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Electron Spin Magnetic Moment in Atomic Hydrogen*

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The ratio of the electron-spin g factor to the proton g factor has been measured in atomic hydrogen by a precision microwave magnetic resonance absorption technique. The result obtained is

$$-g_s/g_p = 658.2277 \pm 0.0002,$$

where g_p refers to protons in molecular hydrogen. The ratio is in good agreement with the atomic-beam result of Koenig, Prodell, and Kusch, and verifies their conclusions concerning the magnitude of the quantum electrodynamical contribution to g_s.

INTRODUCTION

THE evidence for an anomaly in the magnetic moment associated with electron spin began with a discrepancy in the hyperfine splitting in atomic hydrogen.[1] Breit[2] interpreted this as due to a small electron spin moment in addition to that given by the Dirac theory. Schwinger[3] showed that radiation processes in quantum electrodynamics predicted this addition and calculated the correction to the electron spin g factor g_s to the first order in the fine-structure constant. Direct g-factor measurements in one-electron atomic states[4] confirmed the calculated correction.

Although a number of observations with atomic systems have checked these results, no precision measurement of g_s for "free" electrons has been successful thus far, and at the present time measurement of Zeeman level displacements in atoms is the most accurate method of determining g_s. The calculation of g_s from the measured g_J of an atomic state involves corrections due to the purity of the atomic state and relativistic and internal diamagnetic effects. For the hydrogen atom these corrections are known with considerable accuracy, and at the present time they do not limit the accuracy with which g_s is known.

Because of the limited accuracy of magnetic field measurements, g_s is not measured directly in the atomic hydrogen experiments, but rather the ratio $g_s(1s\,^2S_{\frac{1}{2}}$ hydrogen$)/g($proton$)$ is determined and combined with the result of another experiment which determines $g($orbital electron$)/g($proton$)$. The atomic-beam experiment of the Columbia group[5] or the experiment reported here give the ratio g_s/g_p with an error of about one part per million. The Gardner-Purcell experiment[6] determines the ratio g_l/g_p with an error of 12 ppm so that g_s is known with the same error. With a tenfold increase in the accuracy of the Gardner-Purcell experiment, g_s will be known to one ppm, and the exactness of the quantum electrodynamical corrections to a second order in the fine-structure constant[7] can be assessed.

At the time the Columbia atomic hydrogen results were announced,[8] we had developed a microwave absorption experiment[9] for the same purpose. Work was continued and during the past year the experiments were completed. Our strongest reason for continuing with measurements which were not expected to improve significantly on the accuracy of the atomic-beam results was the desire to check an important constant by a different experimental method. At the same time, we could test our method against the highly developed techniques of atomic-beam magnetic resonance. The microwave absorption method developed in the course of this work promises to have general applicability in the

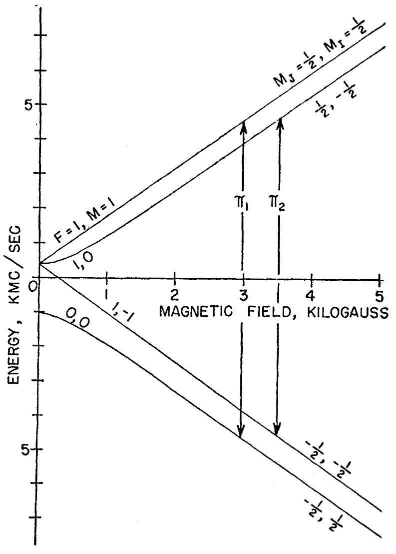

FIG. 1. Magnetic-field dependence of the hyperfine energy levels of the ground state of the hydrogen atom. The arrows indicating the microwave transitions are drawn to scale for the present experiment.

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[1] Nafe, Nelson, and Rabi, Phys. Rev. **71**, 914 (1947); Phys. Rev. **73**, 718 (1948); Nagle, Julian, and Zacharias, Phys. Rev. **72**, 971 (1947).

[2] G. Breit, Phys. Rev. **72**, 984 (1947); **73**, 1410 (1948).

[3] J. Schwinger, Phys. Rev. **73**, 416 (1948).

[4] P. Kusch and H. M. Foley, Phys. Rev. **74**, 250 (1948); and summary in reference 5.

[5] Koenig, Prodell, and Kusch, Phys. Rev. **88**, 191 (1952).

[6] J. H. Gardner, Phys. Rev. **83**, 996 (1951).

[7] R. Karplus and N. M. Kroll, Phys. Rev. **77**, 536 (1950).

[8] Koenig, Prodell, and Kusch, Phys. Rev. **83**, 687 (1951).

[9] R. Beringer and E. B. Rawson, Phys. Rev. **87**, 228 (1952).

measurement of g factors and hyperfine coupling energies in atoms. By avoiding beam detection problems it makes such measurements relatively simple; and the present work shows the accuracy to be comparable with the best atomic beam work.

