in the first scheme has since been established by Langer and Starner[11] through triple coincidence studies.

The present investigation using the summing technique was conducted shortly before the result of the triple coincidence measurements was published. It was hoped that the high efficiency of the summing technique for multiple coincidences might show whether the triple sum peak appears at 0.64+0.72+0.603=1.96 Mev or at 0.71+0.72+0.60=2.04 Mev. However, the normal spectrum shows the possible existence of a gamma ray at 1.36 Mev. This peak, when summed with the 0.603-Mev gamma ray, falls at 1.96 Mev and interferes with the observation of the triple sum peak. It is conceivable that when a larger crystal is used the present method will furnish independent information in this respect. The summing spectrum shown in Fig. 2 does show, however, a positive indication of the coincidence between the 2.06-Mev and the 0.603-Mev gamma rays.

[11] L. M. Langer and J. W. Starner, Phys. Rev. **93**, 253 (1954).

I^{131} [7]

The summing spectrum of the gamma rays following the beta decay of I^{131} is shown in the lower part of Fig. 3. The spectrum clearly indicates that the 0.284-Mev transition is in cascade with the 0.080-Mev transition. Because of the high efficiency of the crystal for detecting the 0.080-Mev gamma ray, the 0.286-Mev peak is greatly diminished through the summing effect. However, no such summing effect is observed for the 0.638-Mev or the 0.722-Mev gamma rays. This is a further confirmation of the decay scheme proposed originally by Metzger and Deutsch[12] and augmented by Emery.[13]

[12] F. Metzger and M. Deutsch, Phys. Rev. **74**, 1640 (1948).
[13] E. W. Emery, Phys. Rev. **83**, 679 (1951).

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Nuclear Levels in Ca^{43}†

TORSTEN LINDQVIST* AND ALLEN C. G. MITCHELL
Physics Department, Indiana University, Bloomington, Indiana
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The nuclear levels in Ca^{43} have been studied in the decay of Sc^{43} by positron emission to Ca^{43}. Levels have been found at 0.369, 0.627, and 0.81 Mev. In an earlier experiment, the decay of K^{43} by negatron emission was studied by the authors and nuclear levels were found in Ca^{43} at 0.627, 1.00, 1.39, and 1.61 Mev. The levels have been given configuration assignments. Theoretical discussion of the $(f_{7/2})^3$ configuration is compared with experimental results.

I. INTRODUCTION

RECENTLY, the decay of K^{43} has been studied by the authors.[1] K^{43} decays by negatron emission to Ca^{43} and nuclear levels were found at 0.627, 1.00, 1.39, and 1.61 Mev. The spin of the ground state of Ca^{43} has been measured by Jeffries[2] and has been found to be 7/2 and of odd parity. Of the levels of Ca^{43} measured in the experiments on K^{43}, the ground state and that at 0.627 Mev were found to have odd parity, while those at 1.00, 1.39, and 1.61 Mev were shown to have even parity. Ca^{43} can be reached by the negatron decay of K^{43} or the positron decay of Sc^{43}. Since K^{43} has 19 protons and Sc^{43} 21 protons, the shell model predicts that the parity of the ground state of K^{43} will be even $(d_{3/2})$ and that of Sc^{43} odd $(f_{7/2})$. For this reason, and also since the spins of the two nuclei differ considerably, it is expected that different levels of Ca^{43} will be observed depending on which parent nucleus is investigated. Therefore, it was decided to reinvestigate the decay of Sc^{43} and to compare the results with those obtained from the decay of K^{43}.

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* On leave from the University of Uppsala, Uppsala, Sweden.
[1] T. Lindqvist and A. C. G. Mitchell, Phys. Rev. **95**, 444 (1954); **95**, 612A (1954).
[2] C. D. Jeffries, Phys. Rev. **90**, 1130 (1953).

II. DECAY OF Sc^{43}

(a) Previous Results

The radiations from Sc^{43} have been investigated by various authors. The most recent investigation by Haskins, Duval, Cheng, and Kurbatov[3] was carried out with a magnetic lens spectrometer and showed a half-life of 3.92±0.02 hr, two positron groups of 1.18. Mev (72 percent) and 0.77 Mev (28 percent), and a gamma ray of energy 0.375 Mev. Nussbaum, van Lieshout, and Wapstra,[4] using a scintillation counter, found a gamma ray of energy 0.375 Mev and a half-life of 4.0 hr.

