Total Cross Section for Charge Exchange Scattering of π^- Mesons by Hydrogen at 42, 30, and 20 Mev*

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A measurement of the total cross section for the reaction $p(\pi^-,\pi^0)n$ has been made at three energies using polyethylene and carbon targets in a difference experiment. The values obtained for the cross sections are $\sigma(42\text{ Mev})=6.9\pm1.2$ mb; $\sigma(30\text{ Mev})=5.7\pm0.9$ mb; and $\sigma(20\text{ Mev})=5.0\pm0.8$ mb. The results are analyzed under the assumption of charge independence in an attempt to determine the energy dependence of the S-wave phase shifts.

INTRODUCTION

CROSS sections for charge-exchange scattering of π^- mesons by hydrogen, $(\pi^-+p\rightarrow\pi^0+n)$, have been measured at 120 and 144 Mev by Anderson *et al.*[1] and at 65 Mev by Bodansky *et al.*[2] The total cross section has also been measured at 34 Mev by Roberts and Tinlot.[3] This experiment was a continuation of the latter work with the intent of measuring the energy dependence of the total cross section at the mean meson energies of 42, 30, and 20 Mev in the laboratory system.

Both single and double photon coincidences were detected from the π^0 decay, $(\pi^0\rightarrow2\gamma)$, but the total cross sections were computed from the much larger yield of single photons. At these meson energies the resultant single-photon angular distribution is only slightly affected by the initial π^0 angular distribution, and this permits measuring the total cross section without a detailed knowledge of the π^0 differential scattering behavior.

In the process of measuring the hydrogen cross section some data were collected concerning charge-exchange scattering of π^- mesons by carbon. Here only those events in which both decay photons were detected could be assigned unambiguously to π^0 decay.

EXPERIMENTAL ARRANGEMENT

The hydrogen cross sections were measured by doing a difference experiment with polyethylene and carbon targets. The experiment utilized an external 50-Mev meson beam produced in the 240-Mev Rochester cyclotron by protons impinging on an aluminum target. This beam was moderated to the appropriate energies by absorbers and was focused both vertically and horizontally by two focusing magnets. Figure 1 shows the general arrangement of the focusing magnets, shielding, and counter assembly with relation to the cyclotron.

* This work was supported by the U. S. Atomic Energy Commission.

[1] Anderson, Fermi, Martin, and Nagle, Phys. Rev. **91**, 155 (1953).

[2] Bodansky, Sachs, and Steinberger, Columbia University, Nevis Cyclotron Laboratories, Nevis 1, 1953, Phys. Rev. **93**, 1367 (1954).

[3] A. Roberts and J. Tinlot, Phys. Rev. **90**, 951 (1953).

Counter Assembly and Targets

The procedure with either target consisted of measuring two counting rates: first, the flux of mesons incident on the target; and second, the number of gamma rays that emerged from the target in coincidence with incoming mesons. A single meson telescope performed the first task, while the gamma rays were detected by three gamma telescopes arranged symmetrically around the target.

Figure 2 shows two views of this counter assembly. The meson telescope consisted of two scintillation counters, No. 1 and No. 2. The three gamma telescopes each consisted of two scintillation counters also and are numbered No. 4 through No. 9. For clarity the side view omits two of these three telescopes.

In order to reduce the background it was necessary to subtract those mesons from the beam which did not interact in the target and to shield the gamma telescopes from ionizing particles produced by meson stars in carbon. Counter No. 3 does this by being placed in anticoincidence with the other counters. It was a "bucket"-shaped scintillation counter that surrounded the target in all but the forward direction.

The targets were chosen to have equal stopping power for π^- mesons. This choice was made because the differ-

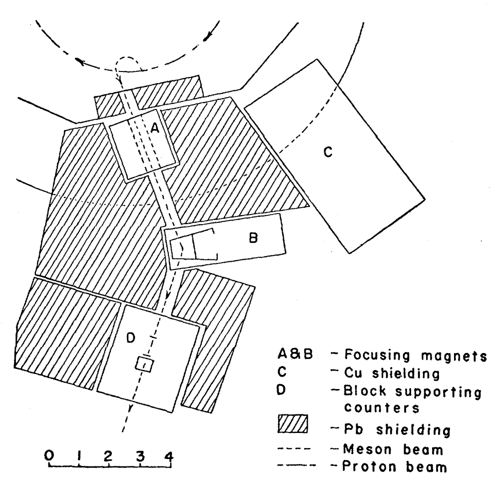

FIG. 1. General arrangement of the apparatus and shielding.