The theory of the atomic-hydrogen experiment is treated in reference 5; we briefly review the reasoning. Suppose hydrogen atoms to be in a strong magnetic field which splits the two states, $M_J=\pm\frac{1}{2}$, by an energy corresponding to microwave frequencies. Let the frequency for the microwave transitions $M_J=\frac{1}{2}\rightarrow -\frac{1}{2}$ and the frequency for proton resonance be measured in the same field. The ratio of these two frequencies essentially determines g_J/g_p, and the accuracy of this ratio is independent of the rather large uncertainty in the proton moment itself. In the actual experiment the deduction of g_J/g_p is complicated by atomic hyperfine-structure. With the assumption that the ground state of atomic hydrogen is pure $^2S_{\frac{1}{2}}$, the level energies W, including magnetic hyperfine interactions, are given by the Breit-Rabi formula:

$$W\left(F=\begin{matrix}1\\0\end{matrix}, M_F\right)$$

$$=-\tfrac{1}{4}\Delta W+g_p\mu_0HM_F\pm\tfrac{1}{2}\Delta W(1+2M_Fx+x^2)^{\frac{1}{2}}$$

$$=-\tfrac{1}{4}h\nu_{\mathrm{H}}-h\nu_pM_F\pm\tfrac{1}{2}h\nu_{\mathrm{H}}(1+2M_Fx+x^2)^{\frac{1}{2}},$$

where

$$x=(g_J-g_p)\mu_0H/\Delta W=(1-g_J/g_p)\nu_p/\nu_{\mathrm{H}}.$$

$\Delta W=h\nu_{\mathrm{H}}$ is the zero-field hyperfine splitting of atomic hydrogen;[10] the field strength H is measured in units of proton resonance frequency where $h\nu_p=-g_p\mu_0H$, g_p refers to protons, and μ_0 is the Bohr magneton. The transitions $\Delta M_J=\pm1$, $\Delta M_I=0$ occur in the microwave region for moderate fields and are conventionally labeled π_1 and π_2, as shown in Fig. 1. Writing the transition energy as $h\nu$, one obtains the ratio g_J/g_p as a function of frequencies only. For the π_1 and π_2 transitions (upper and lower signs, respectively),

$$-\frac{g_J}{g_p}=\frac{\nu}{\nu_p}\left(\frac{\nu+\nu_p\mp\nu_{\mathrm{H}}\mp\frac{1}{2}\nu_p\nu_{\mathrm{H}}/\nu}{\nu+\nu_p\mp\frac{1}{2}\nu_{\mathrm{H}}}\right). \tag{1}$$

There are several diamagnetic effects associated with the interpretation of ν_p and g_p in addition to the bulk demagnetization effects which are discussed later. The first concerns g_p which, in the Breit-Rabi formula written above, refers to protons in atomic hydrogen. That is, the internal diamagnetic corrections to the field at the proton are included in the definition of g_p. The second diamagnetic effect arises from the use of protons in mineral oil for field calibration. A correction for this can be made exactly by substituting into (1) for ν_p the quantity

$$\frac{g_p(\text{atomic hydrogen})}{g_p(\text{oil})}\nu_p(\text{oil}),$$

in which case the ratio calculated in (1) is g_J/g_p(atomic hydrogen). A sufficiently accurate procedure is to ignore the differences in the parenthesis of (1) and to calculate g_J/g_p(oil) using the observed ν_p(oil). The same procedure can be used for other proton-containing samples.

MICROWAVE EXPERIMENT

The determination of g_J/g_p consisted of two parts, a microwave resonance experiment with atomic hydrogen followed by a proton resonance experiment with mineral oil in the same magnetic field. In our apparatus the magnetic field was controlled by the frequency of a proton resonance unit called the *regulator* which was used to vary, reset, and regulate the field in the magnet gap. It did not, however, directly measure the field in the region where atomic hydrogen resonance occurred since its proton coil could not occupy this region. The measurement of proton resonance in this region was accomplished with a second proton unit whose proton coil could be inserted into the microwave cavity in place of the atomic-hydrogen vapor at the conclusion of the microwave experiment. This unit was called the *simulator*.

The experiment used the method of magnetic resonance which was first developed by Rabi, Zacharias, Millman, and Kusch[11] for molecular-beam spectroscopy. Our microwave version was similar to the nuclear magnetic-resonance absorption method of Purcell, Torrey, and Pound[12] in that it detects transitions by the absorption of electromagnetic energy rather than by the trajectories of reoriented atoms. The apparatus was originally developed for the study of stable paramagnetic molecules.[13]

Microwave energy at about 9000 Mc/sec is generated by a klystron oscillator and coupled through a resonant cavity to a detector. The cavity is in a uniform magnetic field of a few thousand gauss and contains the atoms being studied, which must be in states of nonzero total electronic angular momentum. Each atomic state is split into its Zeeman levels which, for the fields used, are separated by microwave frequencies. The magnetic field is then varied until a level separation coincides with the cavity frequency. Absorption and stimulated emission occur and a net lowering of the power at the microwave detector results. The oscillator, detection system, and general experimental details are described in references 13.

In the present work the atomic hydrogen was pro-

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[10] A. G. Prodell and P. Kusch, Phys. Rev. **88**, 184 (1952), who obtained $\nu_{\mathrm{H}}=1420.4051\pm0.0002$ Mc/sec.

