Theory of Electron Multiplication in Silicon and Germanium

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Using methods similar to those employed to treat the gas discharge, a theory is developed to explain the multiplication of electrons (and holes) in Si and Ge junctions. The calculations take into account the effects of electron-phonon and pair-producing collisions on the distribution function. In Si, the only element for which complete measurements have been made, the calculated ionization rate *versus* field curve is in agreement with experiment if one assumes a mean free path of 200A for interactions between electrons and optical phonons.

I.

A RECENT series of experiments by McKay and McAfee[1] have demonstrated that in $p-n$ junctions of Si or Ge it is possible to obtain charge multiplication similar to that observed below breakdown in a gas. In the high-field region of the junction the electrons and holes attain sufficient energy to create additional charge by electron-hole pair production. The multiplication obtained in this way is small for low reverse bias but grows rapidly with increasing voltage and eventually approaches a large value at the critical breakdown potential. This process, as McKay[2] emphasizes, is quite analogous to that taking place in a gas discharge, a fact which permits him to apply a considerable part of the gas discharge theory to the solid state case. In particular, by using the formulas of Townsend's[3] β mechanism, he derives values of the ionization rates per unit path length, α_i, from the data. The resultant curve of α_i *versus* field is one of the most significant features of his work since α_i, though it is used to describe the nature of the discharge, is a quantity which is determined by the properties of the material in which the multiplication takes place. It is the parameter which relates the microscopic solid state properties of the junction to the macroscopic characteristics of the breakdown taking place in it. A theoretical calculation of the α_i *versus* field ($\mathcal{E}$) curve would, therefore, be of special interest since from it one would not only obtain an understanding of breakdown in junctions, but could also infer something about the properties of the materials of which they are made. Such a program is the aim of the present work.

The actual calculation of α_i is straightforward and parallels rather closely similar ones which have been made for ionization rates in gases.[4] Basically, the method consists in solving Boltzmann's equation for the motion of electrons (or holes) in a high field, taking into account the effect of electron-phonon and pair-producing collisions on the distribution functions. Having once obtained the distribution function, it is then a simple matter to calculate α_i, as well as other properties of interest such as the drift velocity. The final expressions contain parameters typical of the solid under consideration, not all of which are known at present. Fortunately, however, it turns out that the only one of these that plays a critical role in determining α_i is the mean free path for electron-phonon scattering. Its value is obtained by matching the theoretical α_i *vs* $\mathcal{E}$ curve to experiment at a single point, the theory then being evaluated by how well the rest of the curve fits the data. As will be seen, reasonably good agreement is obtained in this way, the discrepancies possibly being explainable in terms of fluctuations in donor (or acceptor) density within the junctions.

II.

As outlined in the previous section, the main difficulty in calculating α_i is the solution of the Boltzmann equation for the electron (or hole) velocity distribution functions. However, before one can attack this problem it is necessary to decide what the forces and interactions are that can affect the state of motion of electrons or holes within the solid. In the present case these particles are moving in an electric field and will, of course, be accelerated by it. In addition, they interact with the lattice vibrations of the crystal, emitting and absorbing phonons in such a way that the net effect is a resistive or frictional one opposing the accelerating field. Finally, when the electrostatic field is large enough, they will reach sufficient energy to produce electron-hole pairs in collisions with valence electrons. The question of the exact value of the threshold for this process will be discussed later, but it is clear that it must be at least as great as the forbidden gap in the material in question, i.e., 1.0 ev for Si and 0.7 ev in Ge. Thus pair production and multiplication will only occur when an electron has an appreciable chance of having energy of 1 ev or more. At energies such as these the coupling between electrons and the lattice is quite different from that for slow (thermal) electrons. Experiments of Ryder and Shockley[5] indicate that the fast electrons lose energy principally to the optical modes of the lattice. Few details as

[1] K. G. McKay and K. B. McAfee, Phys. Rev. **91**, 1079 (1953).
[2] K. G. McKay, Phys. Rev. **94**, 877 (1954).
[3] L. B. Loeb, *Fundamental Processes of Electrical Discharge in Gases* (John Wiley and Sons, Inc., New York, 1939).
[4] See, for example, W. P. Allis and Sanborn C. Brown, Phys. Rev. **87**, 419 (1952).
[5] E. J. Ryder and W. Shockley, Phys. Rev. **81**, 139 (1951).

yet are known about this interaction, but the theoretical work of Shockley,[6] as well as that of Seitz,[7] suggests that it is most reasonable to choose a constant mean free path for collisions between electrons and optical phonons. This assumption will be made in the succeeding work of this paper. Actually, the form of the α_i *vs* $\mathcal{E}$ curve appears not to depend critically upon it so that a small velocity dependence of the mean free path should not invalidate the results.