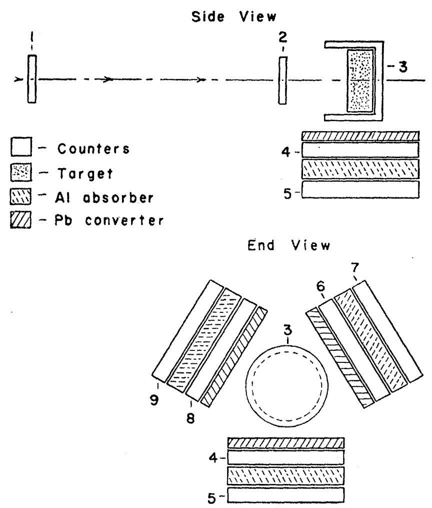

FIG. 2. Counter assembly.

ence in energy loss in passing through CH_2 and C targets with equal numbers of carbon nuclei is not negligible at the mean meson energy of 20 Mev. In this energy range the cross section for charge-exchange scattering in carbon might vary rapidly since the kinematic threshold is 12 Mev in the laboratory system. Table I shows the energy spread through the targets for the various mean meson energies.

Gamma-Ray Telescopes

The gamma-ray telescopes detect photons by virtue of electron showers produced in lead converters. They were designed so that Wilson's[4] "Monte Carlo" results could be applied, that is, the two elements of each telescope were separated by an aluminum absorber that cut-off electrons of less than 8-Mev energy. Using Wilson's results the efficiency of the telescopes (when sensitive to minimum ionization particles) *versus* gamma-ray energy could be approximated closely by

$$\epsilon(E) = -0.494 + 0.228 \ln E,$$

in the energy range of the decay photons in this experiment. E is measured in Mev.

Electronics

The electronics were required to monitor the meson beam and to indicate coincidences between mesons and gamma rays that had been detected in one of the gamma-ray telescopes. Counter No. 3 was placed in anticoincidence with the output of the meson telescope to reduce background counts as described previously. Figure 3 shows a simplified block diagram of the circuits used to form these coincidences. The circuits associated with counters (6, 7) and counters (8, 9) were identical with those for counters (4, 5). The final desired coincidence was among counters $(1, 2, \bar{3})$ plus either (4, 5) or (6, 7) or (8, 9). The bar above 3 indicates anticoincidence.

Variation of the cyclotron beam intensity made it necessary to subtract random coincidences at the same time that the data were being recorded. This was done by connecting the output of each gamma telescope to one input of each of two coincidence circuits. These coincidence circuits had approximate resolving times of 3×10^{-8} and 5×10^{-7} second, respectively. The common $(1, 2, \bar{3})$ output pulse was split also and fed to the other input of each of these circuits. Prior to entering the slow circuit the $(1, 2, \bar{3})$ pulse was delayed sufficiently so that true counts produced no output. The observed counting rate in the slow circuit divided by the known ratio of resolving times between the two circuits gave the random rate in the fast circuit.

When both decay photons from a π^0 meson were detected in two separate gamma-ray telescopes the event was recorded on a separate scaler by using a "threefold and two" Garwin[5] circuit that is not indicated in the simplified block diagram. The output pulses from counters $(1, 2, \bar{3}, 4, 5)$, from $(1, 2, \bar{3}, 6, 7)$, and from counters $(1, 2, \bar{3}, 8, 9)$ were connected to the three input grids of this circuit. This Garwin coincidence circuit was similar to the other coincidence circuits except that the plate load resistances were adjusted to produce an output when any two of the three grids were negatively pulsed. The output from this circuit provided evidence of true π^0 decay and gave the only plausible information about charge-exchange scattering in carbon.

PROCEDURE

Calibration of the Counters

The meson rate was sensitive to the current through the focusing magnets and this was adjusted for a maximum flux at the start of each operating period. The photomultiplier voltages for counters No. 1, No. 2, and No. 3 were then adjusted until plateaus of meson rate *versus* counter voltages were determined for each counter.