[11] Rabi, Zacharias, Millman, and Kusch, Phys. Rev. **53**, 318 (1938).

[12] Purcell, Torrey, and Pound, Phys. Rev. **69**, 37 (1946); Bloembergen, Purcell, and Pound, Phys. Rev. **73**, 679 (1948).

[13] R. Beringer, Annals N. Y. Acad. Sci. **55**, 814 (1952); R. Beringer and J. G. Castle, Jr., Phys. Rev. **78**, 581 (1950); **81**, 82 (1951); Beringer, Rawson, and Henry, Phys. Rev. **94**, 343 (1954).

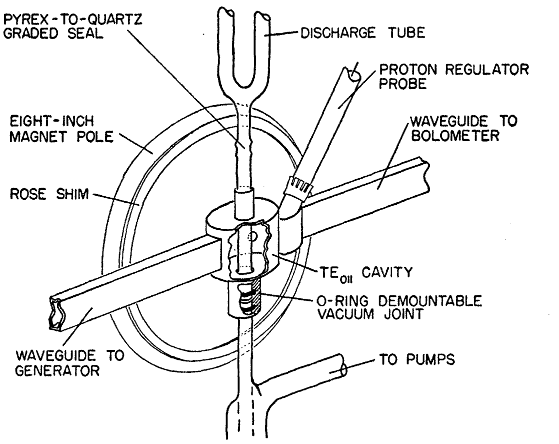

FIG. 2. Cutaway view of atom cavity in magnet gap. The wave guide to the bolometer is offset to permit centering of regulator proton-coil chamber in the magnet gap.

duced in an air-cooled Wood discharge tube.[14] The tube, of 15-mm o.d. Pyrex, was shaped like a narrow, upright U of total length 1.4 m with aluminum electrodes at each end. Tank hydrogen, bubbled through water, was passed through pressure dropping capillaries and led into the Wood tube near each electrode. From the bottom of the U a vertical tube of 13 mm o.d. passed down along the axis of the microwave cavity and coupled through a demountable O-ring vacuum joint to a liquid-air trap and the pumps. The essentials are shown in Fig. 2. The discharge with 20 000 Ω ballast resistance in series operated at a potential of about 2200 v at 50 ma.

The lower part of the discharge tube and the atom-vapor tube were coated with a layer of fused meta-phosphoric acid[15] which inhibits wall recombination of atomic hydrogen. This coating did not extend into the cavity. With a model tube similarly treated we measured atomic concentrations of 50 percent in the atom-vapor tube 20 cm from its connection with the discharge tube with flow rates of 10^{-4} g/sec. A recombination bolometer of tungsten wire was used to determine the atom concentration.

The gas system was designed for fast pumping of the atomic vapor from discharge tube to cavity. A small oil diffusion pump of speed 5 l/sec (with air at our operating pressures), backed with a mechanical pump, was connected to the atom-vapor tube through a high-conductance line and liquid-air trap. Under typical conditions the gas transit time from the base of the U to the cavity was estimated to be 10^{-2} sec. It would be possible to reduce this transit time to 10^{-3} sec without major change of the pumping system. The pressure at the cavity was of the order of 50 microns.

The cavity resonated in the TE_{011} mode which permits concentric holes in the end plates without seriously imparing the Q or perturbing the fields. To lower dielectric losses the axial tube was of fused silica. With the tube in place a typical cavity had an unloaded Q of 15 000 at 9200 Mc/sec and a transmission factor of 3.5 db. Cutoff wave guides surrounding the silica tube at top and bottom suppressed leakage of the cavity fields. Several cavities differing in details were used in the course of the experiments. They were constructed of commercial copper or silver-plated brass.

The arrangement of a TE_{011} mode cavity with an axial atom-vapor tube is particularly suitable for the experiment. The microwave electric field is low in the axial region while the microwave magnetic field is large, being distributed in space much like the magnetic field of a solenoid of length about one-half the cavity height. This field configuration gives good coupling of the microwave radiation to transitions of the $\Delta M_J = \pm 1$ type. It also makes it possible to determine the dc magnetic field in the same region of space with a simulator proton coil the dimensions of which are chosen to yield approximately the same distribution of rf magnetic fields as those in the microwave cavity. This is very desirable since the sampling of inhomogeneities in the magnet field must be closely similar in the atomic hydrogen and simulator experiments. Sampling differences can contribute important systematic errors in the experiment and we have gone to considerable effort to minimize them.

MAGNETIC FIELD

The electromagnet which provided the uniform magnetic field had ribbon-wound coils with edge cooling provided by water-cooled fins. The yoke, winding, and conical poles were designed for high fields (20 000 gauss). The poles tapered to 8-inch diameter at the air gap which was 2 inches long. The pole tips were ground and lapped flat. They rested loosely on the main poles with an oil film between. Final adjustment of the gap was made with brass jacks in the air gap which pressed the tips apart and could rock them about 10^{-4} inch on the oil film.

The tips were flat and parallel, but a raised edge or Rose[16] shim was provided for each. These shims could be slid around to obtain improved homogeneity at the point in the gap selected for the experiment. The shims were held to the pole tips by magnetic forces. Small pieces of nickel sheet were used as local shims.

