Thus $\text{Limit}_{cl}p^{(1)}$ and $f(r^2,z^2)$ are monotone increasing functions of each other and surfaces $f=constant$ surfaces are surfaces of uniform pressure.

The choice $\alpha^{(0)}=\alpha=2$ and $\beta=100$ gives a self-contained solution. Figure 2 shows the trace of the boundary on the positive part ($z>0$) of the $r-z$ half plane. The fluid mass is in the shape—roughly speaking—of an ellipsoid of revolution about the minor axis. It differs from an ellipsoid to the extent that the equation of the boundary is

$$z=\pm 0.269(0.0684-r^2-0.967r^4)^{\frac{1}{2}},$$

whereas the boundary of an ellipsoid with the same axes is

$$z=\pm 0.277(0.0643-r^2)^{\frac{1}{2}}.$$

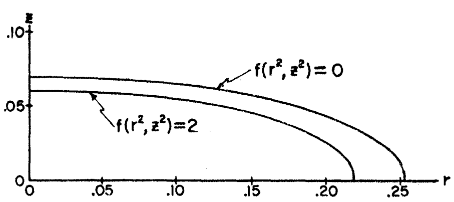

FIG. 2. A self-contained dynamical system in the first approximation.

It has been found that there are other choices of the parameters $\alpha^{(0)}$, α, and β which give non-self-contained solutions in this approximation.

Thus, the classical limit of the first order approximation in general relativity is a theory in which there exist self-contained dynamical systems which perform Born-type rigid motion.

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Comparison of the Cut-Off Meson Theory with Experiment*

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The cut-off theory for the interaction of pions with nonrelativistic nucleons is tested against experiments involving a single nucleon, with and without the presence of an electromagnetic field. It is concluded that most of the existing information about the P-wave pion-nucleon interaction can be understood with a renormalized coupling constant, $f^2=0.058$ and a cut-off energy, $\omega_{\max}=5.6\ \mu$. No light is shed on the S-wave pion-nucleon interaction.

I. INTRODUCTION

THE purpose of this paper is to compare with existing experimental data the so-called cut-off form of the Yukawa theory for the interaction of pions with nucleons. Although this form is not Lorentz-invariant[1] and is appropriate only when the nucleon velocity is small compared to the velocity of light, the meson velocity is unrestricted, so the theory can be applied to a very wide range of experiments. These include pion-nucleon scattering, photo-pion production, nucleon-nucleon scattering, and the ground-state properties of the deuteron, as well as the anomalous electromagnetic properties of nucleons (e.g., magnetic moments). It will be shown here that a large amount of the existing experimental information can be correlated by the meson theory with only two arbitrary parameters: a coupling constant and an energy cutoff.

The theory can most easily be characterized by writing down the interaction energy which it postulates between the pion field and a single fixed nucleon (at the origin of the coordinate system):

$$H_{\text{int}}=(4\pi)^{\frac{1}{2}}(f/\mu)\int d\mathbf{r}\rho(r)\sum_{\lambda=1}^{3}\tau_\lambda\boldsymbol{\sigma}\cdot\nabla\phi_\lambda(\mathbf{r}). \qquad (1)$$

Here f is the dimensionless unrationalized coupling constant ($\hbar=c=1$), μ is the pion mass, $\rho(r)$ the "source" function, normalized so that $\int\rho(r)d\mathbf{r}=1$, $\boldsymbol{\sigma}$ and $\boldsymbol{\tau}$ are the Pauli spin and isotopic spin operators for the nucleon, and the ϕ_λ are the three real components of the pion field. The form (1) is often referred to as "gradient" coupling, but we prefer to call it simply "linear" coupling, since it is the only form compatible with the conservation of angular momentum, parity, and isotopic spin which at the same time is linear in the pion field and does not involve antinucleons. The effective nonrelativistic linear interactions of any field theory (including the γ_5 theory) must reduce to the form (1).

Although (1) has been written for an infinitely heavy nucleon, it is not hard to make the interaction Galilean invariant, that is, to include effects of order v/c, where v is the nucleon velocity. This has been done for some of the calculations discussed below, where it was felt that the accuracy of both experiment and calculation

* Supported by the U. S. Office of Naval Research.
[1] For a general discussion of the cut-off theory and more references, see W. Pauli, *Meson Theory of Nuclear Forces* (Interscience Publishers, Inc., New York—London, 1946), p. 12.

justified the extra care. However, the detailed construction of the Galilean invariant theory will be left for a subsequent paper. The consequences of the v/c corrections have never been found to change qualitative effects or conclusions.