Another question of importance is that of the mean free path for hole-electron pair production. Fortunately, as will become clear later, the exact value of this quantity is not necessary in calculating α_i, it only being required that it be small compared to the mean free path for electron-phonon interaction. For the latter process the value needed to fit the experiments turns out to be about 200A. On the other hand, an estimate of the mean free path for pair production gives 15A for an electron energy 0.5 ev above threshold. This figure is obtained by treating the ionizing collision as a Coulomb scattering, the total cross section being found by integrating Rutherford's formula over values of the scattering angle such that energy greater than the gap width is transferred in the interaction. This value is also a reasonable one from the experimental point of view. Johnson and McKay[8] have shown that secondary electron emission from Ge is similar to that from metals, and for a metal a mean free path for internal secondaries of 15A is about right. Thus the criterion that the electron-phonon mean free path be long compared to that for pair production is satisfied, provided the electron energy is at least a few tenths of an electron volt above threshold. Right at threshold, of course, the pair cross section is zero, but it rises rapidly (quadratically if one counts available phase space) with increasing energy and, as the above figures show, soon surpasses that for electron-phonon scattering.

III.

The next step in the calculation of α_i is the solution of the Boltzmann equation. Since the cross section for pair production is large, this process will have a big effect on the distribution function. This fact will necessitate different treatments in the regions above and below the pair production threshold. In the low-energy region the velocity distribution is simple and can be expressed in terms of tabulated functions. For higher velocities the situation is more complicated, and it is only possible to obtain an explicit solution of the Boltzmann equation for energies far enough above the pair threshold that the mean free path is small compared to that for phonon scattering. The intermediate region is quite difficult and would require the use of numerical techniques. Considering the many uncertainties inherent in this problem such effort hardly seems warranted. Instead, the high- and low-velocity solutions will be extrapolated into the rather narrow intermediate range and there matched. Figure 1 illustrates in a qualitative way the sort of procedure that will be used. This method, in effect, replaces the problem described above by a slightly different one in which the pair-production cross section has a threshold about 0.2 ev higher than formerly but also has a finite value there. The shape of the α_i *vs* $\mathcal{E}$ curve is mainly determined by the ability of the electrons to overcome the phonon drag and reach the threshold. This process will be the same in both these cases so they should give substantially the same results for α_i. Of course, if really accurate values of α_i ever become necessary it is always possible to carry out the numerical integration of the differential equations.

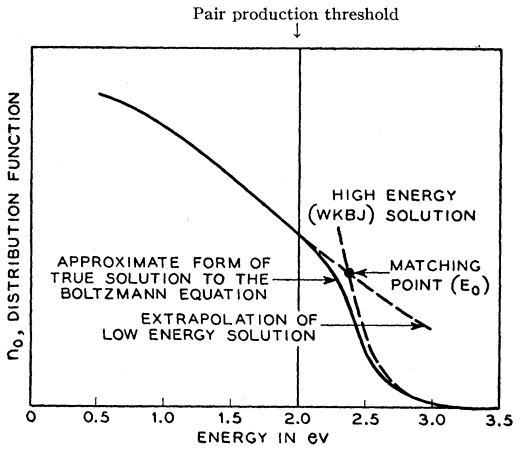

FIG. 1. Approximate form of electron energy distribution.

From Wannier's[9] paper on high-field mobilities it is evident that the solution of the Boltzmann equation is generally difficult. The expansion of the distribution function, which depends on the magnitude and direction of the electron velocity, into spherical harmonics leads to an infinite set of coupled differential equations which is very hard to handle. However, in certain limiting cases the problem simplifies considerably. For instance, if the charges lose only a small fraction of their energy at each collision the velocity distribution is nearly spherically symmetric and only the first two Legendre polynomials are needed in its expansion. The Boltzmann equation then reduces to a pair of fairly manageable, coupled differential equations. The solid-state breakdown being discussed here satisfies this criterion since, as was mentioned earlier, to obtain appreciable multiplication the average electron (or hole) energy must be approximately 1.0 ev, whereas the energy of the optical phonons is of the order of 0.1 ev.