Except for preliminary mechanical alignment of the pole tips all of the shimming was empirical. The simulator proton coil was placed in the cavity and the shims adjusted until the narrowest and most symmetrical proton signal was observed on the oscilloscope. The field distribution varied somewhat from day to day and hour to hour, so that the magnet was reshimmed for each run. While tedious, this procedure had merit in providing a rather random set of field inhomogeneities

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[14] R. W. Wood, Phil. Mag. **44**, 538 (1922).

[15] H. G. Poole, Proc. Roy. Soc. (London) **A163**, 404 (1937).

[16] M. E. Rose, Phys. Rev. **53**, 715 (1938).

for the various runs and hopefully a random error in g_J/g_p resulting from sampling differences between the microwave and the proton experiments.

The magnet was powered by 200-ampere-hour lead storage cells for the 3000-gauss range used in these measurements. The current was adjusted with a series-parallel combination of commercial slide-wire rheostats. In addition to the main coil, two high impedance pole-tip coils were provided. One supplied the magnetic-field modulation (30 cycles/sec) and the second was used for field regulation. The regulator unit provided a feedback signal for this second coil. The regulator proton coil was originally mounted close but not attached to the microwave cavity. In the later experiments the coil was contained in a shield which was rigidly attached to the cavity as close to the experimental region of the cavity as possible. The relative differential field between simulator and regulator proton coils was typically about 30 ppm.

The regulator and simulator units used a nuclear resonance circuit of our design. It is shown in Fig. 3. The chief point of interest is the separation of the oscillator and resonance-detector functions, with the proton coil appearing in the detector circuit. Most of the wide-frequency-coverage nuclear-resonance detectors which have been described put the proton coil in the oscillator tank. This leads to undesirable frequency modulation of the oscillator when resonances with "wiggles" are used. In our circuit the oscillator and detector tanks are tuned together with a dual condenser; small tracking errors are compensated with a trimmer capacitor C_1, and tracking is tested by a rotating capacitor C_2 which produces very slight frequency modulation of the oscillator. The trimmer is adjusted to give zero amplitude modulation output from this frequency modulation, thus assuring that the proton circuit is exactly tuned to the oscillator frequency. With this arrangement and alignment procedure we were assured that asymmetries in the proton resonance pattern were due to field inhomogeneities and we could take steps to reduce them. Other circuits tried were inferior in this respect and led to systematic errors in field measurement due to circuit unbalance. The circuit is quite dependable, easily constructed, and has a fairly good noise figure.

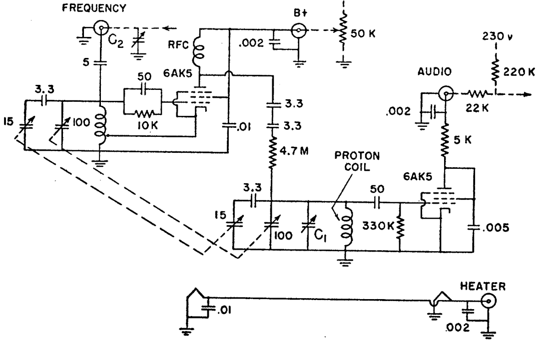

FIG. 3. Proton magnetic resonance absorption circuit with separated oscillator and detector stages.

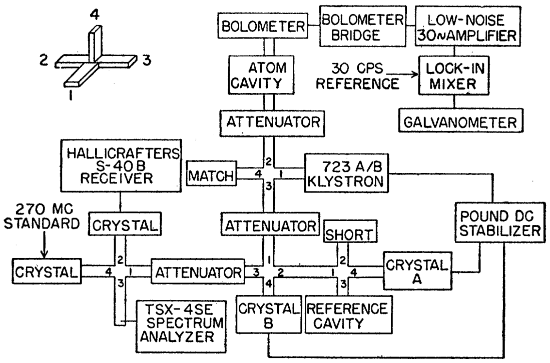

FIG. 4. Microwave magnetic resonance absorption spectrometer, showing frequency stabilization and measurement equipment, and detection system. The atom cavity is in a magnetic field, which is modulated at 30 cycles/sec.

The regulator generated its field-control signal by mixing the amplified proton resonance signal with a 30-cycle/sec reference signal in a lock-in mixer. The mixer output drove a push-pull dc power amplifier with the pole-tip coil as a cathode load. Suitable low-pass filters in the loop prevented oscillation. The feedback gain (gauss per gauss) was about 300, giving short-time (1 sec) field excursions of less than 0.005 gauss and long-time stability at a drift rate determined by the lead storage cells. The current through the pole-tip coil was kept near zero by periodic readjustment of the main magnet current. The over-all field stability and setting accuracy was about equal to the accuracy of centering the simulator resonance with respect to the sweep which was 0.2 ppm (0.0007 gauss).

FREQUENCY SYSTEM

The regulator and simulator proton units were both locked to accurately known frequencies. These frequencies were generated in mixers using appropriate frequencies from a standard frequency system. The microwave frequency used in the atomic hydrogen experiment was measured with the same system. Reference to Eq. (1) shows that g_J/g_p is almost independent of the absolute value of the frequency provided that ν/ν_p is accurately known. However, we periodically standardized our frequencies against radio station WWV to less than half a part per million.