The interaction (1) corresponds to a coupling between the nucleon and P-wave pions *only*. Recoil effects introduce small interactions in states of angular momenta different from 1, but the only way to put a strong S-wave interaction into a nonrelativistic theory is through terms of higher order in the pion field (e.g., terms proportional to ϕ^2). There is considerable arbitrariness in how such terms are to be written, so we have elected to concentrate here on P-wave phenomena. Fortunately for this approach, the dominant experimental effects are almost always in the P wave.

So long as the source function $\rho(r)$ is reasonably chosen, the theory characterized by (1) exists and calculations can in principle be made of any relevant physical experiment. The results of most such calculations may be expressed in terms of integrals over virtual pion momenta which will converge by virtue of the presence of factors $v(k)$, where $v(k)$ is the Fourier transform of the source function

$$v(k)=\int d\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}\rho(r).$$

For reasonable source shapes, $v(k)$ will be closely equal to unity from $k=0$ up to $k\approx k_{\max}$ and will then fall rapidly to zero. So long as $k_{\max}$ is substantially larger than the momenta of any real pions which occur in the problem, the exact way in which $v(k)$ falls to zero is usually unimportant and we may approximate it by a step function, i.e., $v(k)=1$ for $k<k_{\max}$, $v(k)=0$ for $k>k_{\max}$. Occasionally this procedure is dangerous, but in such cases the warning signals are fairly obvious. For the majority of the problems we shall consider, the theory may be said to contain only two constants, f^2 and $k_{\max}$ [or what is equivalent $\omega_{\max}=(k_{\max}^2+\mu^2)^{\frac{1}{2}}$].

It must be emphasized that the theory we are discussing here is not new, having been considered by many previous workers.[1] What is perhaps new is the method of evaluating its predictions. The group at Illinois has applied to it the coupling constant renormalization techniques invented by Dyson[2] and Ward[3] for use in relativistic theories, where there is no cutoff and where sensible results can be obtained *only* after renormalization. As explained in an earlier paper,[4] coupling constant renormalization is not necessary in the cut-off theory, but it makes possible the use of perturbation methods which are inapplicable in its absence.[5] Strictly speaking, the constant f occurring in (1) is not the constant to which we shall have reference hereafter. By our method of calculation only the renormalized f [referred to as f_r in reference (4)] occurs in expressions which are to be compared directly with experiment. We do not know and do not need to know the value of the unrenormalized coupling constant, although we suppose it to be considerably larger than the renormalized constant [see note added in proof].

The relationship of the cut-off theory to a more fundamental underlying theory is not clear. It is possible that the cutoff is nothing more than a rough approximation to damping effects which occur at high frequencies in the local γ_5 theory and whose existence eventually will be demonstrated by some extremely clever theorist. The author tends not to believe this, inclining more to the idea that the cutoff has something to do with the mysterious K particles and hyperons which have been discovered in the past few years, some of which have strong interactions (at least in pairs) with pions and nucleons. In other words we feel that the pion and the nucleon are probably only the lowest lying states in a complex system and that any theory which attempts to isolate them, as this one does, is bound to involve some characteristic energy corresponding to the states which are being ignored. We shall not delve deeper into such questions now, however, but confine our attention to the success of the cut-off theory as it stands.

Although some of the detailed calculations on which the results presented here are based already have been published; in several instances the work is still continuing or at least has not yet been written up. The author feels, however, that the success achieved so far is sufficiently interesting to make a preliminary general report worth while now, with the individual detailed calculations to appear later.

II. PION-NUCLEON SCATTERING

As reported in recent letters[6] the four P-wave phase shifts for pion-nucleon scattering have been calculated for laboratory energies up to 200 Mev. Specifically, the important terms up to fourth order in the sense of reference 4 have been kept and the scattering integral equation solved. Investigations of higher order effects have indicated that these will not be of great importance, even though the fourth-order terms were not negligible compared to those of second order. A detailed justification of these points will be given at a later time.

As already indicated by the variational treatment of second order terms alone,[7] the only phase shift which can become large in our theory is δ_{33}, since only in the

[2] F. J. Dyson, Phys. Rev. **75**, 1736 (1949).
[3] J. C. Ward, Proc. Phys. Soc. (London) **A64**, 54 (1951).
[4] G. F. Chew, Phys. Rev. **94**, 1749 (1954).