Within the approximation mentioned above, the velocity distribution function can be written in the form

$$n(c,\theta)=n_0(c)+n_1(c)\cos\theta, \tag{1}$$

[6] W. Shockley, Bell System Tech. J. **30**, 990 (1951).
[7] F. Seitz, Phys. Rev. **73**, 550 (1948).
[8] J. B. Johnson and K. G. McKay, Phys. Rev. **93**, 668 (1954).
[9] G. H. Wannier, Bell System Tech. J. **32**, 170 (1953).

where c is the magnitude of the electron or hole velocity and θ is the angle it makes with the electric field. When the electron energy is below the pair-production threshold, n_0 and n_1 are solutions of the equations

$$\frac{a}{3}\left(\frac{dn_1}{dc}+\frac{2n_1}{c}\right)=\frac{-n_0}{\tau}+4\pi\int F_0(c,c')\frac{n_0(c')}{\tau(c')}dc' \quad (2)$$

and

$$a\frac{dn_0}{dc}=-\frac{n_1}{\tau}+4\pi\int F_1(c,c')\frac{n_1(c')}{\tau(c')}dc'. \quad (3)$$

Here $a=e\mathcal{E}/m$ is the acceleration, τ the collision time, and

$$F_n(c,c')=\int P_n(\cos\theta)F(c,c',\cos\theta)d\Omega, \quad (4)$$

where $F(c,c',\cos\theta)$ is the probability that in an electron-phonon interaction an electron of velocity c' is scattered through the angle θ with final velocity c. The pair of equations, (2) and (3), are just those given in Wannier's paper for the motion of electrons in a gas.

At room temperatures in Si or Ge very few optical phonons are excited, so an electron will almost always lose energy in an interaction with the lattice vibrations. $F(c,c',\cos\theta)$ then takes the form

$$F(c,c',\cos\theta)=\delta(c-(c'^2-2\hbar\omega/m)^{\frac{1}{2}})g(c,\theta)/c^2, \quad (5)$$

where $\int g(c,\theta)d\Omega=1$, δ is the Dirac delta function, and ω is the frequency of the optical mode which, in the following, will be assumed constant. $F_0(c,c')$ is now given by

$$F_0(c,c')=\delta(c-(c'^2-2\hbar\omega/m)^{\frac{1}{2}})/c^2, \quad (6)$$

and Eq. (2) becomes

$$\frac{a}{3}\left(\frac{dn_1}{dc}+\frac{2n_1}{c}\right)=-\frac{n_0}{\tau}+\int\delta\left(c-\left(c'^2-\frac{2\hbar\omega}{m}\right)^{\frac{1}{2}}\right)\times\frac{n_0(c')}{\tau(c')}\frac{c'^2dc'}{c^2}. \quad (7)$$

After performing the c' integration, advantage may be taken of the fact that $2\hbar\omega/mc^2$ is generally small compared to unity by making a power series expansion in this parameter. The result, to lowest order, is

$$\frac{a}{3}\left(\frac{dn_1}{dc}+\frac{2n_1}{c}\right)=\frac{\hbar\omega}{mc^2}\left[c\frac{d}{dc}\left(\frac{n_0}{\tau}\right)+\frac{n_0}{\tau}\right]. \quad (8)$$

By multiplying through by c^2, this equation can be written in the form

$$\frac{a}{3}\frac{d}{dc}(c^2n_1)=\frac{\hbar\omega}{m}\frac{d}{dc}\left(\frac{cn_0}{\tau}\right). \quad (9)$$

To the same order of approximation, i.e., neglecting $2\hbar\omega/mc^2$ compared to 1, Eq. (3) becomes

$$adn_0/dc=\langle\cos\theta-1\rangle n_1/\tau, \quad (10)$$

where

$$\langle\cos\theta-1\rangle=\int g(c\theta)(\cos\theta-1)d\Omega. \quad (11)$$

Equation (9) integrates immediately to give the relation

$$\tfrac{1}{3}ac^2n_1=(\hbar\omega cn_0/m\tau)+\text{constant}. \quad (12)$$

Substituting into Eq. (10) gives the differential equation

$$\frac{dn_0}{dc}+\left[\frac{3\langle1-\cos\theta\rangle\hbar\omega}{mca^2\tau^2}\right]n_0=\frac{\text{const}\langle\cos\theta-1\rangle}{a\tau c^2}. \quad (13)$$