The local standard was a 100-kc/sec temperature-regulated crystal oscillator.[17] This frequency was multiplied to 270 Mc/sec in the usual way by a chain of amplifiers with plate tanks tuned to harmonics of the grid drive. The 270-Mc/sec harmonics from a silicon-tungsten crystal were used as standard microwave frequencies. The cavities were designed to resonate near a 270-Mc/sec harmonic so that a radio receiver could be used to measure the frequency difference between the

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[17] Bliley model BCS-1A.

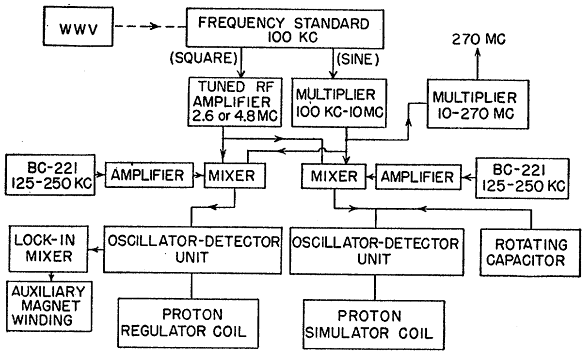

FIG. 5. Standard frequency and proton-resonance field-calibration systems. Both proton signals are displayed on oscilloscopes with coherent 30-cycle/sec sinusoidal sweep.

markers and the klystron oscillator. The receiver was calibrated with a BC-221 frequency meter. This microwave generating and frequency measuring equipment is shown in Fig. 4.

The proton regulator and simulator locking frequencies were generated by mixing the standard 10 Mc/sec with a suitable high-order harmonic of the 100-kc/sec standard and with the 125–250-kc/sec electron coupled oscillator output of a BC-221 frequency meter. Figure 5 shows this equipment. For best operation the line length from the rotating capacitor had to be adjusted so that its impedance variation at the oscillator was purely reactive. With other line lengths some undesirable amplitude modulation was introduced.

PROCEDURE AND RESULTS

An experiment involved the following steps. (a) The field homogeneity was adjusted using the mineral-oil simulator coil mounted in the cavity. Shims were moved until the best proton resonance pattern was obtained. (b) After suitable homogeneity was achieved, a "simulator run" was made. This consisted in setting the field with the proton regulator and tuning the simulator frequency until its pattern was centered on an oscilloscope. The oscilloscope was swept with the 30-cycle/sec field modulation signal. The simulator run consisted of about ten regulator settings, equally spaced in regulator proton frequency and centered at the resonance field for the microwave transition. (c) The simulator coil was then removed and the discharge tube put in place as shown in Fig. 2. Quick change vacuum and gas connections were made, the gas flow adjusted, and the discharge initiated. The microwave oscillator was tuned to the cavity resonance, its frequency measured, and a "microwave run" begun. This consisted in setting the regulator and noting the detected 30-cycle/sec signal output on a galvanometer. The magnetic field was varied point-by-point through the region of the atomic hydrogen resonance. A plot of a microwave run is shown in Fig. 6. (d) The discharge-tube system was quickly dismantled and another simulator run taken to verify step (b).

Throughout an experiment, including the time between the runs, the magnetic field was regulated and kept to values near the microwave resonance and was varied only enough to display the resonances. This control was maintained in order to minimize changes in the distribution of the field in the gap since we relied on the oscillator frequency of the regulator to reproduce magnetic field values in the experimental region of the cavity. Actually, there remained both systematic and random temporal drifts in this reproducibility, as indicated by changes in the frequency difference between regulator and simulator. Although these drifts were random in direction, their average value increased with time. Thus it was essential to take a set of runs, simulator, microwave, simulator, as rapidly as possible. With our apparatus the total elapsed time was at least 45 min.

In all, from the beginning of the experiments, forty-seven microwave runs and associated simulator runs were made. The three earliest runs used crude frequency and field control and were essentially exploratory. They are not included in this paper. The other forty-four runs began in December 1952, and extended through October 1953. During this period modifications were made in the cavities, simulator coils, regulator unit and its mounting, standard-frequency system, and the microwave-oscillator stabilization system. Three cavities were essentially as shown in Fig. 2 and one of these was used in runs which received ninety percent of the total weight; a fourth used reentrant metal tubes surrounding all but a short central region of the atom vapor tube in an attempt to reduce the volume of the experimental region and hence the total field inhomogeneity. Throughout the course of the experiments the data improved as judged by consistency and by the width of the microwave resonance line and the reproducibility of simulator runs. These improvements resulted from better mechanical design, more accurate field control, and reduced microwave power level. The latter largely eliminated saturation line-broadening which was present in the runs previous to June 4, 1953. In view of these improvements we adopted a set of weighting criteria for the data, but did not reject any data *a priori*. However, rejections were later made on statistical grounds.