[5] The criterion for weak coupling in the absence of renormalization is that $f^2(k_{\max}/\mu)^2\ll 1$. After renormalization, it is only $f_r^2(k_{\max}/\mu)\ll 1$.
[6] G. F. Chew, Phys. Rev. **95**, 285 (1954); F. Salzman and J. Snyder, Phys. Rev. **95**, 286 (1954).
[7] G. F. Chew, Phys. Rev. **89**, 591 (1953).

state of $J=\frac{3}{2}, I=\frac{3}{2}$ is our scattering "potential" attractive. Recently Glicksman[8] and Bethe[9] have shown that a satisfactory analysis of the experimentally observed P-wave scattering can be made by neglecting all P phase shifts except δ_{33}, which means roughly that δ_{31}, δ_{13}, and δ_{11} can be assigned any values less than about one-tenth that of δ_{33} without violating observation. A possible interpretation is to say that δ_{33} is determined by the experiments, while the only thing we know about the other P phase shifts is that they are small. On this basis both Glicksman and Bethe find that δ_{33} passes through 90° near a laboratory energy of 200 Mev. This fact has been taken as one of two primary experimental data to determine the two parameters in the theory. The other datum is a phase shift $\delta_{33}=9.1°$ at 65 Mev, published by Bodansky, Sachs, and Steinberger.[10] Requiring our theory to give these two values for δ_{33} at the energies mentioned leads to a coupling constant,

$$f^2=0.058,$$

and an energy cutoff,

$$\omega_{\max}=5.6\,\mu.$$

In Fig. 1, the complete theoretical curves of δ_{33}, δ_{11}, δ_{13}, and δ_{31} (the latter two are always equal in our theory) are shown as a function of energy. The values of δ_{33} arrived at by Glicksman[8] and Bethe[9] as well as the Columbia result[10] are shown for comparison.

The rather substantial deviation of the coupling constant and cut-off used here from the earlier values ($f^2=0.2$, $\omega_{\max}=3.2\mu$) proposed by the author[7] needs some explanation. The major point is that when the earlier analysis was done, the existing experimental data seemed to indicate that δ_{33} did not actually pass through 90°. The fit given, in fact, corresponded to $\delta_{33}=53°$ at 200 Mev. Raising the cutoff was necessary to attain a 90° phase shift at the high energy, but this change required the coupling constant to be reduced to keep a fit at low energies. Also helping to lower the coupling constant were the fourth-order effects not considered in the earlier work and the exact solution of the integral equation. The variational approximation, used previously, systematically underestimates δ_{33}, as shown in detail by Gammel.[11]

It has been verified that for the earlier (low) cutoff the v/c corrections are not important, provided one calculates the phase shift directly and uses the center-of-mass pion energy. It is not clear that with the higher cutoff we may continue to ignore recoil and calculate accurately, but important corrections are unlikely.

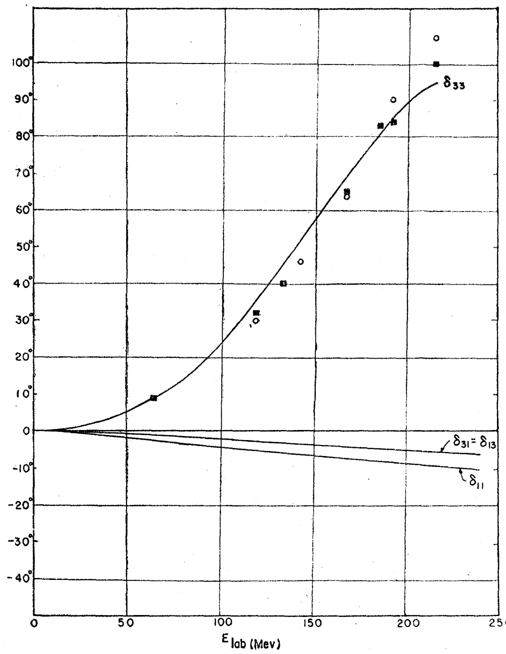

FIG. 1. The P phase shifts for pion-nucleon scattering, calculated from the cut-off theory. The upper six solid squares are Chicago values for δ_{33} given by Glicksman (see reference 8), while the point at 65 Mev was obtained by the Columbia group (see reference 10). The five open circles are due to de Hoffmann *et al.* (see reference 9).

The very complicated behavior of the S phase shifts in pion-nucleon scattering[9] is completely unexplained by the theory as it stands (with a linear interaction only). Recoil effects by themselves lead to results[12] which bear no resemblance to the experimental observations. A major extension of the theory is evidently required to explain the S-wave interaction.

It is not clear to how high an energy our nonrelativistic theory should be expected to be applicable. Calculations of pion-nucleon scattering will certainly become much more complicated as other processes, such as pion production, become energetically allowed. No thought has been given yet to the high energy problem, but it seems fair to say that the theory successfully describes the experimentally observed P-wave scattering up to 200 Mev.