This equation has the integrating factor

$$\exp\left[\int\frac{3\langle1-\cos\theta\rangle\hbar\omega}{ma^2\tau^2}\frac{dc}{c}\right], \quad (14)$$

with which it is possible to construct the general solution, but the result is so complicated as to be almost worthless for practical purposes. At this point, therefore, the approximation of constant mean free path will be made ($\tau=\lambda/c$). In addition, the velocity dependence of the factor $\langle1-\cos\theta\rangle$, which should not be great, will be neglected. With these simplifications the solution of Eq. (13) is

$$n_0=Ae^{-E/W}+\tfrac{1}{2}Be^{-E/W}\mathrm{Ei}(E/\omega), \quad (15)$$

where

$$W=\frac{(e\mathcal{E}\lambda)^2}{3\langle1-\cos\theta\rangle\hbar\omega}, \quad (16)$$

and A and B are constants of integration. The Ei function used here is that defined and tabulated in the WPA tables.[10]

A qualitative understanding of why W is the important parameter can be obtained by considering the average rate of energy gain by electrons in a spherically symmetric velocity distribution. After being accelerated for a time τ, an electron with initial velocity $c\cos\theta$ along the field will have a speed $c\cos\theta+e\mathcal{E}\tau/m$ in this direction. The average energy gain in time τ, obtained by squaring this expression and averaging over θ, is then $(e\mathcal{E}\lambda)^2/2mc^2$. On the other hand, an electron loses energy $\hbar\omega$ in this time. At equilibrium the rates of energy gain and loss must be equal, which gives the condition

$$(e\mathcal{E}\lambda)^2/4E\sim\hbar\omega, \quad \text{or} \quad E/W\sim1, \quad (17)$$

which is exactly the result one would obtain from a consideration of the distribution function, n_0. From this argument one sees that the form of the velocity distribution and the average energy of the electrons are determined by a competition between the process of

[10] *Tables of Sine, Cosine, and Exponential Integrals* (Work Projects Administration for City of New York, New York, 1940), Vol. 1.

energy gain from the field and that of energy loss to the phonons.

IV.

In the high-velocity region, the equations which determine the distribution function are

$$\frac{a}{3}\frac{d}{dc}(n_1c^2)=\frac{\hbar\omega}{m\lambda}\frac{d}{dc}(c^2n_0)-\frac{c^2n_0}{T}, \tag{18}$$

and

$$an_0'=-\langle 1-\cos\theta\rangle\frac{cn_1}{\lambda}-\frac{n_1}{T}, \tag{19}$$

where T is the mean free time for pair production. With the exception of the terms involving T, they are identical with Eqs. (9) and (10). These terms describe a continual loss of high-energy electrons. For each electron so lost it is assumed that another enters the distribution at zero velocity, thus keeping the total number of particles constant. Of course, this is somewhat of an oversimplification, since, in producing a pair, an electron will not drop exactly to zero energy, although, almost invariably, it will have quite a small energy after such an ionization. The gas discharge calculations[4] have shown that such an approximation hardly changes the values of ionization rate that one calculates. Its only effect is to cause an unimportant singularity in the distribution function at $c=0$.

As was pointed out earlier, the solution of Eqs. (18) and (19) for all values of c is very difficult because of the presence of the velocity dependent term $1/T$. This is particularly true near the threshold for pair production, where the percentage change in $1/T$ with velocity is large. For higher values of c, however, the situation is simpler, since the $1/T$ term changes fairly slowly and also becomes big enough to control the behavior of n_0 and n_1. In this range both n_0' and n_1' are very large, so that in terms such as $(d/dc)(c^2n_1)$ it is permissible to neglect the derivative with respect to c^2 since $cn_1'/n_1\sim c/aT\gg 1$ if one uses values of the mean free path for pair production estimated in Sec. II and field strengths below 10^6 volts/cm. With this approximation Eq. (18) becomes

$$\frac{a}{3}n_1'=\frac{\hbar\omega}{m\lambda}n_0'-\frac{n_0}{T}. \tag{20}$$

The other equation is

$$an_0'=\langle\cos\theta-1\rangle\frac{cn_1}{\lambda}-\frac{n_1}{T}\cong-\frac{n_1}{T}. \tag{21}$$