[3] Haskins, Duval, Cheng, and Kurbatov, Phys. Rev. **88**, 876 (1952).
[4] Nussbaum, van Lieshout, and Wapstra, Phys. Rev. **92**, 207 (1953).

TABLE I. Ca isotopes and reactions.

Ca isotopes	Ca^{40}	Ca^{42}	Ca^{43}	Ca^{44}	Ca^{46}	Ca^{48}
Rel. abund. %	96.9	0.64	0.14	2.1	0.0032	0.18
Sc isotopes	Sc^{43} β^+	···	Sc^{46} β^-	Sc^{47} β^-	Sc^{49} β^-	···
from Ca(α,p)Sc	3.9 hr	···	85 days	3.43 days	57 min	···
Sc isotopes	···	Sc^{44} β^+	···	Sc^{46} β^-	Sc^{48} β^-	···
from Ca$(\alpha p,n)$Sc	···	4.0 hr	···	85 days	44 hr	···

(b) Preparation of Sources

The isotope Sc^{43} was prepared by the reaction $Ca^{40}(\alpha,p)Sc^{43}$. Purified CaO was bombarded for an hour with 40 μa of 18–22 Mev alpha particles. Table I shows the relative abundance of the various calcium isotopes and the possible reactions.

Considering the half-lives as well as the relative abundances of the isotopes, the most likely products to appear, besides the dominating Sc^{43}, are Sc^{44} and Sc^{47}. By reducing the beam energy to 17–18 Mev it was possible to suppress the Sc^{44} activity, since the (α,pn) reaction decreases more rapidly with alpha-particle energy than does the (α,p) reaction.

There was a small amount of Sc^{47} produced. During the first 24 hours, when Sc^{43} was measured, the activity from any Sc^{47} was too low to be detected, but after 5 days it was possible to measure the radiation in a scintillation spectrometer. A gamma ray of energy 180 kev was found and believed to belong to Sc^{47} since it had the same half-life. The half-life for Sc^{47} was checked over a two-week period giving $T_{1/2}=82.3$ hr (3.44 days) in agreement with earlier experiments.

The half-life of Sc^{43} was measured in the same run, and after subtraction of the 82.3-hr activity a half-life of 3.8 ± 0.1 hr was obtained for Sc^{43}.

(c) Gamma-Ray Spectrum

The gamma rays were investigated by measuring the photoelectron spectrum from various radiators with the help of a magnetic lens spectrometer. A scintillation spectrometer was also used. Figure 1 shows the photoelectron spectrum from a lead radiator of thickness 8 mg/cm^2. Photoelectron lines corresponding to three gamma rays and the annihilation radiation were found. In the scintillation spectrometer, there was also found a weak gamma ray of higher energy. Table II gives the energies and relative intensities for the gamma rays. In order to confirm that the photoelectron line at "0.627K" really was a K line and not an L line with the corresponding K line hidden under the annihilation radiation, various radiators were used. Uranium as well as indium radiators confirmed the result obtained with a lead radiator. The half-life for the γ rays was checked, and each had a half-life corresponding to the decay of Sc^{43}.

TABLE II. Gamma rays from Sc^{43}.

Energy Mev	Rel. abund.	Measured in
$0.25\ \pm0.01$	0.5	magn. lens
0.369 ± 0.005	8	magn. lens
0.511	100	magn. lens
0.627 ± 0.005	2	magn. lens
$0.84\ \pm0.02$	weak	scint. spectr.

(d) Positron Spectrum

The positron distribution was measured in the magnetic lens and was analyzed by making a Fermi plot of the data. The results are shown in Fig. 2. The spectrum can be analyzed into three groups as shown in the figure and in Table III.

Internal conversion lines were also looked for but none were found, implying that the gamma rays are of low multipole order.