III. PHOTO-PION PRODUCTION NEAR THRESHOLD

The cut-off theory gives a very simple result for charged photo-pion production close to threshold

$$\frac{d\sigma}{d\Omega}=2\frac{e^2f^2}{\mu^2}\frac{k}{\nu}. \tag{2}$$

Here $e^2=1/137$, while k and ν are the momenta of outgoing pion and incident photon, respectively. Formula (2) corresponds to an electric dipole transition with the production of an S-wave pion. Because in our theory

[8] M. Glicksman, Phys. Rev. **95**, 1045 (1954).

[9] H. A. Bethe, Proceedings of the Fourth Annual Rochester Conference on High Energy Nuclear Physics (University of Rochester Press, Rochester, to be published), p. 134; de Hoffmann, Metropolis, Alei, and Bethe, Phys. Rev. **95**, 1586 (1954).

[10] Bodansky, Sachs, and Steinberger, Phys. Rev. **93**, 1367 (1954).

[11] J. Gammel, Phys. Rev. **95**, 209 (1954).

[12] E. Henley and M. Ruderman, Phys. Rev. **90**, 719 (1953).

there is no interaction between S-wave pions and nucleons, there are no higher order corrections to (2) after renormalization. One might worry that a modification of the theory to explain the observed S-wave scattering would spoil the simplicity of (2), i.e., introduce "radiative" corrections. However, a theorem due to Kroll and Ruderman[13] suggests that this will not happen. These workers have proved that for the kind of relativistic theory currently in fashion, the only radiative corrections to photo-pion production at threshold after renormalization are of order μ/M, where M is the nucleon mass. It does not require a great deal of optimism to believe that this feature will be present in the final correct theory which explains S-wave scattering along with everything else.

Since the v/c corrections to (2) are very simple and not quite negligible, we shall list them here. Kinematical effects give rise to a factor, $(1+\nu/M)^{-2}$, in the cross section, while the matrix element itself acquires a factor $(1\pm\nu/2M)$, the plus sign going with negative pion production (from neutrons) and the minus sign with positive pion production (from protons). Thus the $-/+$ ratio is $(1+2\nu/M)$, which is certainly not in serious disagreement with the values obtained so far from measurements near threshold on deuterium. For example, Beneventano, Bernardini, Lee, and Stoppini[14] find an average $-/+$ ratio of 1.5 ± 0.1 at 170 Mev, where the theoretical ratio is 1.3.

Modifying formula (2) by the v/c corrections listed above, we have for positive pion production near threshold,

$$\frac{d\sigma}{d\Omega_+}=2\frac{e^2f^2}{\mu^2}\frac{(1-\nu/M)}{(1+\nu/M)^2}\frac{k}{\nu}. \tag{2'}$$

Bernardini and Goldwasser[15] find that the low-energy S-wave positive photo-pion production from hydrogen can be fitted by formula (2′) if f^2 is taken to be 0.066 ±0.008, a value in satisfactory agreement with that obtained above from the P-wave scattering.

With respect to both the absolute value for photo-pion production and the positive negative ratio near threshold, therefore, the theory seems adequate.

IV. ANOMALOUS NUCLEON MAGNETIC MOMENTS

According to the cut-off theory, the pion field associated with single nucleons gives rise to anomalous magnetic moments which will be positive for protons and negative for neutrons, the absolute value being the same. The second-order formula for these anomalous moments is quite simple. In units of nuclear magnetons,

$$M_2=\pm\frac{8}{3\pi}\frac{M}{\mu}\frac{f^2}{\mu}\int_\mu^{\omega_{\max}}d\omega\frac{k^3}{\omega^3}, \tag{3}$$

and if formula (3) is evaluated with the constants determined from the scattering analysis, one finds $M_2=\pm1.15$. Friedman[16] has derived the formula for the fourth-order magnetic moment, which when evaluated for the same constants gives $M_4=\pm0.33$. Thus, up to fourth order the pion contribution to nucleon magnetic moments is ±1.48.

Experimentally, the total proton moment is $+2.79$ and the neutron moment -1.91 in these units. If one assumes the proton to have an intrinsic moment of one unit and the neutron to have zero intrinsic moment, then the residual or anomalous moments are $+1.79$ and -1.91, respectively. In this way of looking at the problem, therefore, one might say that the cut-off theory for pions explains a large fraction of the anomalous moments. The remainder could easily come from heavier mesons, together with nucleon recoil contributions.