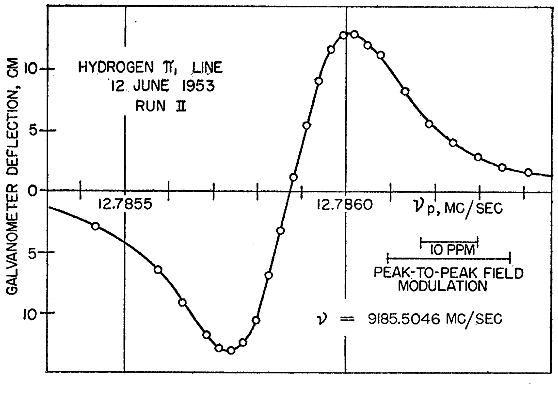

FIG. 6. Plot of the data of a good microwave run.

TABLE I. Summary of experimental data.

Run	Peak-to-peak field modulation (ppm)	Rf power at bolometer (mw)	Peak-to-peak microwave linewidth (ppm)	Assigned linewidth error (ppm)	Assigned simulator error (ppm)	Total normalized weight (%)	$-g_J/g_p$
Hydrogen π_1 line							
1952							
[a]Dec. 19	50	0.5	68	5.5	4.9	0.1	658.2324[g]
20 I	50	0.5	79	3.6	6.4	0.1	.2256[g]
II	50	0.5	74	3.6	6.4	0.1	.2221[g]
22 I	50	0.5	77	4.1	3.0	0.3	.2252[g]
II	50	0.5	74	3.7	3.0	0.4	.2236[g]
1953							
[b]Jan. 20	55	0.3	50	3.16	1.52	0.6	.2166
22 I	33	0.5	33	2.36	1.20	1.1	.2145
II	33	0.5	44	2.87	1.04	0.8	.2157
23 I	33	0.5	37	1.90	2.40	0.8	.2201
II	33	0.5	37	2.20	1.04	1.3	.2166
28 I	33	0.5	46	2.38	2.36	0.7	.2212[g]
II	33	0.5	42	2.15	2.36	0.8	.2220[g]
29 I	33	0.5	42	2.46	1.64	0.9	.2176
II	33	0.5	44	2.70	1.56	0.8	.2173
[c]Mar. 25	36	0.5	29	1.43	2.28	1.1	.2209[g]
[d]May 4 I	37	1.0	40	1.99	1.55	1.2	.2191
II	37	0.5	40	1.92	0.51	2.0	.2187
22	37	0.7	34	2.87	1.84	0.7	.2251[g]
June 3 I	37	1.0	38	2.80	1.32	0.8	.2241[g]
II	37	1.4	38	2.02	1.10	1.5	.2167
III	37	1.4	38	1.81	0.68	2.1	.2194
IV	37	0.35	30	1.55	0.65	2.8	.2171
4 I	26	0.33	23	1.17	1.33	2.5	.2151
II	26	0.10	22	1.07	1.33	2.7	.2152
III	26	0.18	23	1.11	0.93	3.8	.2188
IV	26	0.15	23	1.15	0.48	5.1	.2181
5	26	0.20	20	1.08	0.72	4.6	.2186
12 I	27	0.09	24	1.21	0.60	4.3	.2161
II	21	0.16	20	0.98	0.50	6.4	.2171
III	21	0.09	21	1.04	1.58	2.2	.2195
IV	21	0.13	20	0.99	0.48	6.4	.2179
Hydrogen π_2 line							
[e] 27	18	0.18	23	1.18	2.41	1.1	.2157
30	18	0.14	20	0.97	0.48	6.7	.2168
July 1	18	0.16	21	1.53	0.99	2.4	.2171
4	18	0.15	22	1.28	1.88	1.5	.2174
Sept. 1	27	0.07	26	1.27	1.11	2.8	.2172
2 I	27	0.20	26	1.28	0.91	3.2	.2162
II	27	0.20	27	1.31	0.35	4.3	.2170
9	27	0.23	27	1.32	1.76	1.6	.2168
17	27	0.17	23	1.09	1.26	2.8	.2175
22	23	0.17	20	1.04	1.36	2.7	.2184
26	23	0.17	21	1.03	0.77	4.7	.2175
29	23	0.17	22	1.09	0.73	4.6	.2171
[f]Oct. 3	24	0.17	21	1.03	1.40	2.6	.2172

[a] Cavity *A* (cylindrical) and matching simulator coil *A*.
[b] Cavity *A* modified, and simulator coil *B*.
[c] Cavity *B* (re-entrant sleeve) and simulator coil *C*.
[d] Cavity *C* (cylindrical) and simulator coil *D*.
[e] Simulator coil *E*.
[f] Simulator coil *F*.
[g] Data discarded in final calculation.

A microwave run was analyzed to yield g_J/g_p in the following way. A curve such as Fig. 6 was drawn and the line-center "cross-over" determined graphically. This yielded the field value for microwave resonance in units of the regulator frequency, which was then converted to simulator frequency by using the observed average difference frequency between the regulator and simulator. In obtaining this difference frequency account was taken of field-distribution changes between simulator runs and tracking errors of the two BC-221 meters. The simulator frequency thus obtained is ν_p. This, together with the measured microwave frequency ν was substituted into Eq. (1) to determine g_J/g_p where g_p refers to protons in the cylindrical mineral oil sample.