The energy of the meson beam was measured by range curves with the absorbers placed in front of counter No. 3. The beam was contaminated by several sources and it was not possible to determine accurately either the nature or the amount of beam contamination from range curves alone. This contamination also could have changed from one operating period to the next or

TABLE I. Meson energies at the target, in Mev.

Nominal energy	Incident energy	Outgoing energy
42	45.6	39.0
30	34.0	26.0
20	25.0	14.5

[4] R. Wilson, Phys. Rev. **86**, 261 (1952).

[5] R. Garwin, Rev. Sci. Instr. **21**, 569 (1950).

even during a single period. From the tails of the range curves, however, it was estimated that the total contamination was 18±7 percent, and it was assumed that the associated error was sufficient to account for any fluctuation throughout the experiment.

The sensitivity of the gamma-ray telescopes to minimum ionization particles was determined by their response to cosmic rays. Each counter was first tested for uniform sensitivity over its entire volume by using a collimated gamma-ray source. Each telescope then was calibrated by placing it between two smaller test counters so that every cosmic-ray particle that passed through the two test counters would also pass through it. All of the counters could be set at voltages such that the telescopes were 98±2 percent efficient for minimum ionization particles. In operation these telescopes were in the presence of strong radiation from the cyclotron and checks were made to show that there were no efficiency losses due to dead time or pileup in the circuits.

Reliability of the Data

During each operating period the targets and meson energies were changed at roughly hour intervals and no set pattern was followed. The number of counts for a given energy and target were always compared with the mean of previous runs, and no drifts ever were noticed that could not be explained by statistical variation. A "column" check also was possible since three separate gamma telescopes were used. If the three telescopes remained equally sensitive throughout the experiment then the total number of true counts recorded by any telescope should equal the total recorded by any other. The "column" totals for the three telescopes were equal within statistics.

All of the data also were analyzed for random variation about their mean values. This required plotting the deviation, run by run, of the rate recorded in each run from the mean rate for that target and energy. This was done for all targets and energies and finally all such plots were adjusted to the same scale. The expected second moment of the resulting distribution on this scale was 3.5. The actual second moment of the distribution was 3.6. The first moment of the distribution was zero within statistics. It is believed that there was no long term drift or any erratic excess or deficiency of counts due to unstable operation of the apparatus.

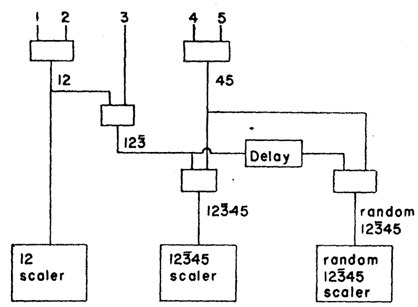

FIG. 3. Simplified block diagram of electronics. Circuits associated with counters (6, 7) and (8, 9) are identical to those used with counters (4, 5).

TABLE II. The corrected experimental data in counts per million mesons.

Period	CH_2	C	0
	42-Mev meson energy		
4	116.0±7.0	83.0±7.0	29.0±12.0
5	124.0±6.0	87.0±6.0	37.0± 7.0
	30-Mev meson energy		
Period	CH_2	C	0
4	85.4±8.5	66.4±7.2	12.0± 7.0
5	86.2±3.8	50.0±4.2	20.0± 5.0
6	88.5±3.9	59.2±4.4	25.0± 4.0
	20-Mev meson energy		
Period	CH_2	C	0
4	67.0±4.9	47.2±3.6	10.0± 4.0
5	76.5±3.3	40.3±3.2	21.0± 5.0
6	68.3±5.7	44.7±6.7	18.0± 8.0

Column headings show the target material and the operating period. CH_2 refers to counts obtained with a CH_2 target. C refers to counts obtained with a C target. 0 refers to counts obtained with no target in place.

ANALYSIS OF THE DATA

The data were analyzed in three steps. First, the counts due to carbon nuclei and other spurious effects were subtracted from the number of counts obtained with the CH_2 target to obtain the total hydrogen effect. Second, the number of counts due to the competing process $p(\pi^-,\gamma)n$ were subtracted from the total hydrogen effect. Finally, the gamma-ray distribution that results from the initial π^0 distribution at 40 Mev[6] was obtained from the kinematics of π^0 decay. This distribution, weighted by the telescope efficiency *versus* gamma-ray energy and integrated over the surfaces of the telescopes, transforms the counting rate for charge exchange to a cross section.