V. PHOTO-PION PRODUCTION AT HIGHER ENERGIES

Above 200 Mev, the P-wave final state rapidly becomes important in photo-pion production and the simple formula (2) or (2′) must be modified. The lowest order matrix element for charged photo-pion production, is

$$\mathcal{H}_2{}^+=2\pi ie\frac{\sqrt{2}f}{\mu}\frac{1}{\nu}\left\{\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}-\frac{2\boldsymbol{\sigma}\cdot(\mathbf{k}-\boldsymbol{\nu})\boldsymbol{\varepsilon}\cdot\mathbf{k}}{\omega_{\mathbf{k}-\boldsymbol{\nu}}{}^2}\right\}, \tag{4}$$

where $\boldsymbol{\varepsilon}$ is the unit polarization vector of the incident photon, and

$$\begin{aligned}\omega_{\mathbf{k}-\boldsymbol{\nu}}{}^2&=\mu^2+(\mathbf{k}-\boldsymbol{\nu})^2\\&=2\nu^2(1-v\cos\theta),\end{aligned} \tag{5}$$

if $v=k/\nu$ is the velocity of the outgoing pion. (In this discussion we shall neglect recoil completely.) The matrix element for neutral photoproduction vanishes in this order.

The first term $\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}$ in the bracket of (4) is the electric dipole matrix element already discussed, which leads to formula (2). The second term results from a mixture of many multipoles and gives infinitely many orders of outgoing pion angular momenta (because of the retardation factor in the denominator). By a remarkable cancellation, however, when (4) is squared and averaged over photon polarization and nucleon spin, interference between the two terms in (4) almost knocks out the square of the second term. The result for the cross

[13] N. Kroll and M. Ruderman, Phys. Rev. **93**, 233 (1954).
[14] Beneventano, Bernardini, Lee, and Stoppini (private communication).
[15] G. Bernardini and E. Goldwasser, Phys. Rev. **94**, 729 (1954) and private communication.
[16] M. H. Friedman (to be published). For a derivation of M_2, see reference 1, p. 38.

section in lowest order is[17]

$$\left(\frac{d\sigma}{d\Omega_2}\right)=\frac{2e^2f^2}{\mu^2}v\left\{1-\frac{\mu^2}{2\nu^2}\frac{v^2\sin^2\theta}{(1-v\cos\theta)^2}\right\}, \qquad (2'')$$

where one easily sees that the new contribution never amounts to more than a small fraction of that already given by (2).

Just as in the scattering problem, however, one must worry about higher order effects if intermediate states can exist which "resonate" with the final state. Salzman[18] has analyzed this problem and finds that those effects are large only if the final state has total angular momentum $\frac{3}{2}$ and total isotopic spin $\frac{3}{2}$.[19] One may say crudely that outgoing pions in this state suffer a strong secondary "scattering" by the nucleon, which amplifies their role tremendously and which must be represented by additional terms. At the same time, terms corresponding to exchange scattering must appear in the matrix element for neutral photo-pion production. It is well known from isotopic spin considerations that these will be exactly $\sqrt{2}$ times the extra terms appearing in the charged matrix element.

The corrected matrix elements now become

$$\mathcal{H}^+=ie\frac{\sqrt{2}f}{\mu}\frac{2\pi}{\nu}\left\{\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}-\frac{2\boldsymbol{\sigma}\cdot(\mathbf{k}-\boldsymbol{\nu})\,\boldsymbol{\varepsilon}\cdot\mathbf{k}}{\omega_{\mathbf{k}-\boldsymbol{\nu}}^2}+2M_1\frac{i\mathbf{k}\cdot(\boldsymbol{\nu}\times\boldsymbol{\varepsilon})}{k\nu}+(M_1+E_2)\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\nu}\,\boldsymbol{\varepsilon}\cdot\mathbf{k}}{k\nu}-(M_1-E_2)\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\,\mathbf{k}\cdot\boldsymbol{\nu}}{k\nu}\right\} \qquad (6)$$

$$\mathcal{H}^0=ie\frac{2f}{\mu}\frac{2\pi}{\nu}\left\{2M_1\frac{i\mathbf{k}\cdot(\boldsymbol{\nu}\times\boldsymbol{\varepsilon})}{k\nu}+(M_1+E_2)\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\nu}\,\boldsymbol{\varepsilon}\cdot\mathbf{k}}{k\nu}-(M_1-E_2)\frac{\boldsymbol{\sigma}\cdot\boldsymbol{\varepsilon}\,\mathbf{k}\cdot\boldsymbol{\nu}}{k\nu}\right\}, \qquad (6')$$

where the symbols M_1 and E_2 represent the effective matrix elements for transitions to the 33 state due to magnetic dipole and electric quadrupole radiation, respectively. These matrix elements will be discussed in detail in the forthcoming paper by Salzman.[18] An evaluation is being carried out using the same kind of approximation employed to get the scattering phase shifts.