(e) Disintegration Scheme of Sc^{43}

The results of the investigation are embodied in the disintegration scheme shown in Fig. 3. The three positron groups establish levels at 0.38 and 0.81 Mev, together with the ground state, and all three groups are of an allowed character. The gamma rays of energies 0.369, 0.627, and 0.25 Mev establish an additional level at 0.627 Mev. The positron transition leading to the 0.627-Mev level is so weak that it has not been found. The weak gamma ray of 0.84 Mev, seen only in a scintillation spectrometer, comes from the level populated by the lowest-energy positron group.

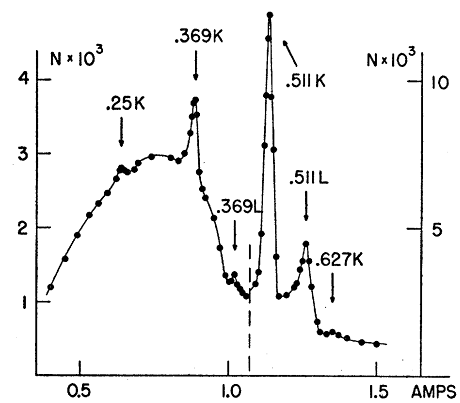

FIG. 1. Photoelectron spectrum from Sc^{43}.

In order to establish the position of the gamma ray at 0.369 Mev, coincidence experiments were performed in which positrons were detected in a scintillation spectrometer using an anthracene crystal and gamma rays in a similar apparatus using a NaI(Tl) crystal. With the scintillation spectrometer detecting gamma rays set on the 0.369-Mev line, the differential pulse-height analyzer of the scintillation spectrometer measuring positrons was swept through the spectrum. No coincidences were obtained for positron energies greater than 0.82 Mev, while coincidences were obtained for energies less than this. This shows that the line at

0.369 Mev is in coincidence with the positron group of 0.82-Mev energy. This experiment has additional importance since, in the decay of K^{43} there is a gamma ray of energy 0.369 Mev which appears to be in a different place in the level scheme.

The assignment of spins and parities to the levels of Ca^{43} was determined essentially by the values of log ft for the positron groups, the measured spin of the ground state of Ca^{43}, and the shell model configuration for the ground state of Sc^{43}. Sc^{43} has 21 protons and 20 neutrons and the shell model predicts unambigously a spin of 7/2 and odd parity for the ground state. The ground state of Ca^{43} has been measured and found to have a spin of 7/2 and odd parity. The positron transitions connecting the ground states of parent and daughter are allowed and are therefore consistent with $\Delta j=0$,no. The states established by the other two allowed positron groups, namely those of energy 0.369 Mev and 0.81 Mev, must have odd parity and spins of 5/2, 7/2, or 9/2. Ca^{43} has the configuration $(f_{7/2})^3$ and according to Kurath[5] the level order is 7/2 (ground state), 5/2, 3/2, 9/2, 11/2 . . ., all of odd parity. The first excited level at 0.369 Mev has therefore been given a spin of 5/2 and the third excited level at 0.81 Mev a spin of 9/2, both consistent with ft values. Further, the second excited level at 0.627 Mev has been given a spin of 3/2, in agreement with the result obtained on K^{43} decay. This will account for the low intensity of the gamma ray at 0.627 Mev and the fact that no positron transition to this level is seen, since the latter would be characterized by $\Delta j=2$,no and would be very weak compared to the allowed transitions to the other states.

TABLE III. Positron groups of Sc^{43}.

Group	E Mev	Relative abundance %	log(ft)	Spin and parity	Transition
I	1.20±0.01	79	5.0	$\Delta j=\pm1,0$,no	Allowed
II	0.82±0.02	17	5.1	$\Delta j=\pm1,0$,no	Allowed
III	0.39±0.03	4	4.9	$\Delta j=\pm1,0$,no	Allowed

III. LEVELS OF Ca^{43}

The levels of Ca^{43} have been determined from three separate experiments: the disintegration of K^{43}, the reaction[6] $Ca^{42}(d,p)Ca^{43}$, and the present experiments on Sc^{43}. In addition the relative spacing and order of occurrence of the levels of Ca^{43} have been calculated according to the $(j-j)$ coupling model by Kurath.[5] The effects of surface coupling for this nucleus have been discussed by Bohr and Mottelson[7] and have been calculated in a weak-coupling approximation by Ford.[8]

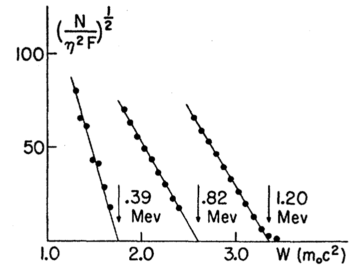

FIG. 2. Fermi plot for Sc^{43}.