Computing the Hydrogen Effect

Table II shows the data obtained during various operating periods after correction for random background counts and beam contamination. Subtracting the counts due to carbon in the polyethylene target is not direct because a carbon target of equal stopping power does not contain the same number of carbon nuclei. The subtraction was done as follows:

$$N_{\mathrm{H}\gamma}=N(\mathrm{CH}_2)-\alpha N(\mathrm{C})-(1-\alpha)N(0).$$

$N_{\mathrm{H}\gamma}$ equals the number of counts per million π^- mesons due to hydrogen in the polyethylene target. α equals the ratio of the number of carbon nuclei in the polyethylene target to the number of carbon nuclei in the carbon target. $N(\mathrm{CH}_2)$ equals the total number of counts from the polyethylene target per million π^- mesons. $N(\mathrm{C})$ equals the similar number from the

[6] A. Roberts and J. Tinlot, Phys. Rev. **94**, 766 (1954).

carbon target and $N(0)$ equals the counts per million mesons recorded with no target in the beam. In this case $N(0)$ is a valid measurement of the number of counts due to interactions of the beam with other sections of the apparatus since the targets remove less than 2.7 percent of the meson flux from the beam. In all cases a lead converter is used with each gamma telescope.

This description has omitted a difficulty in subtracting the counts due to carbon arising from second-order scattering in the target. One such effect is meson-carbon scattering followed by capture in a carbon nucleus. This can occur whenever the scattered meson has insufficient range to escape from the target in its new direction. This effect might invalidate the method described for subtracting counts due to carbon since it is not proportional to the number of carbon nuclei present. It is energy dependent because at low energies a greater fraction of the first scattered mesons will have insufficient range to leave the target. Counter No. 3 discriminated against this, however, by cancelling meson produced stars in carbon, and it was demonstrated during the experiment that with counter No. 3 in operation this effect was negligible.

A similar effect occurs when a scattered meson is subsequently captured by a hydrogen nucleus in the CH_2 target. To determine the magnitude of this effect the number of mesons that scatter and stop in the target was estimated from the data of Byfield, Kessler, and Lederman.[7] The number of such stopped mesons that were captured by a hydrogen nucleus was determined experimentally.[8] This effect produced approximately a 4 percent correction to the observed hydrogen effect.

The observed counting rate from hydrogen also will be increased at the 20-Mev point by those mesons that stop in the target due to ordinary range straggling. The report of Caldwell[9] was used to estimate this effect and the required correction was negligible.

Correcting for the Reaction $p(\pi^-,\gamma)n$

An estimate of the effect of the competing reaction $p(\pi^-,\gamma)n$ was made. The results of Jacobson, Schulz, and White[10] indicate that for the purpose of this correction the cross section for the reaction $p(\gamma,\pi^+)n$ can be considered equal to the cross section for the reaction $n(\gamma,\pi^-)p$. Detailed balancing, this assumption, and the results of Bernardini[11] for the reaction $p(\gamma,\pi^+)n$ at the appropriate energies make it possible to compute the required correction. This process is responsible for approximately a 5 percent correction to the results at each energy.

[7] Byfield, Kessler, and Lederman, Phys. Rev. **86**, 17 (1952).
[8] The observed probability for the capture of a π^- meson by a hydrogen nucleus in CH_2 was 0.3 ± 0.2 percent.
[9] D. Caldwell, Phys. Rev. **88**, 131 (1952).
[10] Jacobson, Schulz, and White, Phys. Rev. **88**, 836 (1952).
[11] G. Bernardini, Phys. Rev. **93**, 930 (1954).