Table I gives the results for the forty-four runs. The observed line width was probably due primarily to inhomogeneity broadening. However, calculations indicated that the contributions due to wall and interatomic collisions and to Doppler effect were of nearly the same order of magnitude. Some saturation broadening was present in the early runs. The assigned errors appearing in Table I were based on two independent measures of

TABLE II. Summary of results by groups of runs.[a]

Group	Number of runs[b]	All data	Retained data
December	5(0)	658.2253±0.0015	...
January	9(7)	658.2177±0.0008	658.2168 ±0.0007
March	1(0)	658.2209±0.0018	...
May–June	6(4)	658.2196±0.0011	658.2186 ±0.0006
June	10	658.2175±0.0004	658.2175 ±0.0004
All π_1	31(21)	658.2180±0.0004	658.2175 ±0.0003
June–July	4	658.2168±0.0002	658.2168 ±0.0002
Sept.–Oct.	9	658.2172±0.0002	658.2172 ±0.0002
All π_2	13	658.2171±0.0002	658.2171 ±0.0002
All runs	44(34)	658.2176±0.0003	$658.2173_4+0.0001_9$

[a] The precision estimate shown is the statistical standard error in the weighted mean.
[b] The figure in parentheses is the number of runs retained in the final calculation.

precision. The first is a measure of the indeterminacy in the microwave resonance field contributed by the microwave run. This was computed from the observed line width, with additions from asymmetry in the line and from crossover indeterminacy caused by noise. A representative, symmetric line of 0.06 gauss "peak-to-peak" width (20 ppm in field, or the equivalent of 170 kc/sec in microwave frequency) was taken as an assigned error of 1.0 ppm. The second is a measure of the indeterminacy in the microwave resonance field contributed by the observed shift in the simulator-regulator field difference before and after a microwave run. This assigned error was increased for long time intervals between simulator runs; minor contributions were assigned from estimates of the mechanical stability of the apparatus in the magnet gap, the dependability of the alignment procedure for the proton-resonance circuits, and the care exerted in evaluating calibration errors and frequency drifts in the BC-221 frequency meters. In later runs, a typical observed shift in the regulator-simulator differential field of 0.004 gauss (1.3 ppm) for simulator runs one hour apart was taken as an error of 0.5 ppm. Suitable lower limits were imposed on the various assigned errors to prevent fortuitous runs from receiving undue weight. The weighting factors used were the reciprocals of the sum-of-squares of the two indices. The highest-weighted run contributed one-fifteenth of the total weight of the 44 runs. Measurement and control of the microwave frequency were sufficiently accurate to provide no contribution to the assigned error.

A summary of the weighted means and standard errors obtained from different groups of runs, representing major modifications in apparatus and procedure, is given in Table II. A weighted histogram of all runs appears in Fig. 7, together with the normal error curve of corresponding mean and standard deviation.

One immediately observes the large number of anomalously high values which were obtained. Generally these runs occurred among the earlier measurements, and consistently received low weights on the basis of the error indices just discussed. Nevertheless, we are unable to account for this obvious asymmetry of wide deviations. In view of the consistency of the more recent runs, we feel justified in rejecting these extreme values as statistically unacceptable, and further we believe that the effects which caused these extreme values were not present in the majority of the runs and do not influence the mean value of the remaining distribution. The value of g_J/g_p above which runs are rejected is somewhat arbitrary, but not very critical; we have chosen to discard the ten runs (5 percent of total weight) above the arrow in Fig. 7. A summary of the data remaining appears in the final column of Table II; the appropriate error curve is shown in Fig. 7.

We take as our best value,

$$-g_J/g_p(\text{mineral oil})=658.21734\pm0.00019,$$

where the proton resonance is observed in our finite-cylinder mineral oil samples. The error shown is the statistical standard error in the weighted mean.

DIAMAGNETIC AND RELATIVISTIC CORRECTIONS

In precision nuclear resonance experiments several corrections to the magnetic field at the nucleus must be considered if accurate values of nuclear moments are sought.[18] In this experiment, nuclear resonance is used as a magnetic field standard and the only relevant considerations are those of reproducibility of the standard. While mineral oil is a very convenient working standard, previous measurements have shown that there are significant variations in the diamagnetic shielding of the protons in different samples of mineral oil.[19] At

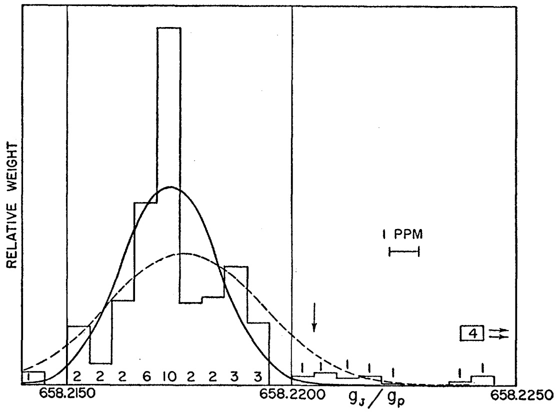

FIG. 7. Weighted histogram of *g*-factor ratio determinations. The number at the base of each column indicates the number of runs falling within that range. The dashed error curve is fitted before discarding data above the arrow; the solid curve, with maximum at 658.2173, after discarding.