Squaring and averaging (6) and (6′), we obtain for the charged and neutral production cross sections

$$\left(\frac{d\sigma}{d\Omega}\right)_+=\frac{2e^2f^2}{\mu^2}v\left\{1-\frac{\mu^2}{2\nu^2}\frac{v^2\sin^2\theta}{(1-v\cos\theta)^2}+|M_1-E_2|^2\cos^2\theta-2\,\mathrm{Re}(M_1-E_2)\left(1-\frac{v^2/2\sin^2\theta}{1-v\cos\theta}\right)\cos\theta+\left[2|M_1|^2+\tfrac{1}{2}|M_1+E_2|^2+v\,\mathrm{Re}(M_1+E_2)\right]\sin^2\theta\right\}, \qquad (7)$$

$$\left(\frac{d\sigma}{d\Omega}\right)_0=\frac{4e^2f^2}{\mu^2}v\left\{|M_1-E_2|^2\cos^2\theta+\left[2|M_1|^2+\tfrac{1}{2}|M_1+E_2|^2\right]\sin^2\theta\right\}. \qquad (7')$$

A preliminary test of these formulas has been made by assuming that

$$M_1=m_1(\mu\nu/k^2)e^{i\delta_{33}}\sin\delta_{33}$$
$$E_2=e_2(\mu\nu/k^2)e^{i\delta_{33}}\sin\delta_{33}, \qquad (8)$$

where m_1 and e_2 are real and energy-independent constants. The assumption (8) is certainly valid in the resonance region and appears, according to Salzman's early work, to be reasonable all the way from 200- to 400-Mev photon energy. Outside of these limits, the secondary scattering is unimportant anyway. By guessing at the values of certain complicated integrals, Salzman estimates $m_1=0.58$ and $e_2=0.18$ from the cut-off theory with $f^2=0.058$ and $\omega_{\max}=5.6\,\mu$. These numbers are subject to revision, but they will be used here to illustrate formulas (7) and (7′). A complete calculation which does not employ the simplifying assumption (8) and which includes states other than the $(\frac{3}{2},\frac{3}{2})$ will be published later by Dr. Salzman.

In Fig. 2 are plotted the theoretical and experimental[15,20–22] cross sections for positive photo-pion production up to 300-Mev photon energy at 90° in the barycentric system, as given by formula (7), with

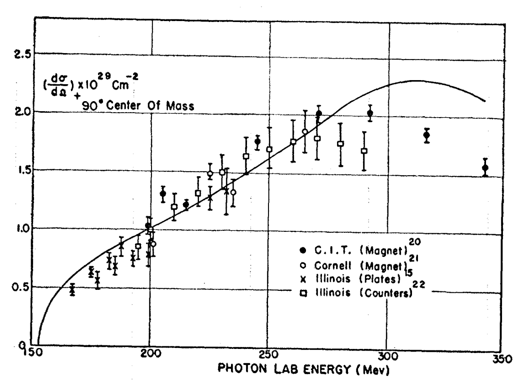

FIG. 2. A plot of formula (7), the theoretical prediction for charged photo-pion production at 90° in the center-of-mass system. The experimental points shown give a fairly representative sample of the more recent work (see references 15, 20–22).

[17] R. E. Marshak, *Meson Physics* (McGraw-Hill Book Company, Inc., New York, 1952), p. 13.

[18] F. Salzman (to be published).

[19] A conjecture that one need consider only the final state with $J=\frac{3}{2}$ and $I=\frac{3}{2}$ is the basis of a phenomenological approach developed by K. A. Brueckner and K. M. Watson, Phys. Rev. **86**, 923 (1952).

[20] V. Z. Peterson (private communication).

[21] Jenkins, Luckey, and Wilson, Phys. Rev. **94**, 755 (1954).

[22] J. E. Leiss and C. S. Robinson (private communication).

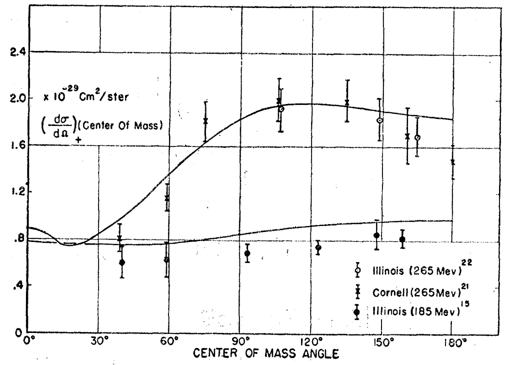

FIG. 3. The theoretical angular distribution for charged photo-pion production, compared to recent experiments at 185- and 265-Mev photon energies (see references 15 and 21).