Kinematics of Charge-Exchange

The resultant gamma-ray distribution was computed for arbitrary S and P wave scattering distributions of the π^0 mesons. If the initial π^0 distribution is expressed in the form

$$J(\theta')d\Omega'=\sum_l a_l P_l(\cos\theta')d\Omega',$$

in the center-of-mass system, it can be shown that the resultant gamma-ray distribution in the center-of-mass system is of the form:

$$I(\theta',E')dE'd\Omega'=\sum_l (2\pi\beta\gamma\mu)^{-1}a_l P_l(\cos\theta') \times P_l\left(\frac{1}{\beta}-\frac{\mu}{2\beta\gamma E'}\right)dE'd\Omega'.$$

The decay gamma rays fall between the energy limits

$$\tfrac{1}{2}\mu\gamma(1-\beta)\leq E'\leq\tfrac{1}{2}\mu\gamma(1+\beta).$$

μ is the rest energy of the π^0 meson ($\mu=138$ Mev). βc is the velocity of the π^0 meson in the center-of-mass system, and $\gamma=1/(1-\beta^2)^{\frac{1}{2}}$. $I(\theta',E')$ is normalized to one π^0 meson (two gamma rays).

The probability of observing a π^0 meson by detecting one of its decay photons in a gamma telescope is given by

$$\text{Eff.}=\int_{\Omega'}d\Omega'\int_{E'}I(\theta',E')\epsilon(E')dE'.$$

All integrations are to be carried out in the center-of-mass system. $\epsilon(E')$ is the detection efficiency of a gamma telescope *versus* gamma-ray energy. In order to reduce the labor involved this integral was approximated by

$$\text{Eff.}=\int_{\Omega}d\Omega\int_{E'}I(\theta,E')\epsilon(E')dE'.$$

The unprimed quantities refer to the laboratory system. The coefficients associated with $I(\theta,E')$ were those of the π^0 distribution as it appeared in the laboratory system, $J(\theta)=\sum_l a_l P_l(\cos\theta)$. These were obtained from the observed $J(\theta')$ for 40-Mev π^- mesons.[6] The approximate solution for a 40-Mev π^- meson differed by less than 1 percent from the exact solution in the center-of-mass system.

The distribution of the meson flux over the target surface normally needs to be applied as a weighting factor in computing telescope efficiencies. In this case, however, due to the location of the gamma telescopes the average solid angle did not vary greatly over the surface of the target. The resultant error introduced by any reasonable flux distribution was less than ±1 percent.

The effect of the actual π^0 distribution measured at 40 Mev has been used in computing the value of the cross section at 42 Mev. The same π^0 distribution applied at 30 Mev and 20 Mev raises these cross sections 1 percent and 0.6 percent, respectively, over the cross sections obtained by assuming an isotropic π^0 distribution. At 30 and 20 Mev the cross sections were computed on the basis of the 40-Mev π^0 distribution and the percentage change between this value and that obtained for an isotropic π^0 distribution was assigned as an additional error.

Errors and Corrections

Two sets of errors must be attached to the cross sections. Each measurement can be used singly as an absolute value or the three values can be considered together as a relative measurement of the energy dependence.

Each cross section as an absolute value was assigned the following major sources of error in addition to those already discussed: (1) Statistical fluctuations contribute a standard deviation of ± 10 percent at each energy. (2) It is assumed that the detection efficiencies of the gamma telescopes *versus* gamma-ray energy are known within ± 10 percent. (3) From range curves it is estimated that the uncertainty in the mean meson energies is ± 2 Mev at each point.

If the three cross sections are considered together as a relative measurement of the energy dependence only the statistical standard deviations produce a significant contribution to the error.

In addition to those previously mentioned the following corrections were also applied to the data: (1) A correction of 3 percent for gamma conversion in the target. (2) A correction of 1.5 percent for the alternate mode of π^0 decay.[12]

Results

With the preceding corrections and assigned errors the total cross sections for hydrogen are:

$$6.9\pm 1.2 \text{ mb at } 42\pm 2 \text{ Mev},$$
$$5.7\pm 0.9 \text{ mb at } 30\pm 2 \text{ Mev},$$
$$5.0\pm 0.8 \text{ mb at } 20\pm 2 \text{ Mev}.$$

The errors refer to each value as an absolute measurement.

Charge-Exchange Scattering in Carbon

A rough estimate of charge-exchange scattering in carbon could be determined from the number of double photon coincidences observed with a carbon target. It can be estimated roughly that the ratio of the number of detected double photons from charge exchange in carbon to the total cross section in carbon is the same as the similar ratio for scattering in hydrogen. Using the ratio observed for hydrogen the order of magnitude of the total cross section in carbon is 8 mb at 42 Mev, 6 mb at 30 Mev, and 3 mb at 20 Mev.