It is, therefore, of interest to discuss the experiments in the light of the theory.

Figure 4 shows the levels obtained from the three experiments separately, together with the combined level scheme. No one method gives all the levels of Ca^{43}, but this is to be expected on account of selection rules. The experiment of Braams gives all of the levels up to 1.40 Mev, with the exception of that at 0.81 Mev, and, as shown in Fig. 4(b), the energies are in reasonably good agreement with those determined spectroscopically from radioactive nuclei. In this experiment, no assignments of spin and parity were given.

The work on K^{43} confirmed the shell model prediction for the ground state of K^{43}, spin 3/2 and even parity. Therefore the allowed beta transitions found there lead to the levels of even parity in Ca^{43}, shown in Fig. 4(a). On the basis of ft values and internal conversion coefficients, the 0.627-Mev level was given the assignment 5/2− or 3/2−.

The present experiment on Sc^{43} gives the levels shown in Fig. 4(c). The assignments of the levels have been discussed in the foregoing. The absence of transitions from Sc^{43} to the even parity states of Ca^{43} is accounted for both on energy and parity considerations.

No beta ray corresponding to a state at 0.369 Mev in Ca^{43} was found in the decay of K^{43}, although such a transition should have a character $\Delta j=\pm1,0$ yes and should easily compete with the transition to the 0.627-Mev state. This might have been missed on account of the low intensity of the transitions to all odd states.

In Fig. 4(d) is shown the level scheme obtained by combining the results of the two decay experiments. It is reasonable to consider that the low-lying, odd levels

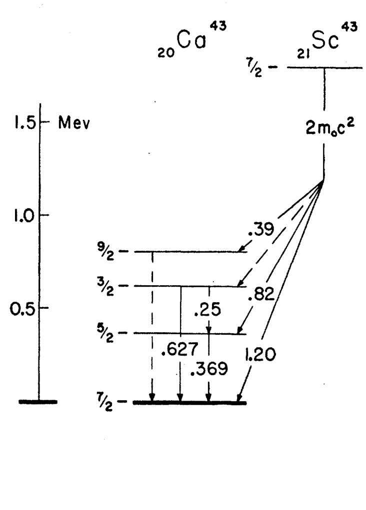

FIG. 3. Disintegration scheme of Sc^{43}.

[5] D. Kurath, Phys. Rev. **91**, 1430 (1953).
[6] C. M. Braams, Phys. Rev. **95**, 650 (1954).
[7] A. Bohr and B. R. Mottelson, Kgl. Danske Videnskal. Selskab, Mat.-fys. Medd **27**, No. 16 (1953).
[8] K. W. Ford (private communication).

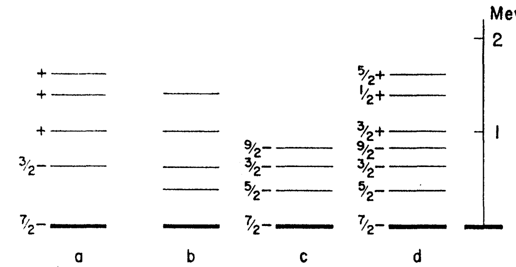

FIG. 4. Nuclear levels in Ca^{43}: (*a*) from K^{43} decay, (*b*) from Ca^{42} (d,p) Ca^{43}, (*c*) from Sc^{43} decay, (*d*) combined from (*a*) and (*c*).

arise from the $(f_{7/2})^3$ configuration. The even states of higher energy can be explained only by assuming a neutron from a lower shell being excited, thus giving a $(f_{7/2})^4$ configuration+a neutron hole in the preceeding shell. Therefore the even states have been given the spins 3/2, 1/2, and 5/2, in order of energy, as is shown in Fig. 4(*d*). This level order was chosen as the inverse of the observed order of filling of the states in the previous shell ($d_{5/2}$, $s_{1/2}$, $d_{3/2}$). There is, however, some possibility of inversions in the level order.