Differentiating gives the relation

$$n_1'\cong-aTn_0''-aT'n_0', \tag{22}$$

which, to the same approximation as used above, can be written

$$n_1'\cong-aTn_0''. \tag{23}$$

Combining this result with Eq. (20) then gives

$$-\frac{a^2}{3}Tn_0''=\left(\frac{\hbar\omega}{m\lambda}\right)n_0'-\frac{n_0}{T}. \tag{24}$$

This equation has a solution of the form $n_0=e^S$ where, because of the slow percentagewise change in $1/T$ for energies more than a few tenths of an electron volt above threshold, $S''<(S')^2$. Using the quadratic threshold law discussed at the end of Sec. II, one estimates that this approximation is valid for energies greater than 0.3 ev above threshold. In this range S is then determined by the equation

$$-\frac{a^2T}{3}(S')^2-\frac{\hbar\omega}{m\lambda}(S')+\frac{1}{T}=0. \tag{25}$$

From this formula S' is found to be

$$S'=-\Delta/T, \tag{26}$$

where

$$\Delta=\frac{1}{2}\left\{\frac{3\hbar\omega}{ma^2\lambda}+\left[\frac{9}{a^4}\left(\frac{\hbar\omega}{m\lambda}\right)^2+\frac{12}{a^2}\right]^{\frac{1}{2}}\right\}. \tag{27}$$

The quantity n_0 is then given by the relation

$$n_0=D\exp\left(-\Delta\int_{c_0}^{c}\frac{dc}{T}\right), \tag{28}$$

where c_0 is the velocity at the matching point and D an integration constant. It will be recognized that the method of solution used here is entirely analogous to the WKBJ method, T/Δ representing a large and relatively slowly varying "wavelength."

Having found forms for the velocity distribution, the final step in the calculation of α_i is the matching of these solutions at $c=c_0$. The constants A, B, and D must be determined such that n_0 and n_1 are continuous through the matching point and such that the total number of particles is unity. Calling the two independent solutions in the low-velocity region $\psi_1(c)$ and $\psi_2(c)$, respectively, one obtains the following set of algebraic equations to determine A, B, and D:

$$\begin{aligned}&A\psi_1(c_0)+B\psi_2(c_0)-D=0,\\ &A\psi_1'(c_0)+B\psi_2'(c_0)+Ac_0D/\lambda=0,\\ &A\int_0^{c_0}c^2\psi_1(c)dc+B\int_0^{c_0}c^2\psi_2(c)dc=1.\end{aligned} \tag{29}$$

The contribution of the high-velocity region to the normalization has been neglected in the last equation since n_0 falls very rapidly in this range (see Fig. 1).

The ionization rate per unit time, I, is given by

$$I=\int_{c_0}^{\infty}\frac{c^2n_0dc}{T}, \tag{30}$$

which, from Eq. (18), equals

$$I=\frac{\hbar\omega}{m\lambda}c_0^2n_0(c_0)-\frac{a}{3}c_0^2n_1(c_0). \tag{31}$$

Using formulas (10) and (15), this expression becomes

$$I=\frac{a^2\tau c_0}{3\langle 1-\cos\theta\rangle}B=\frac{a^2\lambda}{3\langle 1-\cos\theta\rangle}B. \tag{32}$$

α_i, the ionization rate per unit distance travelled along the field, is obtained by dividing I by the drift velocity, $\bar{c}$. This quantity is given by

$$\bar{c}=\frac{1}{3}\int n_1c^3dc=-\frac{1}{3}\int\frac{\lambda a n_0'c^2dc}{\langle 1-\cos\theta\rangle}, \tag{33}$$

where, for simplicity, the small number of electrons having velocity above c_0 is neglected in evaluating $\bar{c}$. Integrating by parts then yields

$$\bar{c}=\frac{2}{3}\int\frac{\lambda a n_0 c dc}{\langle 1-\cos\theta\rangle}, \tag{34}$$

or

$$\bar{c}=\frac{2}{3}\int_0^{c_0}\frac{\lambda a c}{\langle 1-\cos\theta\rangle}[A\psi_1(c)+B\psi_2(c)]dc. \tag{35}$$