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[18] W. C. Dickinson, Phys. Rev. **81**, 717 (1951).

[19] H. A. Thomas, Phys. Rev. **80**, 901 (1950); H. S. Gutowsky and R. E. McClure, Phys. Rev. **81**, 276 (1951). They obtain shifts from H_2 of 1.6 and 3.7 ppm, respectively, for spherical samples.

present, molecular hydrogen is probably the most reproducible standard for field measurements. It is also a substance for which reliable diamagnetic corrections have been calculated.[20]

We compared the frequency of proton resonance in a finite-cylindrical sample of our mineral oil (Squibb Heavy Californian Liquid Petrolatum) with that in molecular hydrogen at about 70 atmospheres pressure. This comparison was made in a thick-walled brass chamber fitted with a plunger by means of which the mineral oil sample (in a polystyrene capsule) could be alternately inserted in the proton-detector coil or withdrawn, leaving only H_2 in the rf field of the coil. The oil samples were of the same size and shape as in our simulator coils (length/diameter = 3.1). Thus the proton frequency shift between oil and H_2 can be applied directly to give g_J/g_p(spherical H_2). We measured, for our sample and geometry, a relative field shift,

$$\Delta H/H = (2.1\pm0.1)\text{ ppm}.$$

The sign is such that the field at the protons in oil is lower than in H_2 for a given external field.

Table III summarizes the numerical values we obtain for various relevant corrections. The entries for proton resonance in a finite-cylinder of mineral oil and for a spherical H_2 sample are direct measurements from our experiments. The entry for a spherical mineral oil sample is included for comparison with the Columbia results[5] and uses their value for the susceptibility of oil in making the bulk demagnetization correction. This correction was not measured by us and the g_J/g_p(spherical oil) ratio is less reliable than other entries in this table.

TABLE III. Summary of results for particular proton samples.

Proton resonance environment	Magnitude of shift from spherical H_2 (ppm)	$-g_J/g_p$	$-g_s/g_p$
Finite-cylinder mineral oil	$+2.1\pm0.1$[a]	658.2173[a] ±0.0002	
Spherical mineral oil	$+3.2\pm0.3$[b]	658.2181 ±0.0003	658.2298 ±0.0003
Spherical H_2	...	658.2160 ±0.0002	658.2277 ±0.0002
Unshielded protons	-26.6 ± 0.3[c]	658.1984 ±0.0003	658.2101 ±0.0003

[a] Primary experimental measurements.
[b] Directly comparable with the measurements cited in reference 19.
[c] Taken from Newell, reference 20.

The electron g-factor observed in this experiment is $g_J(^2S_{\frac{1}{2}}\,\mathrm{H})$. To obtain g_s for a free electron one must apply a relativistic correction on account of the binding of the electron in the hydrogen atom,[21]

$$\begin{aligned} g_J(^2S_{\frac{1}{2}}\,\mathrm{H})/g_s &= \tfrac{1}{3}[2(1-\alpha^2)^{\frac{1}{2}}+1],\\ &= 1-\alpha^2/3-\alpha^4/12-\cdots,\\ &= 1-17.75\times10^{-6}. \end{aligned}$$

This modification is included in the final column of Table III. A thorough discussion of other considerations in reducing g_J to g_s is given in reference 5.

DISCUSSION

The primary quantity measured by Koenig, Prodell, and Kusch[5] is

$$-g_J/g_p(\text{spherical mineral oil}) = 658.2171\pm0.0004.$$

(The precision estimate shown here with their result is again the statistical standard error.) For this same quantity we obtain from Table III,

$$658.2181\pm0.0003.$$

The difference of 1.5 ppm is consistent with the estimated precision of the two measurements, especially when allowance is made for possible differences between mineral oil samples.

Gardner's measurement[6] of

$$-2g_l/g_p = 657.475\pm0.004$$

was made using proton samples of water and mineral oil of presumably spherical geometry. Our g_s/g_p result most comparable to Gardner's conditions is 658.2298. Combining the two results, we obtain

$$\begin{aligned} g_s &= 2(g_s/g_p)(g_p/2g_l)\\ &= 2(1.001148\pm0.000006), \end{aligned}$$

taking g_l as unity and again expressing precision as a standard error. The value obtained by Karplus and Kroll from quantum electrodynamical calculations was[7]

$$g_s = 2(1.00114536).$$

The agreement is remarkable although unchanged, of course, from the results of Koenig, Prodell, and Kusch on account of the limited precision of Gardner's measurement.

The authors wish to acknowledge the assistance of Dr. E. B. Rawson in the development of the apparatus and techniques used here.

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[20] N. F. Ramsey, Phys. Rev. **78**, 699 (1950); G. F. Newell, Phys. Rev. **80**, 476 (1950).

[21] G. Breit, Nature **122**, 649 (1928); H. Margenau, Phys. Rev. **57**, 383 (1940); N. F. Mott and H. S. W. Massey, *Theory of Atomic Collisions* (Clarendon Press, Oxford, 1949), second edition, p. 72.