[12] Lindenfield, Sacks, and Steinberger, Phys. Rev. **89**, 531 (1953).

DISCUSSION OF THE RESULTS

Phase Shift Analysis

The experimental results for hydrogen can be discussed in terms of a phase shift analysis. Two assumptions are made: (1) only S- and P-wave scattering contribute appreciably and (2) the total isotopic spin is a good quantum number. Under these assumptions all meson-nucleon interactions in the appropriate energy range can be explained in terms of six phase shifts. The notation of Ashkin[13] is used to describe these phase shifts. In this notation all states of isotopic spin $T=\frac{3}{2}$ are designated by $\beta_J{}^L$ and all states with $T=\frac{1}{2}$ by $\alpha_J{}^L$. $\alpha_{\frac{3}{2}}{}^1$ is the $T=\frac{1}{2}$, $J=\frac{3}{2}$, and $L=1$ state.

Anderson *et al.*[1] analyzed their data in the region of 120 Mev in terms of a phase shift analysis and gave "first solutions" for the six phase shifts at each of their energies. These solutions have an energy dependence for $\beta_{\frac{3}{2}}{}^1$ of $16° \eta'^3$ while $\beta_{\frac{1}{2}}{}^1$, $\alpha_{\frac{1}{2}}{}^0$, and $\alpha_{\frac{3}{2}}{}^1$ have values less than $\pm 6°$. $\eta'=p'c/\mu_0c^2$, μ_0c^2 is the rest energy of a $\pi^\pm$ meson, and p' is the momentum of the π^- meson in the center-of-mass system. If these solutions are assumed to be correct in the region of 120 Mev then the following approximations can be made at lower energies: (1) Since $\beta_{\frac{1}{2}}{}^1$, $\alpha_{\frac{1}{2}}{}^1$, and $\alpha_{\frac{3}{2}}{}^1$ are small it is a reasonable first approximation to set these three phase shifts equal to zero in the energy range between 20 and 42 Mev. (2) It is assumed that $\beta_{\frac{3}{2}}{}^1=16° \eta'^3$ in this lower-energy interval. (3) With this assumed energy dependence for $\beta_{\frac{3}{2}}{}^1$ it is a consistent further assumption that $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ is proportional to η' at sufficiently low energies.

With these assumptions the cross sections from this experiment were used to solve for the energy dependence of $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$. The values obtained at 20, 30, and 42 Mev were $-(16.2°)\eta'$, $-(16.7°)\eta'$, and $-(16.6°)\eta'$, respectively, with an average value of $(-16.5°\pm 1.5°)\eta'$. The limits assigned to the average value correspond to the two values of $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ for which the resulting predicted cross sections fall outside the assigned experimental errors of any cross section. A total cross section for charge exchange of $(4.4\pm 0.9)v_0/v$ millibarns is predicted if this energy dependence of the phase shifts is extrapolated to very low energies. v_0 is the relative velocity of the π^0 meson and the neutron while v is the relative velocity of the π^- meson and the proton.

CONCLUSIONS

A discrepancy exists if the present results are extrapolated to the energy of Panofsky's[14] experiment under the assumption that $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ remains proportional to η'. This has been discussed by Fermi[15] for an extrapolation of $\beta_{\frac{1}{2}}{}^0$ and $\alpha_{\frac{1}{2}}{}^0$ under the same assumption. The extrapolation predicts a capture rate for charge exchange from the lowest Bohr orbit of hydrogen. From

[13] J. Ashkin and S. Vosko, Phys. Rev. **91**, 1248 (1953).
[14] Aamodt, Hadley, and Panofsky, Phys. Rev. **81**, 565 (1951).
[15] H. Anderson and E. Fermi, Phys. Rev. **86**, 794 (1952).

the results of this experiment $R=\sigma_0 v/\pi b^3=(1.0\pm0.2)\times10^{15}$ sec^{-1} if the limits placed on $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ are taken seriously. This assumes the radius of the mesonic Bohr orbit $b=2.2\times10^{-11}$ cm and that $v_0=8\times10^9$ cm/sec; $\sigma_0=(8\pi/9k^2)(v_0/v)(\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0)^2$. Panofsky's experiment shows that this capture rate should equal the capture rate for the competing process $p(\pi^-,\gamma)n$.