A and B are given by the following expressions obtained from Eq. (29):

$$A=\frac{1}{(\text{det.})}\left(\frac{\psi_2(c_0)\Delta c_0\langle 1-\cos\theta\rangle}{\lambda}+\psi_2'(c_0)\right), \tag{36}$$

$$B=\frac{-1}{(\text{det.})}\left(\frac{\psi_1(c_0)\Delta c_0\langle 1-\cos\theta\rangle}{\lambda}+\psi_1'(c_0)\right).$$

The factor (det.) is the determinant of the coefficients in Eqs. (29). Both of these formulas can be simplified by noting that

$$\Delta c_0\langle 1-\cos\theta\rangle/\lambda=\left(\frac{\Omega E_0}{Wc_0}\right),$$

where Ω is the dimensionless parameter

$$\Omega=[1+(1+4\langle 1-\cos\theta\rangle W/\hbar\omega)^{\frac{1}{2}}], \tag{37}$$

and W is defined in Eq. (16). The following expressions for A and B are obtained:

$$A=\frac{1}{c_0}\left[\frac{E_0}{W}(\Omega-2)\psi_2(c_0)+1\right]\frac{1}{(\text{det.})}, \tag{38}$$

$$B=-\frac{(\Omega-2)E_0\psi_1(c_0)}{Wc_0(\text{det.})}. \tag{39}$$

The formula for α_i now takes a form which is amenable to numerical calculation, namely,

$$\alpha_i=\tfrac{1}{2}aB\Big/\left[A\int_0^{c_0}c\psi_1(c)dc+B\int_0^{c}c\psi_2(c)dc\right]. \tag{40}$$

Notice that to evaluate α_i it is not necessary to calculate the denominator of A or B since this term enters in the same way in I and $\bar{c}$ and thus cancels in the expression for α_i. With Eq. (40) the calculation for α_i is complete and it is only necessary to compute to obtain its values. These will be presented and discussed in the next section.

V.

Before using Eq. (40) to evaluate α_i it is necessary to choose a value for E_0, the energy at which the high and low velocity solution of the Boltzmann equation are matched. The discussion of Sec. II indicates that E_0 should be taken a few tenths of an electron volt greater than the threshold for pair production. Unfortunately, however, this threshold is a quantity whose value is difficult to ascertain with any accuracy. It is true that if one assumes spherical, nondegenerate energy bands with equal masses for electrons and holes that the conservation laws for energy and crystal momentum require that the threshold should fall at one and a half times the gap width, but the band structures in Ge and Si are certainly more complicated than this and, in addition, such a calculation takes no account of umklapp or phonon processes. The latter certainly permit some pair production at somewhat lower energies but they probably have little importance in determining the energy of principal importance here, namely the energy at which the mean free path for pair production becomes smaller than that for the electron-phonon interaction. On the other hand, the form of the energy bands has a marked influence on this value as can be seen by repeating the energy-momentum argument mentioned above with different band shapes or different effective mass ratios. In silicon (the element on which most of McKay and McAfee's work has been done) the shape of the energy bands is in doubt. Calculations of Herman and Callaway[11] suggest that the valence band has a triply degenerate maximum at $k=0$ and that the conduction band edge occurs away from $k=0$, but the details of this structure are still obscure. This lack of knowledge makes any estimate of the pair-production threshold quite risky, but calculations using the Herman-type bands with the minimum in the conduction band halfway to the edge of the Brillouin zone suggest that, unless the effective masses have rather special values, the threshold obtained from the conservation laws will be somewhat greater than one and a half times the gap width, perhaps somewhat over 2.0 ev in Si. Clearly, this figure is not a very reliable one but, without more information about the bands in

[11] Frank Herman and Joseph Callaway, Phys. Rev. **89**, 518 (1953).

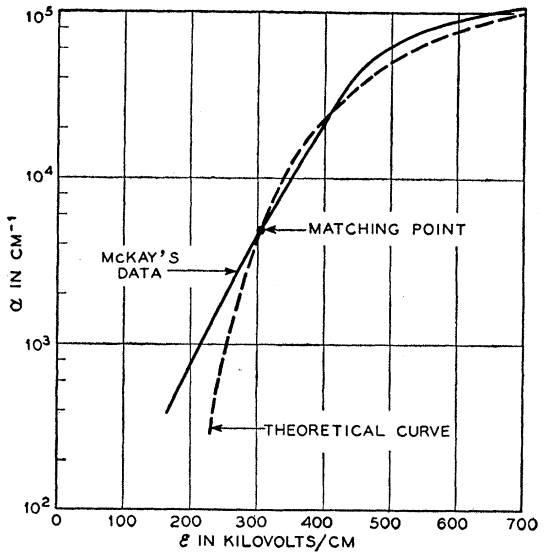

FIG. 2. Ionization rate *versus* electric field for silicon.