$f^2=0.058$, and M_1 and E_2 approximated by formula (8). In other words, this is the prediction of the cut-off theory. There are no arbitrary constants once f^2 and $\omega_{\max}$ are fixed, as they have been by the scattering. It is seen that the fit to experiment is adequate. The fact that the maximum occurs at too high an energy might be attributed to complete neglect of recoil effects such as those given in Eq. (2′) for the electric dipole term alone. The problem of including recoil in the secondary scattering terms is under investigation.

Further indication of the success of the theory is given by the comparison of theoretical and experimental angular distributions for charged photo-pions shown in Fig. 3. Remember that this is an absolute comparison; the normalization of the theoretical curves has not been adjusted.

In Fig. 4 the neutral photo-pion cross section at 90° is compared to the experiment of Silverman and Stearns.[23] It should be said here that more recent and still unpublished experiments at other laboratories indicate larger absolute values for the neutral cross sections. If these turn out to be correct, our agreement with the Cornell values is meaningless. One would have to hope, in that case, that the more accurate evaluation of M_1 and E_2 will yield larger numbers than have been used here.

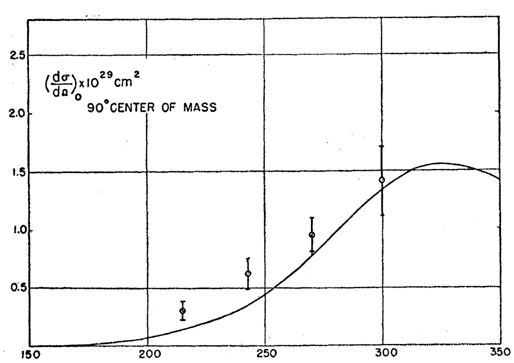

FIG. 4. The theoretical prediction for neutral photo-pion production at 90° in the center-of-mass system, compared to the experiment of Silverman and Stearns (see reference 23).

The angular distribution for neutral photo-pion production has also been measured. The results can be given in terms of ratios, A_0/A_2 and A_1/A_2, which refer to the angular distribution as expressed in the form

$$(d\sigma/d\Omega_0)=A_0+A_1\cos\theta+A_2\sin^2\theta.$$

A recent compilation of data,[21] based on results from Cornell and M.I.T.,[24] leads to $A_0/A_2=0.10\pm0.12$ and $A_1/A_2=-0.10\pm0.09$ in the energy range from 230 to 310 Mev. Formula (7′) predicts that $A_1=0$ and that with our guess for M_1 and E_2, $A_0/A_2=0.2$, with no energy dependence.

Although much more can be done with formulas (7) and (7′) in the way of comparison with experiment, especially at higher energies, we feel that until nucleon recoil is included only rough tests are justified. In our judgment the comparisons described above indicate that the cut-off theory can describe photo-pion production with an accuracy of about 20 percent in the region of a few hundred Mev. In view of the neglect or incomplete treatment of nucleon recoil, this is the most that can be expected.

Before passing to the next subject it should be pointed out that even viewed as empirical formula, the expressions (7) and (7′) contain two qualitative features that distinguish them from certain earlier theoretical formulas which have been proposed.[19,25] The first is the inclusion of E_2 as well as M_1, which tends to make the neutral angular distribution almost a pure $\sin^2\theta$ and reduces the asymmetric $\cos\theta$ term in the charged angular distribution. The second feature is the inclusion of angular momenta greater than 1 in the outgoing charged pion wave. The most important consequence of the higher components is an added term in the $\sin^2\theta$ coefficient, due to interference with the scattered P wave. This term has its maximum for $\delta_{33}=45°$ rather than 90° and thereby causes the over-all maximum in the charged cross section to occur at a lower energy than that in the neutral.

VI. THE NEUTRON-ELECTRON INTERACTION

Another experimental datum to which one might be tempted to apply our theory is the interaction between electrons and slow neutrons. The most recently published result gives a depth, -3860 ± 370 ev,[26] for the effective neutron-electron potential of radius equal to the classical electron radius. Foldy[27] has pointed out

[23] A. Silverman and M. Stearns, Phys. Rev. **88**, 1228 (1952).

[24] G. Cocconi and A. Silverman, Phys. Rev. **88**, 1230 (1952); Goldschmidt-Clermont, Osborne, and Scott, Phys. Rev. **89**, 329 (1953).

[25] B. T. Feld, Phys. Rev. **89**, 330 (1953). Recently M. Ross, Phys. Rev. **94**, 454 (1954) has published a treatment which includes effects of the kind mentioned here.

[26] Hughes, Harvey, Goldberg, and Stafne, Phys. Rev. **90**, 497 (1953).