Bernardini[11] has discussed the cross sections for $p(\gamma,\pi^+)n$, $d(\gamma,\pi^+)2n$, and $d(\gamma,\pi^-)2p$ for E_γ between 170 and 190 Mev in the laboratory system. If it is assumed that the ratio of π^- to π^+ production obtained from the second reaction is the same as the photoproduction ratio between the free neutron and free proton then the principle of detailed balance and this ratio can be used to predict the corresponding cross sections for $p(\pi^-,\gamma)n$. If these cross sections are extrapolated to the energy of Panofsky's experiment Bernardini obtains a capture rate that requires the initial slope of $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ to be $\pm(9.2°)\eta'$ in contrast to the value of $-(16.5°)\eta'$ obtained from this experiment.

Bethe and Noyes[16] have given an argument to explain this discrepancy in terms of Marshak's[17] suggestion. In brief this argument assumes that the slope of $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ obtained from this experiment cannot be extrapolated to the energy of Panofsky's experiment, but that this initial slope is $\pm(9.2°)\eta'$. They then fit this initial slope and the data of this experiment with a smooth curve for $\beta_{\frac{1}{2}}{}^0-\alpha_{\frac{1}{2}}{}^0$ *versus* η'. When the values of $\beta_{\frac{1}{2}}{}^0$ and $\alpha_{\frac{1}{2}}{}^0$ from higher energies are extrapolated with this restriction on their difference, it is difficult to fit the data without assuming that $\alpha_{\frac{1}{2}}{}^0$ varies less rapidly than η' and that $\beta_{\frac{1}{2}}{}^0$ varies more rapidly than η' in the energy region between 20 and 42 Mev. For the most probable fit under these assumptions $\beta_{\frac{1}{2}}{}^0$ changes sign between 20 and 30 Mev. This energy dependence for $\beta_{\frac{1}{2}}{}^0$ suggests a Jastrow[18] potential for this phase shift.

[16] H. Bethe and H. P. Noyes, Proceedings of the Fourth Annual Rochester Conference, 1954, University of Rochester.
[17] R. Marshak, Phys. Rev. **88**, 1208 (1952).

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[18] R. Jastrow, Phys. Rev. **81**, 1165 (1951).

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Quantum Electrodynamics at Small Distances*

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The renormalized propagation functions D_{FC} and S_{FC} for photons and electrons, respectively, are investigated for momenta much greater than the mass of the electron. It is found that in this region the individual terms of the perturbation series to all orders in the coupling constant take on very simple asymptotic forms. An attempt to sum the entire series is only partially successful. It is found that the series satisfy certain functional equations by virtue of the renormalizability of the theory. If photon self-energy parts are omitted from the series, so that $D_{FC}=D_F$, then S_{FC} has the asymptotic form $A[p^2/m^2]^n[i\gamma\cdot p]^{-1}$, where $A=A(e_1^2)$ and $n=n(e_1^2)$. When all diagrams are included, less specific results are found. One conclusion is that the *shape* of the charge distribution surrounding a test charge in the vacuum does not, at small distances, depend on the coupling constant except through a scale factor. The behavior of the propagation functions for large momenta is related to the magnitude of the renormalization constants in the theory. Thus it is shown that the unrenormalized coupling constant $e_0^2/4\pi\hbar c$, which appears in perturbation theory as a power series in the renormalized coupling constant $e_1^2/4\pi\hbar c$ with divergent coefficients, may behave in either of two ways:

(a) It may really be infinite as perturbation theory indicates;
(b) It may be a finite number independent of $e_1^2/4\pi\hbar c$.

1. INTRODUCTION

It is a well-known fact that according to quantum electrodynamics the electrostatic potential between two classical test charges in the vacuum is not given exactly by Coulomb's law. The deviations are due to vacuum polarization. They were calculated to first order in the coupling constant α by Serber[1] and Uehling[2] shortly after the first discussion of vacuum polarization by Dirac[3] and Heisenberg.[4] We may express their results by writing a formula for the potential energy be-

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[1] R. Serber, Phys. Rev. **48**, 49 (1935).
[2] A. E. Uehling, Phys. Rev. **48**, 55 (1935).
[3] P. A. M. Dirac, Proc. Cambridge Phil. Soc. **30**, 150 (1934).
[4] W. Heisenberg, Z. Physik **90**, 209 (1934).