Si, it is hard to do better. The value of over 2.0 ev seems high but there is a certain amount of experimental evidence to support it. McKay and McAfee[1] have done experiments in which they bombarded Si with α particles and observed the amount of energy required to produce a hole-electron pair. The result was 3.6 ev per pair, a surprisingly large figure compared to the gap width of 1.0 ev, suggesting that the pair-production threshold is, indeed, abnormally high in silicon. As was mentioned earlier, the value of E_0 is a few tenths of an electron volt greater than the pair threshold and consequently will be taken to be 2.3 ev. The shape of the α_i *vs* $\mathcal{E}$ curve is not too dependent on this value of E_0, but a change in it would affect the value of λ deduced from the experiments.

Using the value for E_0 of 2.3 ev, the best fit of Eq. (40) to experiment is obtained with a mean free path, λ, of about 200A. This value is uncertain mainly because of lack of knowledge of E_0. An estimate of what sort of error to expect can be made by fitting to experiment with various values for E_0. Outside limits for this quantity are $1.5 \text{ ev} < E_0 < 3.5$ ev from which one obtains the mean-free-path range, $160\text{A} < \lambda < 260\text{A}$.

The α_i *versus* $\mathcal{E}$ curve is shown in Fig. 2. Agreement between theory and experiment is good in the high-field region but considerably poorer for lower values of $\mathcal{E}$.* Experimental error is large in this region but, in addition, there may be another factor which makes the observed α_i values larger than those calculated. In the experiments of McKay and McAfee it was assumed that the electric field could be calculated from the average donor and acceptor densities. However, small local fluctuations in these quantities cause considerable deviations of the field from the mean, and, since α_i varies so rapidly with $\mathcal{E}$, it is possible that the observed α_i values are typical of an electric field that is larger than the average. Unfortunately, any quantitative estimate of this effect is exceedingly difficult since variations in donor density, besides being caused by statistical fluctuations, may also be produced during the growing of the junctions.

Although there is no simple expression for α_i as a function of $\mathcal{E}$, the numerical calculations show that the general form of the curve is determined by the factor $e^{-E/W}$ which appears in Eq. (15). Physically this means that α_i is limited by the inability of electrons to go rapidly from low to high energy against the drag of the phonon field. In the neighborhood of $\mathcal{E}=6\times10^5$ volts/cm, however, the field becomes large enough that there is an appreciable chance for an electron to be accelerated from zero velocity to the pair-production threshold before it makes a collision with a phonon. At this point the phonon field ceases to play an important role and α_i is approximately given by the inverse of the distance an electron has to travel in the electric field to go from zero energy to 2.3 ev. In this high-field region the variation of α_i with $\mathcal{E}$ is linear as compared with the exponential dependence at lower $\mathcal{E}$. This fact explains why the α_i curve rises rapidly for low $\mathcal{E}$ and then appears to saturate for field strengths in the neighborhood of 6×10^5 volts/cm.

VI.

In the preceding sections of this paper, gas discharge theory has been used to describe the breakdown process in Si and Ge junctions. These calculations explain the course of the α_i *vs* $\mathcal{E}$ curve and thus support the interpretation McKay has placed on his data as well as giving one confidence in the application of gas theory to the solid-state discharge. A by-product of this investigation is the value of the mean free path for electron-phonon scattering in Si at 2.3 ev. The figure of 200A quoted in the text is uncertain mainly because of lack of knowledge of E_0 and could be in error by about 30 percent. Finally, it is suggested that certain deviations of theory from experiment are explainable in terms of fluctuations in donor or acceptor density within the junctions.

In conclusion the author would like to express his thanks to Dr. K. G. McKay and Dr. D. J. Rose with whom he has had numerous fruitful conversations on the topics set forth here. He would also like to express his thanks to them and to Conyers Herring for reading and commenting on the manuscript.

* *Note added in proof.*—In a private conversation Dr. McKay has informed the author that his latest experiments give somewhat smaller values of α_i at low fields than those plotted in Fig. 2. This change will considerably improve the agreement between theory and experiment.