The results for the lower-energy states of odd parity can be compared with the theoretical calculations for the level energies of the configuration $(f_{7/2})^3$ obtained by Kurath.[5] His calculations are based on the "Majorana plus Bartlett" interaction and a strong spin-orbit coupling is assumed. In his calculations, no surface effects are considered. The energies of the states are given in units proportional to A_0, the Gaussian well-depth for the "Majorana plus Bartlett" interaction, and, as a function of ρ, the ratio of the nuclear size parameter to the range of nuclear forces. The experimental data were fitted to the theoretical curves by normalizing to the 0.81-Mev energy difference (7/2 to 9/2) in Ca^{43}, i.e., the largest energy difference determined experimentally. Table IV and Fig. 5 give this

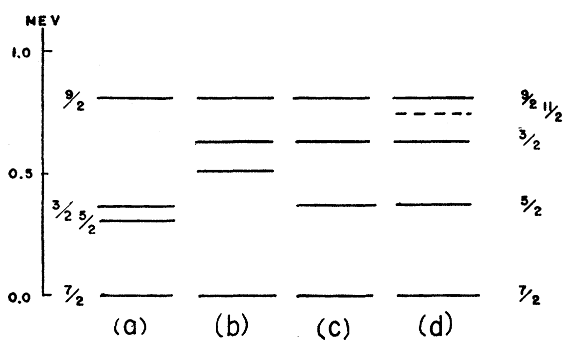

FIG. 5. Position of levels of Ca^{43}: (*a*) according to Kurath $(f_{7/2})^3$ without surface coupling $\rho=1.0$, (*b*) according to Kurath $(f_{7/2})^3$ without surface coupling $\rho=1.3$, (*c*) experimental, (*d*) with surface coupling (Ford); $\rho=1.35$, $A_0=28$ Mev, $\hbar\omega=15$ Mev.

comparison for several values of ρ. It will be seen that the spacing between the 5/2 and 3/2 states is small for any of the values of ρ chosen, while the experimental value is large. A fit can be obtained for the 7/2–5/2 difference for $\rho=1.0$ or for the 3/2–9/2 difference for $\rho=1.3$, but no fit can be found for all four levels for any value of ρ.

The discrepancy between the observed level spacing and the predictions of Kurath can be explained in terms of an additional weak coupling of the particles to the nuclear surface. The surface interaction depresses the 5/2 state relative to the 3/2. For sufficiently strong coupling, this may result in spin 5/2 for the ground state of the $(f_{7/2})^3$ configuration[7] (e.g., Mn^{55}). In the present example, only a weak coupling is expected because of the double-closed-shell core, and indeed only a very weak surface coupling is required to fit the observed level spacing. Ford[8] fits the observed levels with

TABLE IV. Comparison of experimental energy level spacings.

	Without surface coupling Kurath $(f_{7/2})^3$		Experimental	With surface coupling Ford	
	$\rho=1.0$	$\rho=1.3$	···	$\rho=1.35$	$\hbar\omega=15$ Mev
7/2	0	0	0	0	7/2
5/2	0.31	0.51	0.369	0.369	5/2
3/2	0.37	0.63	0.625	0.625	3/2
				0.74	11/2
9/2	0.81	0.81	0.81	0.81	9/2
				1.07	15/2
	$A_0=29$ Mev	$A_0=39$ Mev		$A_0=28$ Mev	

Kurath's potential depth $A_0=28$ Mev, range parameter $\rho=1.35$, and surface vibration energy $\hbar\omega=15$ Mev. The theoretical level diagram corresponding to this choice of parameters is shown in Fig. 5. Since three parameters are used to fit three excited states, it is not surprising that the theoretical diagram fits the observed levels exactly. A test of the model could be made, however, if the remaining two states of spins 11/2 and 15/2 could be observed. It should be noted that, since the coupling to the surface causes considerable shifts in the levels, the normalization to the experimental data was made using the 7/2–5/2 difference (0.375 Mev) rather than the 7/2–9/2 difference (0.81 Mev) employed in the comparison of the experiments with Kurath's theory.

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