[27] L. Foldy, Phys. Rev. **83**, 688 (1951).

from relativistic considerations that the existence of a neutron magnetic moment implies an associated neutron-electron interaction of -4000 ev. Since the Foldy term is wholly relativistic (and completely outside our theory), one sees that the order of magnitude of the entire experimental effect is no larger than that of relativistic contributions to it.

One might conclude, then, that there is no point in applying a nonrelativistic theory to this problem. It is necessary to verify, however, that our theory does not predict a nonrelativistic contribution to the *n-e* interaction which is so much larger than the experimental value that one cannot reasonably expect it to be brought into agreement by uncalculated and unknown relativistic terms. Calculations by Salzman[28] show that the nonrelativistic contribution from the meson charge cloud is -8 kev. One will have to hope that a positive contribution of equal size will be forthcoming from the nucleonic core charge cloud when its effect can be calculated.

VII. NUCLEAR FORCES

No quantitative results have yet emerged from the application of the cut-off theory to the two nucleon problem, but it is worth pointing out that published calculations performed in other connections suggest that our theory may not fail this test. Almost all forms of meson theory attribute the majority of the tensor force in the deuteron to the exchange of a single (P wave) pion. Without cutoff, this exchange leads to the well known tensor force,[29]

$$V_t(r)=f^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\left(\frac{3\boldsymbol{\sigma}_1\cdot\mathbf{r}\boldsymbol{\sigma}_2\cdot\mathbf{r}}{r^2}-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\times\left(\frac{1}{\mu^2r^2}+\frac{1}{\mu r}+\frac{1}{3}\right)\frac{e^{-\mu r}}{r}. \quad (9)$$

With cutoff, the tensor potential is still given by (9) for $r\gg 2/k_{\max}$, but for $r\lesssim 2/k_{\max}$ the cutoff causes the potential to change sign (central part as well as tensor). Since Levy[30] was able to fit the observed properties of the deuteron using the tensor force (9) with $f^2=0.055$ and a repulsive core of radius, $0.38/\mu$, it is not unreasonable to hope that our potential, which already is determined by the constants $f^2=0.058$ and $\omega_{\max}=5.6\mu$, will give at least a rough fit to experiment $(2/k_{\max}=0.36/\mu)$.

It is less probable although not impossible that the central force also will be explained by this most primitive form of the cut-off theory. The difficulty here of course is that if S-wave pions, so far ignored, have an important interaction with nucleons they may be exchanged in pairs and make an important contribution to the central part of the two-nucleon interaction. At the very least, pairs of P-wave pions (i.e., fourth-order terms in f) must be considered,[31] since for a reason not well understood the exchange of single pions contributes almost nothing except a repulsive core to the central force.

[28] G. Salzman (private communication).

[29] See reference 1, p. 6.

[30] M. Levy, Phys. Rev. **86**, 806 (1952).

VIII. CONCLUSION

It has been claimed that a renormalized coupling constant and cutoff can be found for the linear meson theory which successfully correlate a significant number of experiments involving nonrelativistic nucleons. The author believes the meaning of this success to be that, in some sense, *single* pion emission and absorption (real and virtual) are the dominant processes at energies below the nucleon rest energy. All our theory really amounts to is this assumption plus the observance of well known conservation laws. Heavier mesons and antinucleons seem not to intrude on the "low-energy" scene, except perhaps to help determine the magnitude of the cutoff.

The chief value of this kind of approach in the end probably will be to differentiate between those low-energy pion phenomena which give a real clue to the next big theoretical development and those which tell us nothing essentially new. For example, the low-energy P-wave phenomena seem to form a self-consistent group which may well lead nowhere in particular. We have the impression, on the other hand, that the low-energy S-wave pion-nucleon interaction cannot be disassociated from intrinsically high-energy questions such as the existence of antinucleons. Thus one might conclude that attention should be concentrated on the S wave, with the expectation that when a successful theory is constructed there, the P wave will follow in a natural fashion.

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Note added in proof.—T. D. Lee has recently communicated to the author a general proof that the renormalized coupling constant is smaller than the unrenormalized. A discussion of this point is to be published in *The Physical Review.*

[31] Calculations by K. Brueckner and K. Watson, Phys. Rev. **92**, 1023 (1953), by E. Henley and M. Ruderman, Phys. Rev. **92**, 1036 (1953), as well as by M. Taketani *et al.* Progr. Theoret. Phys. (Japan) **7**, 45 (1952), suggest that the central force due to pairs of P-wave pions may be sufficient without any S-wave pairs. These calculations, however, do not consider a cutoff in the sense used here.