reasonable to suppose that approximately 7 of the 15 tridents reported are not, in fact, attributable to the direct pair creation process. It has been pointed out previously that this may be regarded as a minimum estimate. Consideration of this large correction factor leads to a trident cross section more nearly in agreement with theory. It can be shown that even if the minimum resolution distances are halved, our conclusions remain substantially the same; a similar remark can be made with regard to halving the primary energy. We therefore consider the contributions of bremsstrahlung conversion to constitute a major source of error in trident experiments at great energies.

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Decay Scheme of the τ Meson

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A statistical examination of the data at present available on τ-meson decay shows that it is consistent with a two-body decay scheme: $\tau\rightarrow\pi+X^0\rightarrow\pi+\pi^++\pi^-$. The mass of the intermediate particle X^0 would have to be about $595m_e$ and the Q value for its decay about 22 Mev.

As is well known, the τ meson exhibits the characteristic decay scheme $\tau^\pm\rightarrow\pi^\pm+\pi^++\pi^-$ with a Q value of about 76 Mev. This is usually regarded as a spontaneous 3-body decay, and various theories have been worked out on this basis,[1] but no definite conclusions as to the validity of these theories have been reached, because of the scarcity of the observations.

We have considered whether the data now available are consistent with the decay scheme, $\tau^\pm\rightarrow\pi^\pm+X^0$, where X^0 denotes a hypothetical neutral particle which after a very short lifetime decays into $\pi^++\pi^-$. We were led to this supposition by the fact that several authors[2] have proposed the existence of neutral particles which decay into two π mesons, although their proposals have not been widely accepted.

For the analysis of this process, mechanics alone suffices. In the rest frame of the τ meson, we expect an energy-distribution function consisting of a sharp line at an energy E for the first π meson and for each of the remaining π mesons a uniform distribution between the limits

$$E_{\max,\min}=\tfrac{1}{2}(M-2m-E)\pm\tfrac{1}{2}(E^2-m^2)^{\frac{1}{2}}\cdot\{(M^2-2ME-3m^2)/(M^2-2ME+m^2)\}^{\frac{1}{2}},\quad(1)$$

where M, m are the masses of the τ and π mesons, respectively. The mass of the X^0 particle is given by

$$x=(M^2-2ME+m^2)^{\frac{1}{2}}.\quad(2)$$

In order to test the predictions of our hypothesis, we have made a statistical analysis of fourteen events, eleven of which are summarized in a paper by Amaldi *et al.*[3] and three more by Lal *et al.*[4] In most of the cases we have used standard deviations derived directly from the experimenters' estimates of errors; for the

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